REINFORCED CONCRETE BRIDGES

Fig 163 Warrington Bridge 134 ft Span

Frontispiece

Plate I

REINFORCED CONCRETE BRIDGES

THE PRACTICAL DESIGN OF MODERN REINFORCED CONCRETE BRIDGES INCLUDING NOTES ON TEMPERATURE AND SHRINKAGE EFFECTS

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THIRD EDITION REVISED AND ENLARGED

LONDON

THE TECHNICAL PRESS

5 AVE MARIA LANE LUDGATE HILL E.C.

1931

PREFACE

Although in the large number of books available upon the subject of reinforced concrete, bridge work is in many cases dealt with, there appears to be no reference available in English which is wholly devoted to these structures, or which covers the subject in a manner sufficiently comprehensive for the practical use of the engineer.

This book is an attempt to meet this need, and is intended to amplify the information contained in existing works rather than to displace it.

It has not been possible, owing to its scope, to cover the subject exhaustively. Upon the other hand, the information comprises all that is essential to guide an engineer in the adoption and the design of a bridge to suit any case ordinarily met with.

In addition, this work comprises the best and most recent practice available, much of which is given in English for the first time. It is based upon experience gained from the design and erection of hundreds of reinforced concrete bridges, embracing every type.

The illustrations given in Chapter XII. of some of these latter structures form the best criterion of the efficacy of the above information.

The author is greatly indebted to MM. Pelnard-Considère, A. Caquot; Messrs. C. G. Mitchell, B.Sc.Eng., A.M.Inst.C.E.; H. E. Steinberg, M.Inst.C.E.; D. B. Steinman, M.AmS.C.E., Ph.D.; E. S. Needham, Assoc. Mem. Am.S.C.E.; J. Norbeck and C. W. J. Spicer, for the valuable assistance and information given him in the preparation of this book.

The following works of reference have been consulted :—

"Plain and Reinforced Concrete Arches," by Professor J. Melan.

"La Statique Graphique," by M. Maurice Levy.
"Reinforced Concrete Construction" (Vol. III.), by G. A. Hool.
"Eisenbetonbau," by Professor Mörsch.
"Manuel Théorique et Pratique du Constructeur en Ciment Armé," by N. de Tedesco and V. Forestier.
"Eisenbetonbau, Rahmen und Gewölbe," by H. Schlüter.
"Reinforced Concrete," by A. Considère.
"Annales des Ponts et Chaussées Français."

PREFACE TO THE THIRD EDITION

MAINTAINING the original object of supplementing, and not supplanting, other text-books upon the subject, effort has been made, in preparing the third edition, to confine the new matter to useful information not readily obtainable elsewhere.

The chapter on loading has been revised and enlarged, and a new chapter has been added dealing with the design of long-span arch bridges. The recent innovation involving the use of hydraulic jacks to eliminate secondary arch stresses is fully explained, with photographic illustrations.

Deck slab design, according to the method of Monsieur Pigeaud, is now becoming general, and is, therefore, more fully dealt with.

The standard specification has been revised and brought up to date. A description, with several photographic illustrations, is given of the most noteworthy bridge yet constructed in reinforced concrete —the Plougastel Bridge, spanning the Elorn River, France.

The author acknowledges his indebtedness to Mr. C. J. Wood, A.M.I.Struct.E., for his general assistance and the preparation of many of the additional illustrations.

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REINFORCED CONCRETE BRIDGES

CHAPTER I

INTRODUCTION

1. Historical Résumé.—The use of reinforced concrete as a structural material, like many other engineering developments of importance, did not originate with those engaged in the engineering professions. In fact, it was not until a considerable amount of work had been done that qualified engineers turned their attention to the possibilities of the new material.

As is well known, the early application of reinforced concrete was for the construction of large tubs, and later for pipes and other work of small importance. Following on this, its use began to develop for forming beams and floor slabs, although there was at that time no scientific theory underlying the design, and the formulæ by which were determined the sections of the reinforcement and the thickness required for the concrete slabs were of the most empirical and rule-of-thumb character and could only be regarded as approximately accurate over a very limited range.

Nevertheless, under these rough and ready rules, beams, girders, and even bridges were put up, and many of these early structures have proved entirely satisfactory.

On the other hand, some of these pioneer works have, after the lapse of years, developed weaknesses, and in some cases have had to be demolished and entirely reconstructed.

These examples of early work have been of the greatest assistance to engineers engaged in this class of work, in improving the design in general, and in avoiding and eliminating troubles of a similar nature.

2. Resistance of Reinforced Concrete to Destruction.—During the war a large number of important reinforced concrete structures were partially or wholly demolished, and these provided European engineers with valuable experience of the amazing resistance which this material offers to complete destruction.

The general devastation wrought by the war included many examples of all classes of reinforced concrete structures, and in all cases where the work was of good quality and based upon sound design the resistance to destruction was extraordinary.

The demolition by explosive charges of the Château Thierry

Bridge (Fig. 1) forms an excellent example of this. It was found to be a matter of extreme difficulty effectively to destroy this bridge, and when one span was blown up, causing the platform to assume the position shown in Fig. 2, the ribs and decking remained practically intact. This bridge has since been reconstructed to the original design with complete success.

Other examples were bowstring girder bridges, which were caused to drop by the destruction of the supports at one end, with similar results, and in other cases two or three of the vertical suspenders were destroyed without causing any apparent reduction in the bearing capacity of the roadway itself.

The inherent strength and tenacity of these structures caused surprise, even amongst those engineers versed in the capabilities of reinforced concrete, and these facts cannot fail to enhance the confidence of civil engineers who may evince doubt concerning the capability of this material to withstand shock.

A much earlier example of the destruction of a bridge was that constructed and tested to failure in France in 1907. This was a bowstring girder bridge, and was 65·6 feet span, designed to carry a safe load of 60 tons. Failure took place at 241 tons. Even at this load, which produced enormous stresses, failure resulted from a construction defect, which, with ordinary care and supervision, would be a most unlikely occurrence in this country. This test, which illustrates the reserve of strength possessed by a reinforced concrete bridge, was made by M. Considère, and is fully reported in his book on reinforced concrete.

3. Application of General Assumptions and Methods.— As in structures constructed with other materials. the accuracy of the calculated stresses in a reinforced concrete bridge depends upon the degree to which the assumed conditions approximate to those actually realised in practice. For reinforced concrete work this approximation is dependent to a certain extent upon the human element and other factors outside the direct control of the designer.

For this reason the general assumptions and methods employed for reinforced concrete design may be adopted for use in bridge work. These assumptions are briefly as follows :—

(1) Elastic modulus of concrete constant within limits of calculated stresses.

(2) Plane sections remain plane after bending.

(3) Steel reinforcement takes all tension.

(4) Perfect adhesion between steel and concrete.

Although none of these assumptions may be regarded as strictly true, their adoption acts for the most part on the side of safety, and

Fig. 1.—Château Thierry Bridge before Destruction.

Fig. 2.—Château Thierry Bridge after Destruction.

PLATE II.

[*To face page* 2.

it has been amply proved that reinforced concrete structures of all classes erected to these accepted methods of design behave satisfactorily and give test results which compare favourably with those calculated.

In addition—and this is a very important point for those engineers responsible for the actual design of reinforced concrete work—the above assumptions permit the application of greatly simplified methods.

More complicated and laborious methods have been suggested from time to time with a view to theoretical perfection of treatment, but these provide no corresponding advantage in practice, since the requisite degree of approximation in design is governed by the practical limitations mentioned above.

4. Absence of Regulations.—No general regulations governing the design of reinforced concrete bridge work have yet been published in this country. The existing regulations of the London County Council and the recommendations of the Royal Institute of British Architects, while a useful standard for the class of buildings they are designed to cover, do not, nor are they intended to, cover reinforced concrete bridge design and construction.

In the absence of any reliable instructions or regulations regarding design in this connection, it is necessary to go further afield and follow the regulations of Continental authorities who have devoted a great deal of study and expense to investigating this subject. The allowances and assumptions regarding design made in the following pages have the support of extensive practical tests, and may be taken as covering the best practice.

5. Principal Classes of Bridges.—In considering this subject, it is convenient to divide bridges into their two fundamental classes, that is, the Arch Type, giving inclined reactions at their supports, and the Beam Type, giving wholly vertical reactions. Either of these classes may, of course, be composed of single or continuous spans, the latter being either of a number of equal or unequal spans.

In the following pages various kinds of the above classes of bridges will be treated.

The exact arrangement economically suitable for adoption in a particular case depends upon the natural conditions and specified requirements as to width, loading, architectural treatment, headroom or waterway, etc., and engineers must give careful consideration to the problem, if necessary by investigation and elimination, before finally deciding upon a particular type.

It is with regard to this aspect of the problem that experience is invaluable, especially where the structure may be complicated by abnormal conditions, either natural or specified.

6. Particular Types.—A great many arch bridges have been constructed with considerable rise span ratios, principally in the United States, south of France and Switzerland, where, owing to natural conditions, these are particularly applicable. This type of bridge, illustrated in Fig. 165, presents no special difficulty as regards design.

The temporary staging and general methods of construction, however, are factors frequently demanding careful study.

There has recently been constructed on the Continent a number of bridges of a type and size that is entirely without example both in regard to theoretical perfection and architectural grace.

Some of these are what is known as the bowstring girder type of bridge, and others are beam bridges, in which the side spans are semi-cantilevers. These are of comparatively recent development, brought about by the experience and knowledge gained since the erection of the earlier types of reinforced concrete bridges, and are in a large measure the result of the studies of M. Caquot, a member of the well-known French firm of consulting engineers, MM. Pelnard-Considère, Caquot et Cie, of Paris.

In Chapter XII. descriptions and illustrations of various structures are given. These examples cover the principal types of reinforced concrete bridges as well as some of outstanding interest. Included in the descriptions are the arrangements and scantlings of the important members and the reason for their adoption.

7. Methods of Calculation.—For the solution of main supporting members of a bridge it is usual to employ graphical means. These will be used in the following pages, and by the use of typical examples the methods of design will be presented in detail and in as straightforward a manner as possible.

Few engineers responsible for the actual design and erection of bridges ordinarily met with in practice are disposed to introduce more than an absolute minimum of mathematical study, and, with this object in view, every effort has been made to simplify the following methods of design. These methods can, of course, be adopted without investigation of their somewhat lengthy mathematical proofs, but where this is desired any of the very able works already published upon the subject can be consulted. In the case of some of the larger and more important bridges, there are found peculiar circumstances which necessitate the investigation of a bridge being made from "first principles." These circumstances may be so varied and numerous that it is obviously impossible to suggest any general methods of procedure. For example, it may be that in a particular arch bridge the supports are such that their elastic properties have to be provided for, or that the form of the

beam or arch is such that it is impossible to apply with sufficient accuracy the usual assumptions essential for simple treatment.

In cases such as these, very considerable practice and experience is necessary, in the absence of which engineers should consult specialist designers who have access to essential data and possess the requisite experience.

It has been assumed that the reader is acquainted with the design of ordinary reinforced concrete construction, and it is not, therefore, proposed in this book to treat the various details where these do not affect the main supporting members. For instance, the details of design and construction of the columns, deck slab and beams of an arch or beam bridge are to a large extent made in the usual manner. There are certain exceptions, such as the method of determining the bending moment upon a slab due to wheel loads, which are explained in detail and numerical examples given.

8. Criticism of Bad Design.—In cases where certain arrangements of bridge members are recommended as essential, these, of course, refer to economically designed structures and not to those types having a large surplus of material which may provide sufficient reserve strength to offset their defective design. It is obviously possible to design a bridge, if no proper regard is paid to economy, in such a manner that most of its objectionable features are offset by the extravagant use of materials. It is not, however, proposed to discuss such examples, and the reader's attention is called to this point only that the existence of such bridges may not be held to disprove the necessity for the adoption of the particular features or arrangements advocated.

The difficulty of arched design becomes acute when the rise in proportion to the span is limited. When this ratio passes a certain limit, satisfactory design is not possible, since the changes of temperature, shrinkage and settlement alone induce stresses which may be themselves in excess of those permissible upon the materials employed. A more detailed explanation of this is given in Art. 29, Chapter III., page 32.

CHAPTER II

LOADING AND WIND PRESSURE

9. General.—Road bridges comprise by far the largest class of bridge constructed in reinforced concrete in this country.

Another type frequently constructed in this material is that providing for pedestrian traffic over railway lines, etc. These should be simple in character, and being of narrow construction with a platform at some considerable height above the founded level, stability against wind pressure is a factor requiring attention.

Main line railway bridges of reinforced concrete have not as yet been attempted to any extent in this country, although there is ample evidence of their suitability and efficiency available in other countries. Railway engineers do not favour the adoption of any form of construction about which there is an absence of experience extending over a considerable period, and prefer to continue construction in steelwork, the limitations of which are known to them.

There is also the question of speed of erection, important in bridge extensions and renewals, which constitute the largest class of railway bridge work in this country. The generally accepted view that steelwork is capable of greater rapidity of erection than reinforced concrete is only true for certain types of small structures. This is not necessarily the case for bridges of considerable size, and, with the many favourable factors upon the side of reinforced concrete, it can only be a question of time for their suitability to be more widely appreciated and their consequent adoption ensured.

As a general rule, in bridges having spans exceeding 40 or 50 feet the principal load is provided by the dead weight of the bridge, road surfacing, etc. In the majority of bridges the rolling or live load becomes of secondary importance so far as deformation is concerned, since any increase in the specified live load results in the dead weight of the bridge being increased correspondingly.

10. British Standard Loading for Road Bridges.—The live load to be carried by road bridges, of course, varies with the locality and type of bridge adopted, but for all road bridges subject to the approval of the Ministry of Transport the standard loading drawn up by that department has to be allowed for.

There is only one class of load laid down by the Ministry of Transport, the contention of the department being that roads which are at the moment considered as second class may become in

the future first-class roads, and the cost of renewing all the bridges on any such road, if designed for lighter vehicles than those adopted in the more important class, would be practically prohibitive.

The standard loading of the Ministry of Transport (dated April, 1927) consists of an engine weighing 20 tons, followed by three 13-ton trailers.

A diagram of this standard loading is given in Fig. 3, together with certain relevant notes dealing with the number and arrangement of standard trains of loads to be assumed on bridges of various widths and lengths.

This train of loading, although heavy, is a possible case, and represents a class of load which may be well reached on the more important trade routes. The severity of the loading as affecting bridge design, however, is marked in the number of simultaneous loads specified in the above note.

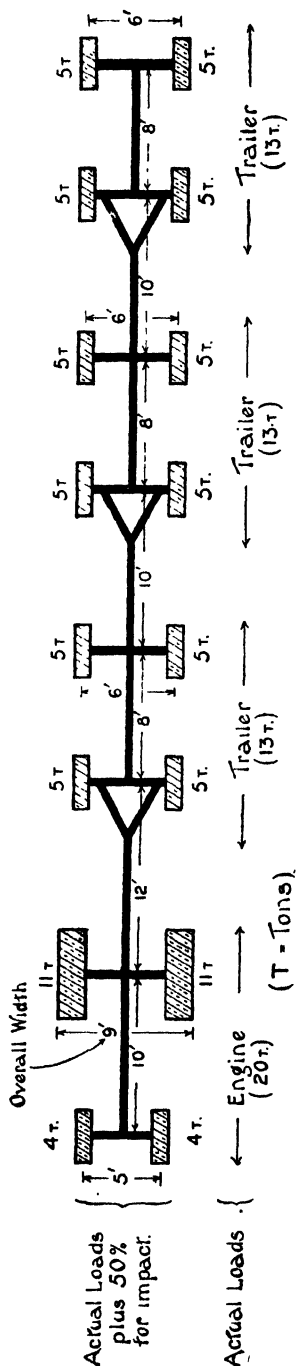
In order to comply with the requirements of the standard loading, two or more parallel trains of loads have to be assumed to be passing over the bridge with a minimum of 12-inch clearance between the driving wheels. Also it is clear from the above note that this standard loading, which must be a practical maximum for many

NOTE.—The bridge shall be assumed to be loaded with such standard trains or parts of standard trains as will produce the maximum stress in any bridge member, provided that in any line of trains there shall not be more than one engine per 75 ft. of the span of the bridge, and each standard train shall occupy a width of 10 ft. Where the width of the carriage-way exceeds a multiple of 10 ft., such excess shall be assumed to be loaded with a fraction of the *axle* loads of a standard train. The fraction to be used shall be the excess width in feet divided by ten.

FIG. 3.—British Standard Load for Highway Bridges.

years to come, is not regarded as an occasional load even for bridges having a roadway of less than 20 feet in width. A bridge roadway has to be considered as entirely covered by such trains. This condition is never likely to be realised in fact, and rarely approached except for cases in or near large industrial centres.

The effect of a double line of the specified wheel loads simultaneously loading any main longitudinal beam in a girder bridge may be very serious, and result in the necessary introduction of heavy members.

The following method of distribution upon roadway beams required to carry the British Standard Loading has the merit of simplicity, and is acceptable to the Ministry of Transport.

Considering, for example, the driving wheels of the tractor comprising two concentrated loads of 11 tons each or an axle load of 22 tons (see Fig. 3), the minimum width of roadway interested in accommodating this axle load is 10 feet, and it therefore imposes an equivalent load of 2·2 tons per foot width of roadway. Using this figure as a basis, the maximum load to be carried by any arrangement of longitudinal supporting beams for these particular loads can be readily ascertained.

Other wheel loads can be dealt with similarly, and although this loading can conceivably be exceeded by placing two wheels from adjacent vehicles with the minimum specified clearance of 12 inches directly over a beam, this contingency is so unlikely in practice that any increase resultant from it can quite well be considered as covered by the generous impact allowance and factor of safety adopted. Where this method of load distribution is employed, it should, for the sake of uniformity, be used for all longitudinal roadway beams, thereby facilitating future widening should this become necessary.

Moreover, bridges in other countries and those in this country constructed some years ago were designed for much lighter loading, and yet, after careful investigation, there is no evidence of properly designed reinforced concrete structures showing defects through overloading, although in most cases vehicles of all classes pass over these older bridges.

The reader is referred to a paper read before the Public Works Roads and Transport Congress, 1923, "General Construction of Bridges," by Lieut.-Colonel J. F. Hawkins, O.B.E., M.Inst.C.E. (County Surveyor of Berkshire), and Captain C. G. Mitchell, A.C.G.I., B.Sc.(Eng.), A.M.Inst.C.E. (Ministry of Transport), explaining briefly and concisely the reason for the adoption of the Ministry of Transport's Standard Loading, in addition to giving other relevant and interesting matter.

11. Abnormal Rolling Loads.—In several specifications for reinforced concrete bridges constructed in the Glasgow district, a superimposed rolling load greatly in excess of the Ministry of Transport's Standard Loading has been adopted.

An extract from one of the above specifications states that the bridge is to be capable of carrying, in addition to the dead weight of the structure, roadway, etc., and other distributed loading :—

> "An occasional moving load of 120 tons carried on a bogie having two axles, 9 feet apart centres, with a gauge of 9 feet centre to centre of wheels, and drawn by a train of five tractors weighing 15 tons each ; 5 tons on front axles and 10 tons on rear axles ; the distance between axles of each tractor being 10 feet, with a gauge of 8 feet between centres of wheels, and the distance apart of the successive vehicles, measured between nearest axles, being 9 feet.
>
> "In calculation for the design of the floor slabs of roadway, an addition of 20 per cent. is to be made to the foregoing figures of moving load to obtain the equivalent stationary load."

This loading is given by the larger type of marine boilers required to be transported from the works to the various shipyards in the district.

It will be seen that this loading gives an equivalent maximum static wheel load upon the roadway slab of 36 tons.

Little information is obtainable regarding the precise details of the vehicle giving this loading, but it would certainly seem that such loads should be subjected to some control regarding the intensity of wheel pressures and an increase either in the width or number of wheels stipulated.

The desirability of some such control as mentioned above may be seen by calculating the pressure upon the road surfacing material in contact with the wheel. This, with the stipulated width of wheel of 15 inches and the customary spread in the direction of travel of 3 inches, gives a pressure of 115 tons per square foot upon the road surface.

Such a vehicle would only occasionally pass across a bridge, and the increase of cost due to covering such cases in the design of bridges should be reduced to a minimum.

12. Impact.—The percentage increase of loading to cover impact stresses specified by the Ministry of Transport in the case of large span bridges is somewhat high. This increase of 50 per cent. (see Fig. 3) is that usually applied to reinforced concrete railway bridges, and in other countries one half this amount (25 per cent.) is generally adopted as the impact allowance for ordinary rolling loads on reinforced concrete structures. This amount is further greatly

reduced, or even excluded entirely, for all members other than those comprising the actual deck construction.

The question of impact introduces a time factor which in turn inevitably involves the mass or inertia of the structure subjected to it. For this reason, and owing to the molecular structure of concrete itself, vibratory stresses are less in a concrete bridge than in a steel structure of the same strength. The speed of the vehicle causing impact is also obviously of the greatest importance, and, while lighter and faster vehicles may cause higher impact stresses, the standard loading in question must always be of comparatively low speed and the percentage increase therefore relatively small.

Furthermore, the proximity of a member under investigation to the point of load application is important as regards the effect of impact upon the stresses produced in the member. The method adopted by the Ministry of Transport of increasing the static load by 50 per cent. results in this increased load being applied to all members comprising a bridge of any type, and consequently some of the members remote from the load are designed to resist impact stresses which cannot conceivably affect them. The Continental method of specifying a decreased permissible working stress varying in amount according to the proximity of the members to the load is considered by many responsible engineers to be the most efficient and proper method of dealing with impact stresses.

It should be noted that in some small bridges an impact allowance of 50 per cent. may be, if anything, on the low side, especially for bridges of the girder type. Reinforced concrete slabs, if properly designed, possess remarkable resistance to loading of all kinds, and it is for the beam members of such bridges that care should be taken with regard to impact stresses.

13. Maximum Standard Wheel Loads.—In the design of the deck slab to a reinforced concrete bridge, the maximum equivalent static wheel load of 11 tons is usually considered to be placed in the centre of the slab for spans up to 6 or 7 feet, due allowance of course being made for the distribution of the loads through the slab and filling (if any). This question is dealt with fully in the design of girder bridges, Chapter VII.

The maximum wheel loads adopted in the following countries may be usefully given :—

Great Britain .	24,640 lbs. = 11 tons, including 50 per cent. increase for impact.
United States .	16,000 lbs. = 7·15 tons.
France . .	13,227 lbs. = 5·9 tons.

14. American Loading for Highway Bridges.—There is no centralised control in the United States governing the design and

construction of highway bridges of reinforced concrete, and it is not possible, therefore, to give definite standard loadings.

The following extracts from the March, 1929, "Conference Specifications," which contain the revisions to the "Standard Specification for Highway Bridges and Incidental Structures" by the American Association of State Highway Officials, gives the loadings now generally in use.

Classification of Bridges.—The classification of bridges with reference to traffic shall be as follows:—

> *Class AA.*—Bridges for specially heavy traffic units in locations where the passage of such loads is frequent.
>
> *Class A.*—Bridges for normally heavy traffic units and the occasional passage of specially heavy loads.
>
> *Class B.*—Bridges for light traffic units and the occasional passage of normally heavy loads. Class B bridges shall be considered as temporary or semi-temporary structures.
>
> *Class C.*—Bridges for electric railway traffic in addition to highway traffic. The latter may correspond to any one of the classes described above.

Highway Live Loads.—The highway live load on the roadway portion of the bridge shall consist of trains of motor trucks, or equivalent loads, as hereinafter specified. Each loading is designated by the letter H, followed by a numeral indicating the gross weight in (short) tons of the heaviest loaded truck in the train.

Traffic Lanes.—The truck trains or equivalent loads shall be assumed to occupy traffic lanes, each having a width of 9 feet, corresponding to the standard truck clearance width. Within the curb to curb width of the roadway, the traffic lanes shall be assumed to occupy any position which will produce the maximum stress, but which will not involve overlapping of the adjacent lanes, nor place the centre of the lane nearer than 4 feet 6 inches to the roadway face of the curb.

Trucks.—The wheel spacing, weight distribution, and clearance of the trucks used for design purposes shall be as shown in Fig. 4.

Highway Loading.—The highway loading shall be of three classes: namely, H20, H15 and H10, and may be either truck train loadings or equivalent loadings. Loadings H15 and H10 are 75 per cent. and 50 per cent., respectively, of Loading H20.

(*a*) *Truck Train Loadings.*—The truck train loading shall be used for loaded lengths of less than 60 feet. It shall consist of one truck of the gross weight indicated by the loading class followed by, or preceded by, or both followed and preceded by, a line of trucks of indefinite length, each of the following

or preceding trucks having a gross weight of three-fourths of the gross weight indicated by the loading class. (Trucks shall be at 30-foot axle spacing.)

Trucks in adjacent lanes shall be considered as headed in the same direction.

(b) *Equivalent Loading.*—The equivalent loading shall be used only for loaded lengths of 60 feet or greater. It shall consist of a uniform load per linear foot of traffic lane, combined with a single concentrated load so placed on the span as to produce maximum stress. The concentrated load shall be

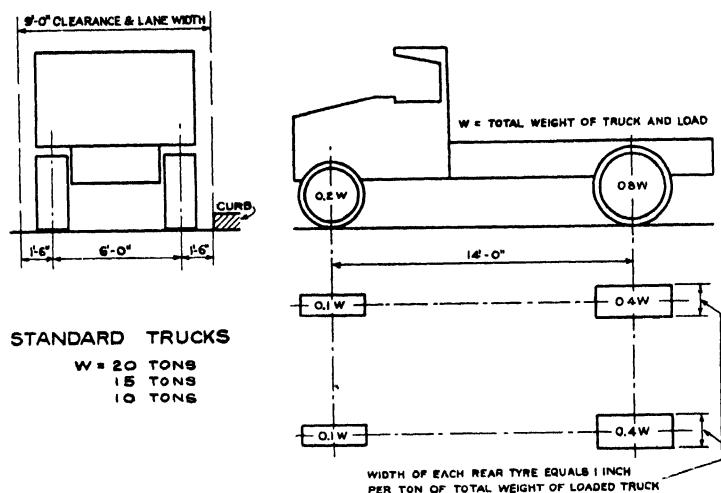



FIG. 4.—American Standard Trucks for Road Bridge Loading.

considered as uniformly distributed across the lane on a line normal to the centre line of the lane. For the computation of moments and shears, different concentrated loads shall be used as indicated.

Loading.	Uniform Load per linear ft. of Lane.	Concentrated Load, for Moment.	for Shear.
	lbs.	lbs.	lbs.
H20	640	18,000	26,000
H15	480	13,500	19,500
H10	320	9,000	13,000

EQUIVALENT LOADING FOR LOADED LENGTH OF 60 FEET AND OVER

Selection of Loadings.—Bridges of the different classes shall be designed for the loadings as follows :—

Class of Bridge.		Loading.
AA	...	H20
A	...	H15
B	...	H10

Impact.—Live load stresses, except those due to sidewalk loads and centrifugal, tractive, and wind forces, shall be increased by an allowance for dynamic, vibratory and impact effects.

$I = \frac{50}{L + 125}$ in which :

I = impact fraction.

L = the length in feet of the portion of the span which is loaded to produce the maximum stress in the member considered.

The amount of this allowance or increment is expressed as a fraction of the live load stress, and for both electric railway and highway loadings shall be determined by the formula given above.

15. French Loading for Road Bridges.—The design of bridges on the premier roads in France (routes nationales) is con-

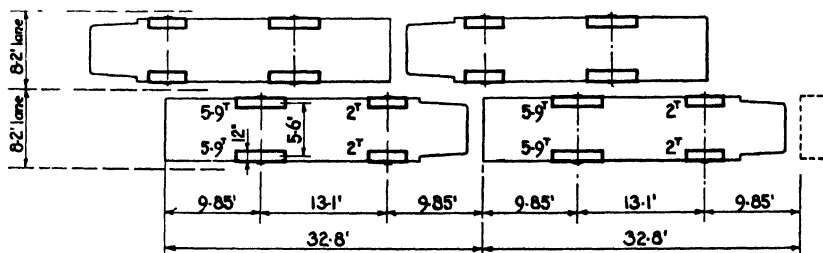



FIG. 5.—French "Standard Lorry" Loading.

trolled by the Ministère des Travaux Publics under regulations dated May 10th, 1927. For the main structure a uniformly distributed superload is specified which varies according to the length of span. Individual members of the bridge must in addition be calculated to carry as many trains of two lorries as the width of roadway will accommodate, and these trains are to be so placed as to produce the maximum stresses possible. Particulars of this standard lorry loading are given in Fig. 5.

In certain cases military convoys have to be provided for, and for these there are three types laid down. The heaviest consists of a large trailer having four wheels per axle and drawn by two traction engines. The axle spacing and wheel loading is shown in Fig. 6.

16. Application of French Loading.—The standard lorry loading is in general only applicable to members less than 65 feet in length. Should they exceed this, only the uniformly distributed superload need be used in the calculations. This is taken as being applied over the entire area of road surface with the exception of the

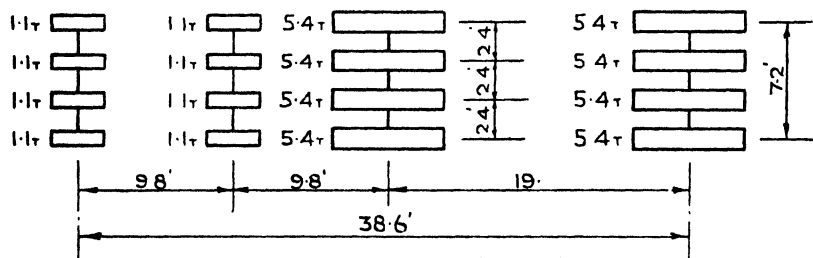



Fig. 6.—Loading for French Military Convoy.

pavements (for which the superload is 82 lbs. per square foot), and its value is found by the following formula :—

$$p = 168 - 0{\cdot}25\,L$$

where p = superload in lbs. per square foot
and L = length of span in feet.

The value of p decreases with the increase in span, and is limited to a minimum of 102 lbs. (= 500 kg. per square metre), reached when L equals or exceeds 262 feet.

The wheel and distributed loads given above are exclusive of impact, and to cover for same must be multiplied by a coefficient which takes into account the dead and live load ratios, as also the span of bridge or member. This coefficient is as follows :—

$$1 + \frac{0{\cdot}4}{1 + 0{\cdot}06\,L} + \frac{0{\cdot}6}{1 + 4\frac{P}{S}}$$

where L = span of member in feet
P = total dead weight
S = total of super and/or wheel loads.

17. Distribution of Loading for Spandril Filled Arches.—For spandril filled arches, which comprise a curved slab, it is customary to consider a longitudinal strip of slab 1 foot wide. It should be noted that in this type of bridge the concentrated wheel loads will spread through the filling, and where this is of a sufficient minimum thickness at the crown, say, 4 feet from point of contact to the intrados of the arch slab, the standard train of wheel loads may be considered to be uniformly applied. The equivalent distri-

buted loading for such cases can be taken as 300 lbs. per square foot for a length of 11 feet under the heavier loads, and 160 lbs. per square foot over the remainder of the bridge.

Moreover, the increased pressure under the driving wheel area need only be considered for the central portion of the bridge where the depth for distribution is limited. The above equivalent distributed loading does not include any allowance for impact which is absorbed in a bridge of this type and weight before affecting the reinforced concrete arch

In flat arches having less than the above stated minimum depth

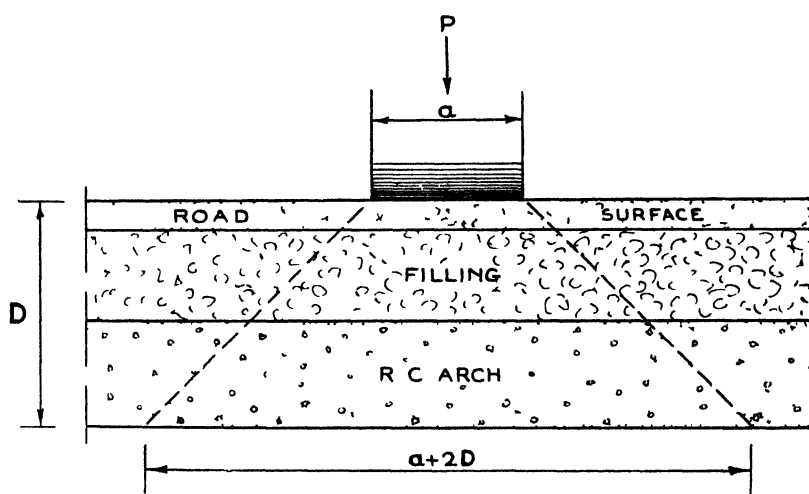



FIG 7 —Distribution of Wheel Loads.

of filling at the crown, the proportion of each point load coming upon a longitudinal strip of arch amounts to $\frac{P}{a + 2D}$ (Fig. 7). In this case no allowance is made for longitudinal distribution for the central portion of the arch, the point loads being applied to the influence lines in the usual manner. If desired, the distributed loading of 160 lbs. per square foot given above may be employed over the spandrils of the arch, although, for the sake of uniformity, the form of loading employed for the central portion is frequently adopted throughout. It should be noted that the above length $a + 2D$ must not exceed one-half of the distance between centres of load trains.

18. Loading for Footbridges and Light Country Bridges. —Footbridges and pavements required to take only passenger traffic are usually specified to be capable of carrying an evenly distributed

superload of from 84 to 150 lbs. per square foot. If not otherwise specified, a safe figure is 112 lbs. per square foot. This figure covers all practical possibilities of loading, and is seldom, if ever, exceeded.

Loading of this class does not require any addition for dynamic or impact effects, and may be regarded as a loading very gradually applied, which it is in fact.

The possibility of sudden and simultaneous movements of a crowd having a density assumed by the above loading is so remote that it may be ignored. Furthermore, the working stresses used in this class of structure may safely be exceeded if, in rare circumstances, such a condition as the above should be ever realised.

In the case of light country bridges for which no specified loading is given, a figure of 112 lbs. per square foot may be used.

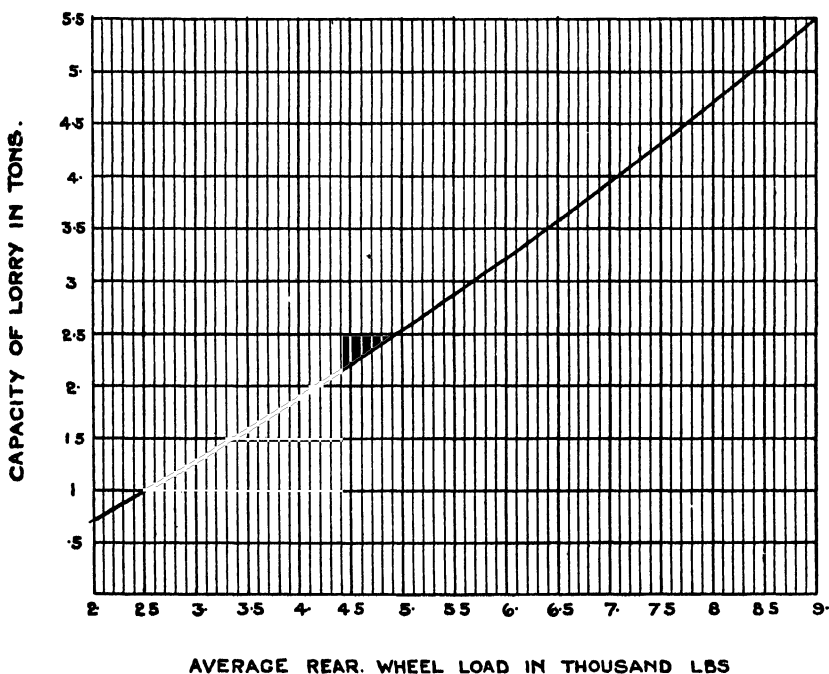



Fig. 8.—Maximum Wheel Loads for Lorries of Various Capacities.

Where a roadway is required to be capable of carrying a specified vehicle, the deck slab should be calculated for the maximum concentrated wheel load. Fig. 8 gives the maximum wheel loads from vehicles of varying carrying capacities.

Deck beams, where required to carry such vehicles, may be calculated for an equivalent distributed load from 1 to 2 cwts. per square foot for vehicles of 1 to 6 tons capacity respectively.

19. Types of Vehicles.—A selection of vehicles of various types using the roads of Great Britain is given on pages 18 to 20. During recent years the progress made in the manufacture of mechanically-propelled vehicles has been considerable, the sizes and weights in many instances approaching the permissible limits controlling their construction. It will be noted that in all cases the maximum wheel loads are considerably less than that in the Ministry of Transport's standard loading.

Page 18 is devoted to passenger-carrying vehicles, and represents fast-moving traffic. The average weight per square foot of road surface actually occupied is only slightly greater than 1 cwt., even in the case of the largest double-decker omnibus, whilst large private cars give about one-half of this amount. Single-decker omnibuses and charabancs can be considered similar to the "luxury" coach (Fig. 10).

There is great diversity in the types of petrol-driven goods vehicles, many being purpose-built for the conveyance of special commodities. The majority fall into one of the three groups illustrated on page 19, *i.e.*, four-wheeled, six-wheeled, or eight-wheeled lorries, according to the loads they carry. They are slower than the passenger vehicles, but are nevertheless capable of relatively fast speeds, in most cases being fitted with pneumatic tyres. The laden weight for this class is usually between 100 and 200 lbs. per square foot of area covered, and seldom exceeds the latter figure.

Steam-propelled vehicles are illustrated on page 20, which gives examples of a four-wheeled wagon, a six-wheeler, and a steam roller. For steam lorries the average distributed weight imposed on the roadway varies between 1½ to 2 cwts. per square foot, whilst for a heavy steam roller it may amount to 2½ cwts.

The above average distributed loading has been calculated upon the nett area of deck occupied by the vehicles, no allowance having been made for clearances between them, such as would necessarily exist even under the most congested traffic conditions.

If such clearances were taken into account, a substantial reduction on the average loading would result, this, of course, varying with the number, type and arrangement of vehicles considered.

It will be noted that more severe individual wheel loads may be imparted by a four-wheeled vehicle than by a six-wheeler of larger capacity, but with the latter the juxtaposition of adjacent wheels should not be overlooked.

In the illustrations (Figs. 9 to 17), the loads given are the total per axle for the vehicles fully laden. These profiles have been prepared from particulars supplied by a number of leading manufacturers, and are representative of standard practice.

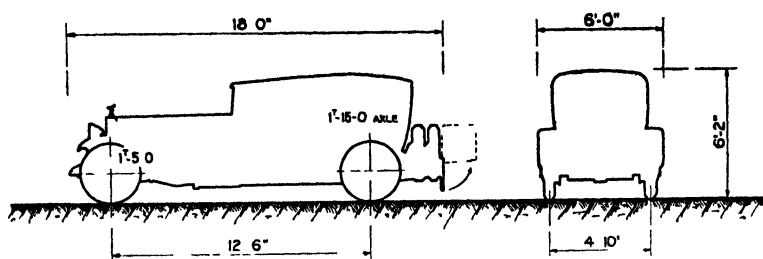



FIG. 9.—Large Saloon Car (Rolls Royce, Ltd.). Gross Weight, 3 Tons.

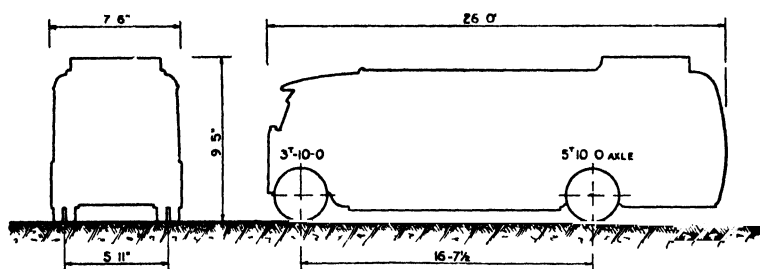



FIG. 10.—28-Seater Luxury Coach (Crossley Motors, Ltd.).
Gross Weight, 9 Tons.

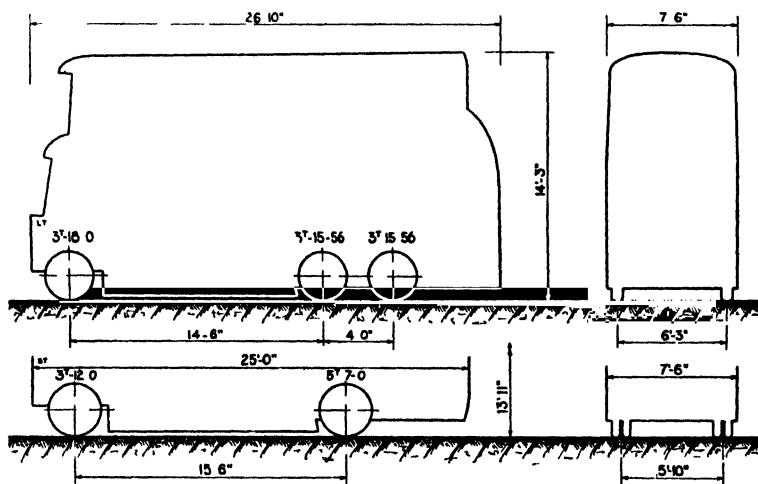



FIG. 11.—Six-wheeled Omnibus. Gross Weight, 11 Tons 9 Cwts.
Four-Wheeled Omnibus. Gross Weight, 8 Tons 19 Cwts.
(London General Omnibus Co., Ltd.).

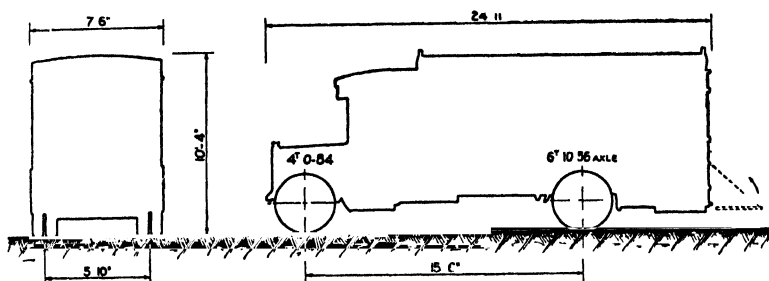



Fig. 12.—6-Ton Van (John I. Thornycroft & Co., Ltd.).
Gross Weight, 10 Tons, 11 Cwts. 1 Qr.

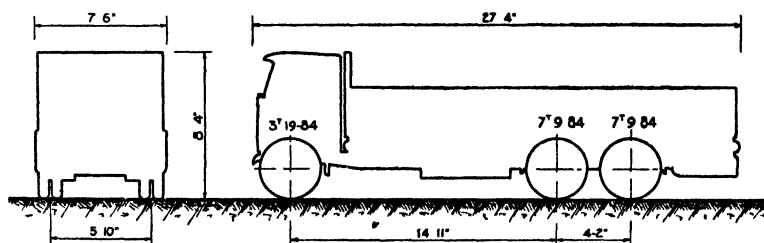



Fig. 13.—12-Ton Six-Wheeled Lorry (Leyland Motors, Ltd.).
Gross Weight, 18 Tons 19 Cwts. 1 Qr.

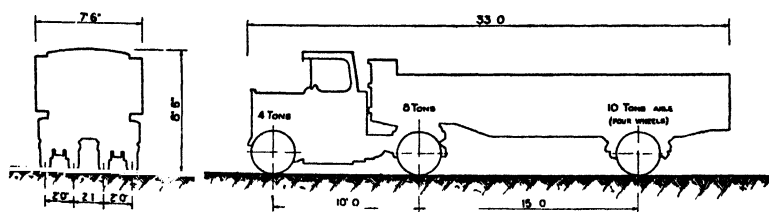



Fig. 14.—Eight-Wheeler, 15-Ton Lorry (Scammell Lorries, Ltd.).
Gross Weight, 22 Tons.

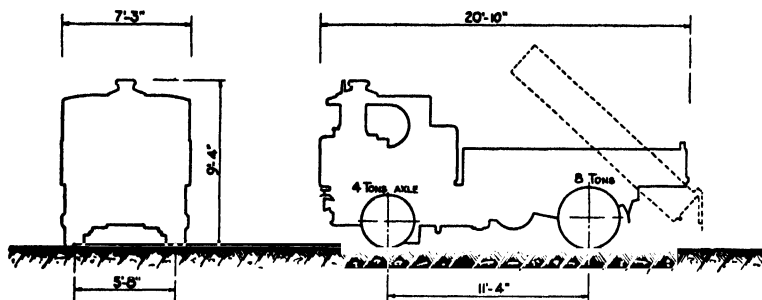



Fig. 15.—6-Ton Steam Tipping Wagon (R. Garrett & Sons, Ltd.). Gross Weight, 12 Tons.

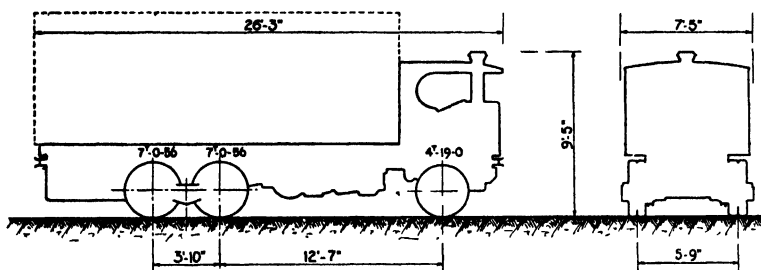



Fig. 16.—Six-wheeler, 12/15 Ton Steam Lorry (The "Sentinel" Waggon Works, Ltd.). Gross Weight, 19 Tons.

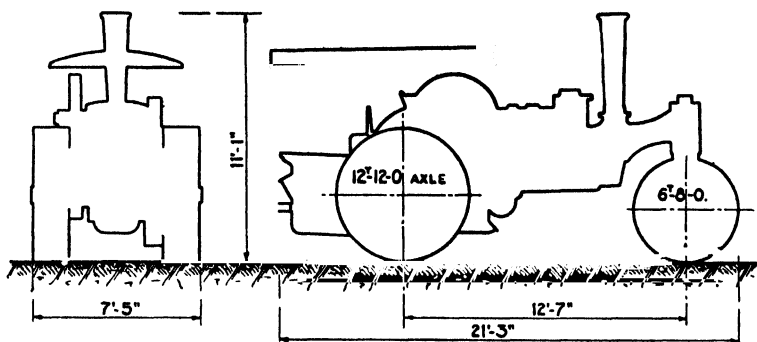



Fig. 17.—Steam Roller (Aveling & Porter, Ltd.). Gross Weight, 19 Tons.

20. Loadings for British Railway Bridges.*—As with steel bridges, reinforced concrete railway bridges are designed for an equivalent uniformly distributed load (E.U.D.L.) derived from the

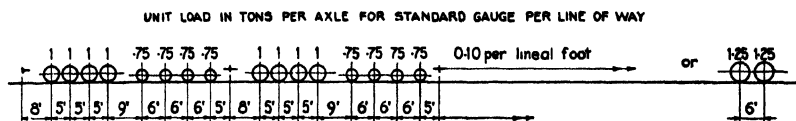



Fig. 18.—Diagram of British Standard Unit Loading for Railway Bridges.

maximum bending moments produced by the actual loads passing over the bridge. Allowances have also to be made for impact,

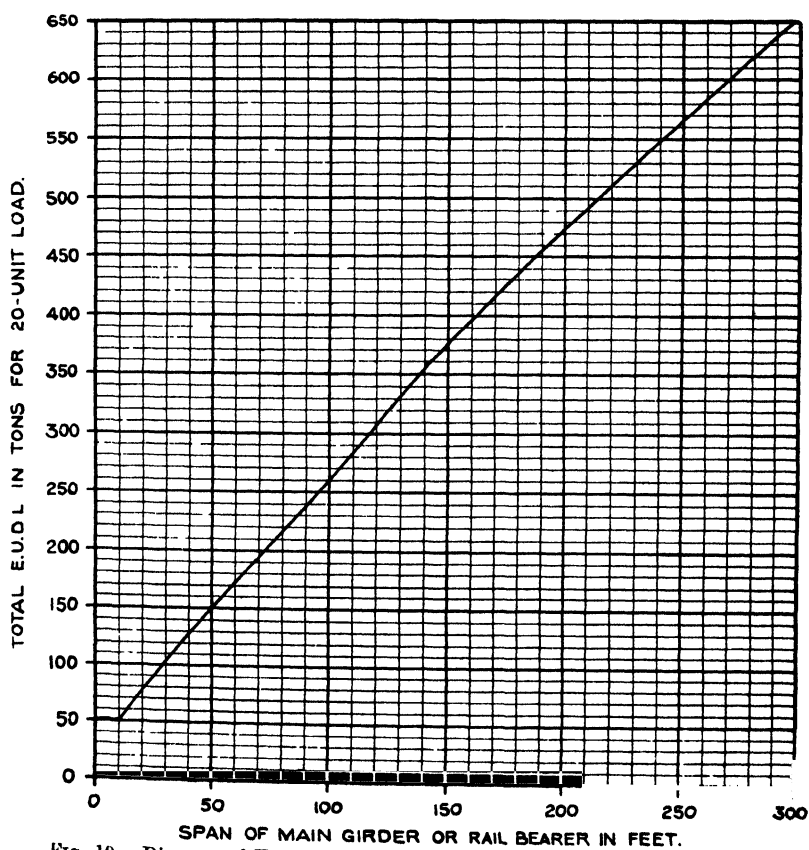



Fig. 19.—Diagram of Equivalent Uniformly Distributed Load (20-Unit Load).

longitudinal forces, due to acceleration and braking, and for centrifugal forces if the railway is on a curve at the bridge.

* See also British Engineering Standards Association, Specification No. 153, Parts 3, 4 and 5, 1923; and "Report of the Bridge Stress Committee, 1928 (issued by the Stationery Office) of the Department of Scientific and Industrial Research."

Standard Loading.—The British Standard unit load for railway bridges is shown in Fig. 18. A multiple of this loading is selected to suit the circumstances of the design.

The Ministry of Transport "Requirements for Passenger Lines and Recommendations for Goods Lines," recommends the adoption of this loading with a multiplier of 20 (20-units loading), for heavy main lines. This practice will probably become general, although several main-line railways still use 18-units loading.

The E.U.D.L. for 20 units of the above load is shown in Fig. 19.

The loads on the cross-girders and for very short spans of rail-bearers or main girders (under 10 feet), are calculated on the alternative load given in Fig. 18, which consists of two heavy driving axles 6 feet apart.

The equivalent loads per track (20-units loading) on cross-girders are thus as follows :—

Spacing in Feet.	Load in Tons.	Spacing in Feet.	Load in Tons.	Spacing in Feet.	Load in Tons.	Spacing in Feet.	Load in Tons.
5	25·00	14	51·42	23	70·44	32	90·00
6	26·66	15	54·34	24	72·70	33	92·42
7	31·42	16	56·88	25	74·80	34	94·70
8	35·00	17	59·12	26	76·74	35	96·86
9	37·78	18	61·12	27	79·08	36	99·30
10	40·00	19	62·90	28	81·26	37	101·62
11	43·64	20	64·50	29	83·28	38	103·82
12	46·66	21	66·66	30	85·66	39	105·90
13	49·24	22	68·64	31	87·90	40	107·88

Impact.—(*a*) The older and approximate method of estimating the impact effect is to increase the static E.U.D.L. by a fraction of its value. This fraction is known as the "Impact Factor" (I), and is calculated thus :—

$$I = \frac{120}{90 + \frac{n+1}{2} L}$$

where L = the loaded length in feet of the track or tracks supported producing the maximum stress in the member under consideration,

n = the number of tracks supported by the member.

Note that for a single track span of 30 feet, I is unity and the impact allowance is equal to the static E.U.D.L., which requires, therefore, to be doubled to allow for impact.

(*b*) The Report of the Bridge Stress Committee of the Department of Scientific and Industrial Research in 1928 (*q.v.*) presents a more complicated but more accurate view of the allowance necessary for impact.

This is found to be dependent on the hammer-blow due to the unbalanced moving parts of the locomotive, and to a resonance effect which occurs when the conditions are such that the frequency of the hammer-blow is in tune with the natural frequency of vibration of the bridge span. Other causes of impact considered in the Report are "lurching effect" and "rail-joint effect." To allow for existing locomotives and for future designs, three loadings are laid down, viz.:—

LOADING A.—20 *units of the standard load with a total hammer-blow on all axles of* $0{\cdot}2N^2$. (This loading corresponds to a heavy but well-balanced modern 4-cylinder locomotive.)

LOADING B.—16 *units of the standard load with a total hammer-blow on all axles of* $0{\cdot}5N^2$. (An intermediate class corresponding to a modern 2-cylinder locomotive.)

LOADING C.—15 *units of the standard load with a total hammer-blow on all axles of* $0{\cdot}6N^2$. (This loading corresponds to an old, badly balanced 2-cylinder locomotive.)

In the above loadings, $N =$ the number of revolutions per second of the driving wheels, and for main lines the maximum value of N is 6 r.p.s., and the maximum hammer-blow for loading C becomes 21·6 tons (total on all axles).

To allow for the resonance effect referred to, the hammer-blow must be multiplied by a factor known as the "Dynamic Magnifier" (K). For spans up to 30 feet K may be taken to be unity. For other spans K is calculable from the natural frequency of vibration, and mass of the span in question, allowing for damping and for the unsprung weight of the locomotive in accordance with an established formula. (See page 99 of the Report.)

Combined E.U.D.L. curves, including impact, have been plotted for steel bridges of various types, and an enveloping curve arrived at (see Fig. 75 of the Report), which agrees well with actual tests on various bridges. It will be seen that the most severe impact effects occur on spans of from 100 to 200 feet. In the extreme case of synchronism, the allowance for impact is 100 per cent. increase of load, but under 100 feet or over 200 feet span, $33\frac{1}{3}$ per cent. increase is generally ample. The impact factor will be observed to follow an entirely different law from the empirical rule enumerated in section (*a*) of this paragraph. Also it should be noted that loading C, though lighter than loading A, is the more severe for those spans where synchronism occurs.

It is hoped to eliminate in time the undesirable types of loco-

motive represented by loading C, but the designer should assume for the present that all three types of loading have to be catered for.

It will be noted that the above consideration of impact is entirely for steel construction. No information is given, and practically none is available, as to the relative mass and frequency of reinforced concrete to steel bridges comparable in length and type of construction. It appears to be generally accepted, however, that the newer material is affected less by impact than steelwork, owing to its greater mass and to the fact that reinforced concrete is not to the same extent elastic.

Lurching Effect.—This is another source of vibration. It is considered in the Bridge Stress Report that, to allow for lurching of the locomotive, the weight on any wheel should be taken as five-eighths of the axle weight.

The loads on the different members require to be suitably modified accordingly, except that for the centre beam of a two-track bridge, it is justifiable to ignore the effect.

Proportion of Hammer-blows on a Given Member.—The hammer-blow on a rail-bearer will not be one-half the total hammer-blow per track, owing to the " out of phase " of the wheel blows.

For loading A, assume on each rail six-fifths of total hammer-blow.

For loadings B and C, assume on each rail five-sixths of total hammer-blow.

Similarly in a three-girder bridge an allowance must be made when estimating the hammer-blow on any one girder. (See page 124 of the Report.)

Rail-joint Effect.—On spans up to 60 feet, rail joints can and should be avoided in most cases. To cover the effect of rail joints and general irregularities of track, it is well to allow an extra E.U.D.L. of $\frac{N^2}{3}$ tons on each track (N being the number of revolutions per second as before).

Loads on Cross-girders, including impact and rail-joint effect (but excluding any allowance for lurching), are shown in Table on page 25.

Longitudinal Forces.—The longitudinal force to be allowed :—

(*a*) for tractive effort (per track) is

$$1{\cdot}75F\left[\frac{20}{L + 75}\right]$$

where F = max. end shear without impact,
and L = effective span.

(*b*) for braking effect (per track) is

$$1{\cdot}75F\left[\frac{12}{L + 90} + 0{\cdot}075\right]$$

TABULATED TOTAL CONCENTRATED LOADS ON CROSS-GIRDERS (ALL LOADINGS)

Spacing (Feet).	Concentrated Loads (tons per track).			Spacing (Feet).	Concentrated Loads (tons per track).		
	3 r.p.s.	4·5 r.p.s.	6 r.p.s.		3 r.p.s.	4·5 r.p.s.	6 r.p.s.
3	28	31	36	17	62	66	71
4	28	31	36	18	64	68	73
5	28	31	36	19	66	70	75
6	29	33	37	20	68	72	77
7	34	38	42	21	70	74	79
8	38	41	46	22	72	76	81
9	41	44	49	23	74	78	83
10	43	47	52	24	75	79	85
11	47	50	55	25	77	81	86
12	50	54	59	26	78	82	88
13	52	56	61	27	81	84	90
14	54	58	64	28	83	87	92
15	57	61	67	29	86	89	94
16	60	64	69	30	88	92	97

In both formulæ a proviso is made that the bracketed term need not exceed 0·15.

Centrifugal Effect.—This is considered as a moving load acting at a height of 6 feet above the rails of value

$$C = \frac{WV^2}{15R} \text{ per lineal foot,}$$

where W = E.U.D.L. per lineal foot without impact,
V = Maximum speed of train in miles per hour,
R = Radius of curve in feet.

Dead Load.—In all cases the dead loading must be added independently of the live loads enumerated in this section.

21. Wind Pressure.—The wind pressure usually specified for bridges is 30 lbs. per square foot, which covers the case of moderate-sized structures subjected to gusts. This figure is occasionally increased to 40 lbs. for very exposed structures. From experiments carried out, it has been found that these figures are never likely to be exceeded in this country.

The consideration of wind pressure is very important on narrow

bridges, and should be taken into account when calculating the strength of the individual members affected and the general stability of the structure.

As mentioned above, it is imperative in light bridges of the trestle type, and occasionally in heavier bridges in which the bridge platform is at a considerable height above the foundation level, to take carefully into account pressure of wind.

The size of a structure does not affect the maximum wind pressure when the wind is steady, but makes a considerable difference when the wind is variable. From experiments carried out it has been found that as the surface area is increased the effects of gusts are felt less and less, until, on a large area approximating in size to that of a 1,000-foot span bridge, the increase of pressure due to gusts would be negligible.

When taking into account wind pressure upon the parapets of a narrow footbridge, it should be noted that the windward face of the windward parapet is subject to actual pressure from the wind, while the leeward parapet may also be stressed in the same direction by the resultant vacuum produced on its leeward face.

Until further experiments have been made, and knowledge gained as to the effect of this vacuum on such structures, the figure of 30 lbs. per square foot may be safely assumed, and parapets to bridges of whatever width designed for the above lateral wind pressure applied to either face.

A useful formula * for determining the wind pressure from a given wind velocity is as follows :—

$$p = 0{\cdot}0032\,V^2,$$

where p = pressure in pounds per square foot,
V = velocity in miles per hour.

It will be seen that the usual wind pressure of 30 lbs. per square foot given above is resultant, according to this formula, from a velocity of 97 miles per hour, and that this assumed wind pressure can safely be considered to cover the maximum pressure likely to be produced by gusts during the highest winds upon the smallest structures subjected to them.

In bridges where the parapet members form the main supports and are of open construction, such as trussed or bowstring girders, the lateral pressure due to wind should be taken into account. In such cases wind pressure will act upon both the windward and leeward girders, but obviously to a greater extent upon the former than upon the latter.

* "National Physical Laboratory Experiments," *Proc. Inst. C.E*, 171.

The following pressures are assumed :—

Total pressure on windward girder . $P_w = p \, . \, b,$

Total pressure on leeward girder . . $P_l = p \, . \, b\left(1 - \frac{b}{a}\right),$

Total lateral wind pressure on bridge . $P_W = p \, . \, b\left(2 - \frac{b}{a}\right),$

where p = assumed wind pressure per square foot,
a = gross surface area of girder in square feet,
b = nett surface area of girder in square feet.

A paper by Mr. D. H. Remfry, A.M.Inst.C.E., on "Wind Pressures and Stresses caused by Wind on Bridges," read before the Institution of Civil Engineers in 1921, gives some useful data upon this subject.

CHAPTER III

TEMPERATURE AND SHRINKAGE EFFECTS

22. Importance of Temperature Stresses.—The coefficient of expansion and contraction due to changes of temperature is practically the same for concrete and steel, and it is this very important fact that enables reinforced concrete to be employed as a constructional material.

Any slight differences of stress in the concrete and adjacent steel, due to change of temperature, of these two materials are of no practical importance, and are neglected in the design of reinforced concrete work.

Quite distinct from the above question, however, are stresses which may be developed in reinforced concrete structures owing to external restraint against temperature or shrinkage movements. Such stresses become very serious indeed in structures of considerable uninterrupted length unless adequate provision is made for linear variations.

Generally speaking, all reinforced concrete bridges over 100 feet in length should be provided with transversal expansion joints. In addition to the above expansion joints, suitable provision must be made to permit the structure to expand or contract. This may be in the form of sliding plates, roller bearings, or elastic supports, the suitability of any particular device depending upon the type of bridge and form of support.

For girder bridges having rigid end supports, such as walls retaining earth filling, it is necessary to provide expansion joints where the total length may exceed 70 or 80 feet. Experience shows that where the deck platform of a bridge is connected to abutment supports of the above type excessive stresses are produced in the deck members. This may result in the latter members cracking, unless the resistance of the abutment connection is overcome and fracture occurs there.

Arched members may be considered as exceptions to the above general rule, and within reasonable limits can be designed to take temperature and shrinkage movements either by elastic deformation or by the introduction of temporary hinges.

The reader should note that expansion joints are always necessary to the deck platform of an arch bridge. These are placed over one, or preferably both, of the abutment supports.

23. Range of Temperature Variations.—Where the stresses produced by temperature are required to be taken into account a range of 30° F. above and below the assumed normal temperature is usually allowed for in this country.

Test results on the coefficient of expansion for concrete vary from 0·000005 to 0·0000065 per degree Fahrenheit. A mean figure of 0·000006 is usually adopted. This coefficient is almost identical with that of mild steel, the latter having an average value of 0·0000062.

Many bridges are protected from the direct effect of the sun's rays and therefore do not expand to the same extent as more exposed works. The temperature increase in these cases affecting the bridge is probably the average of the daily temperatures. Low temperatures, on the other hand, probably affect the covered bridge equally with other types.

Tests indicate that, while the internal temperature of concrete will reach that of the atmosphere for low values, it appears to "lag" for normal and higher temperatures.

The total variation in temperature of concrete bridge members may be taken to be from 70 to 75 per cent. of the corresponding atmospheric variation in this country. This range may be increased or decreased according as the structure is exposed to, or protected from, high winds. From this it will be seen that the customary figure of 30° above and below an assumed mean may easily be realised.

In considering this point, it will be appreciated that this mean temperature may vary from that assumed owing to practical reasons not under the designer's control.

In latitudes where greater variations in temperature are met with, the range of temperature allowed for is correspondingly greater. For instance, in the south of France variations between the limits of 11° and 106° F. have been assumed.

24. Expansion Joints in Wing Walls.—Where bridges are terminated by abutment retaining walls having parallel or angular wing walls, it may be desirable to introduce vertical expansion joints. The maximum length of wall without joints should not exceed 80 feet, and it is usually found convenient to provide them at the junction of the abutment and wing walls. These are usually arranged just outside the parapets and are frequently masked by piers or pilasters.

25. Three-hinged Arches.—Arches containing three permanent hinges or articulations do not require treatment for temperature stresses, since the hinges allow the structure to rise and fall under expansion and contraction without the introduction of any

appreciable internal stresses. In this type of arch a hinge is located near each of the springings and one at the crown.

The difficulty of satisfactorily masking expansion joints without detracting from the appearance of the bridge, particularly at the centre or crown of the arch, frequently causes designers to avoid permanent hinges, and the type of bridge with three permanent hinges is replaced by the fixed type, in which temporary hinges are employed during construction. By this method the advantages of articulations are obtained without the disadvantages enumerated above.

With regard to the deck platform, apart from the question of lateral expansion, transverse expansion joints should be provided in order that no interference with the free rib movements may be made.

It will, of course, be understood that included with the deck construction are the parapets which require the provision of expansion joints equally with the deck slab and longitudinal beams.

In earth-filled arches, spandril walls are necessary, and these also require to be provided with vertical expansion joints immediately above each hinge.

26. Two-hinged Arches.—Arches having two hinges—one at each of the springings—are sometimes employed. With this class of bridge, movements due to temperature or other linear variations induce stresses in the arch which are a maximum at the crown.

The equivalent horizontal thrust or "pull" resultant from an assumed alteration in length of the arch is calculated (see Art. 80) and considered to be applied at the hinges or springings.

The maximum bending moment produced at any arch cross section is therefore the product of the thrust and the height of the section above the springings or hinges. The direct horizontal force, causing an increase or decrease in compression upon a section, is constant throughout the length of the arch.

The maximum bending moments resultant from variations in length of the arch rib do not, however, increase directly with an increase of rise of the arch, as might be inferred from this latter statement, since the thrust itself varies practically inversely with the square of the arch rise. In other words, the flatter the arch the greater the resultant temperature stresses, as would be expected.

This being the case, there is a certain limit within which the span rise ratio must be kept in order that the temperature and other stresses shall not become excessive.

Suitable forms of permanent hinges for both slab and ribbed types of arch bridges are dealt with in Chapter IX.

27. Hingeless Arches.—In hingeless arches the total assumed temperature variations have to be taken into account, whether temporary hinges are employed or not (see Art. 72).

The total equivalent horizontal thrust for this type of arch acts at the elastic centre of the arch. For symmetrical arches of parabolic form the elastic centre is situated at the centre of the span, at a height between two-thirds and three-quarters of the rise of the arch. The resultant moments are therefore a maximum at the springings.

For hingeless arches having spandril walls, it is necessary to form vertical expansion joints in these walls over the springings for all excepting very moderate spans, in order to avoid the appearance of unsightly cracks.

Also, it is essential to form transverse expansion joints in the deck platform over the springings, as explained for the case of hinged arches. This may be attained by providing sliding joints or thin flexible columns against the abutment walls, as explained for girder bridges (Art. 30).

28. Linear Variations of Statically Indeterminate Arches.—The following effects, producing deformation in all arches having less than three hinges, are important and require to be given consideration in design :—

(1) *Shrinkage of Concrete during Setting and Hardening.*—The amount of arch shortening resultant from shrinkage, of course, differs according to the quality and mixing of the concrete employed, and also the manner in which it is placed in the work and allowed to mature. An average figure for the contraction of an arch rib, if placed in one operation, is approximately 0·0004 of the length of the span.

In view of the fact, however, that most ribs are placed in short lengths, the amount of arch shortening due to shrinkage after the ribs are continuous and connected to the abutments is reduced to a figure of from 0·0001 to 0·0002 of its length.

(2) *Compression due to Permanent and Superimposed Loading.*—Arch shortening from this cause varies from 0·0001 to 0·00025 of its length, the exact amount depending upon the resultant average compressive stress upon the sections.

(3) *The Shortening of the Arch Axis due to the Maximum Drop in Temperature.*—The customary figure for this is 30° F. and produces a shortening of 0·00018.

(4) *Settlement of Abutment Supports when the Horizontal Thrust from the Arch is brought upon them.*—This settlement, although not causing the arch to shorten, produces moments and stresses in it of the same sense. There is not sufficient reliable data to enable any

definite figures to be given for this, but for arches having a rise of 1/10 to 1/12 of the span this equivalent arch shortening may become the most serious of the above effects.

These factors considered individually are not very serious, but they are cumulative in effect, and tend to produce particular deformation in the arch, *i.e.*, a flattening at the crown.

Moreover, this deformation is produced by a definite strain, and the resultant stresses are therefore practically independent of the strength of the member subjected to it.

29. Stresses produced by Arch Shortening.—A hingeless arch not provided with temporary hinges will be subjected to deformation due to shrinkage, compression and fall of temperature. It may also be deformed in a similar manner owing to the span increasing under thrust.

In order to investigate the probable compressive stress induced by the combined action of the above effects, figures for the actual or equivalent arch shortening will be assumed as follows :—

Proportion of shrinkage producing arch shortening	0·0001
Compressive strain from permanent and superimposed loading producing arch shortening.	0·0001
Assumed equivalent shortening of arch due to settlement of abutments	0·0001
Shortening of arch axis due to drop in temperature of 30° F.	0·0002
Total equivalent shortening of arch or span .	0·0005

Given this total figure, it is possible to compute the horizontal force required to produce it as follows :—

$$H = \frac{45\,E.Ic.n}{4f^2} \quad \text{(see Art. 72, Formula 58)}$$

where n = above equivalent shortening of span.

Note.—The value $\frac{45.Ic}{Av}$ in Formula 58 is unimportant, and is neglected in ascertaining the thrust.

The force H acts through the elastic centre of arch, and where this is situated at two-thirds of the rise produces bending moments in the arch as follows :—

At springings $\frac{2H.f}{3}$

At crown $\frac{H.f}{3}$

The maximum compressive stress in the concrete at the crown due to bending alone is

$$S\ \text{max.} = \frac{Mc.y}{Ic} \quad \text{(where } y \text{ is the distance from the neutral axis to the extreme fibre)}$$

$$= \frac{45.n.E.y}{12f}$$

For flat arches with rise span ratios in the neighbourhood of 1/10 the value of y is found to be about $\frac{f}{10}$, and, adopting this figure, the variable in the above formula can be eliminated thus :—

$$S\ \text{max.} = \frac{9.n.E}{24}$$

Adopting a value of 2,000,000 lbs. per square inch for E and the above value of 0·0005 for n,

S max. = 375 lbs. per square inch.

At the springings the above stress may be increased up to 100 per cent., the exact figure depending upon the cross section at these points.

The stresses produced by the deformation resultant from arch shortening are quite distinct from, and additional to, the uniform compressive stresses due to the normal thrust produced by the weight of the bridge and superimposed loading and those induced by the bending moments from the latter loading.

Furthermore, the stresses produced by the arch shortening are a result of a particular deformation causing definite strains in the various parts of the arch, and consequently for any given case the stress is practically independent of the amount of longitudinal reinforcement that may be provided.

It will be seen from the above that there is, for an ordinary hingeless arch constructed as such, a fixed limit for the rise in respect to its span beyond which satisfactory design is not possible.

By introducing spiral reinforcement, which greatly increases the compressive strength of the concrete, with a proportionate increase in its elastic modulus, an arch can be constructed with a rise span ratio of 1/10. If, in addition, temporary hinges or other devices (see Art. 128) are introduced to eliminate or neutralise the effect of arch shortening, it is possible to design and construct satisfactorily an arch having a rise span ratio of 1/12. It should be noted, however, that flat arches of these proportions should not be employed except where, from some exigency of the site, arches of more economical proportion are prohibitive.

30. Provision for Temperature Movements in Girder

Bridges.—In girder bridges having rigid supports at their extremities such as those supported upon retaining walls, earth-filled at the back, provision for expansion should be made when the uninterrupted length exceeds 70 to 80 feet.

It is common practice to consider abutment walls subjected to lateral earth pressure as being supported horizontally by the bridge deck platform.

With the introduction of ordinary types of expansion joints at the top of the walls this support is destroyed. For this reason expansion joints are frequently omitted and the main longitudinal beams made monolithic with the abutment construction.

Experience has shown that if this is done the following danger may develop: With the maximum fall of temperature the bridge contracts and pulls the abutment retaining wall with it. During the period of warm weather the bridge expands, and being unable to force the wall back owing to the consolidation of the filling behind it, a crack is formed, and the deck slides over the wall.

With the next fall of temperature the wall is again pulled forward, owing to its resistance being less than the friction between the deck and the wall. This progressive creeping of the wall becomes very serious, and immediate steps to remedy the trouble are imperative.

It may be, of course, that the abutments are sufficiently rigid to prevent the above action. In this case it is probable that fracture will take place either in some of the overstressed deck members or at the abutment, when a primitive joint will be formed. In order to avoid any of these undesirable results, proper joints should be provided.

For bridges over 100 feet long allowance for movement at an abutment should always be made. As a general rule, for overall lengths up to 120 feet expansion provision is essential only at one end of the bridge.

In the case of a trestle bridge having several intermediate supports, and being of a sufficient length to require expansion provision, no bracing should be introduced in the longitudinal direction unless the height is such as to make this essential. In the latter case the longitudinal stiffness should be provided at mid-length where no movement occurs. The remaining supports can usually then be made sufficiently elastic to permit without undue strain the small but inevitable movements due to rise and fall of temperature. In cases where the anticipated movement may cause excessive flexure in the end trestle supports, sliding joints or roller bearings, of the type described below, can be employed.

Probably the most common method of providing for temperature movements is to form a horizontal joint entirely separating the bridge deck from the abutment walls. If this type of joint be

employed the surfaces in contact should be formed on the underside of the main longitudinal supporting beams. These sliding joints should be formed of thin metal plates, as explained in Art. 35. Care should also be taken to protect the end of the bridge deck from any considerable resistance to expansion due to earth thrust, etc.

There are certain objections to the above form of joint. In cases where the span of the bridge may be considerable, its end reactions will also be large and the resultant friction on the sliding plates so great that their object may be defeated.

This type of joint is particularly suitable, however, to take the ends of bridges having end spans in the form of semi-cantilevers. This form of bridge is often introduced where the bridge abutments are existing, but of insufficient strength to take their full share of load from the new work.

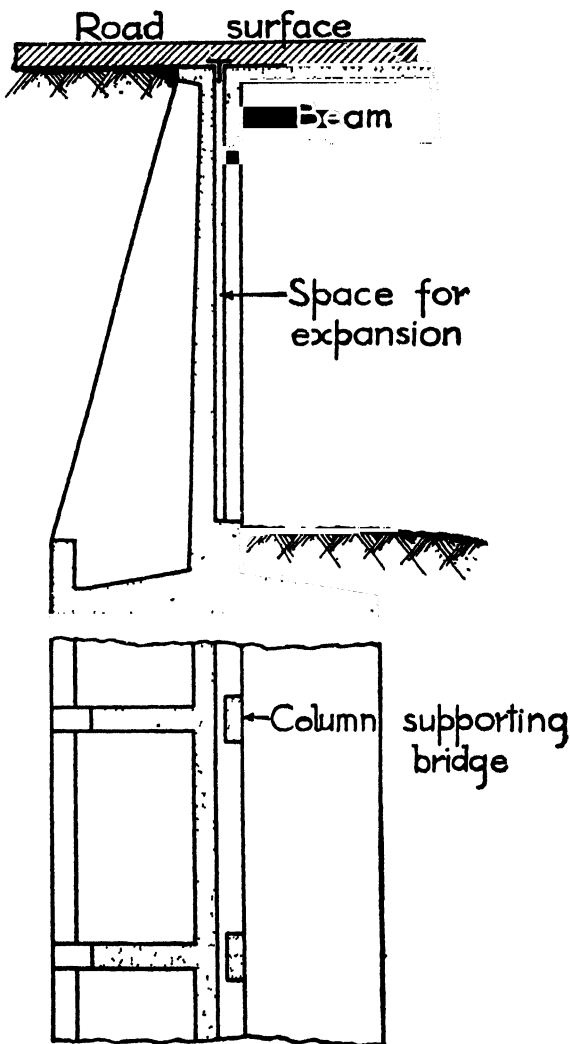



FIG. 20.—Flexible Column supporting End of Girder Bridge.

Other methods sometimes found suitable for use are roller or rocker bearings. For the heaviest loading roller bearings formed of cast steel are usually employed, while for ordinary loads reinforced concrete rocker bearings may be economically employed.

These bearings are placed under the main longitudinal beams, and are usually wholly or partially recessed into the transverse abutment wall. Alternatively they may be arranged on a bracket formed on the bridge face of the wall and constructed with it.

Where the supports may be intermediate, the rocker bearings can be placed on top of the columns or transverse beams comprising the supporting members. For the latter case (and others where the supports are not subjected to any lateral thrusts) the bearings are arranged vertically, but where the supports may be composed of a retaining wall under lateral pressure from the earth filling behind, the rockers are usually inclined at an angle towards the bridge. The horizontal component thrust resulting from this inclination is then considered as wholly or partially resisting the overturning effort of the earth pressure acting at the top of the abutment retaining walls.

An alternative method to the above, where sufficient height is available and also where the appearance of the underside of the bridge is important, is the construction of slender columns arranged in front of the abutment retaining walls upon which the entire vertical reactions of the bridge are carried (see Fig. 20).

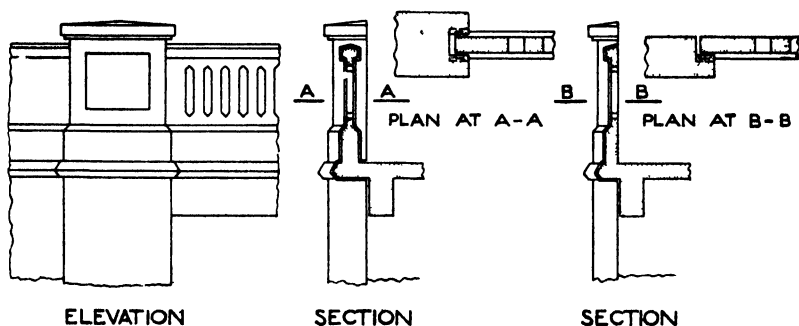



Fig. 21.—Expansion Joints in Parapet.

These columns are designed so that they bend when the deck construction varies in length, producing a displacement of the top of the column relative to its base.

In addition to the provision of any of the above arrangements permitting the beam ends to move horizontally, the complete bridge platform must be severed from the abutment construction in the vertical direction. This, of course, includes the parapets, deck slab, etc., and the construction must permit a movement of these members equal to the maximum anticipated amount.

31. Expansion Joints in Parapets.—The usual method of dealing with the parapet at abutment expansion joints is as shown in Fig. 21.

The abutment pilaster is singly or doubly rebated, as shown, and the end of the parapet to the bridge properly recessed into it.

32. Expansion Joints in Decking.—In highway bridges it is unusual to have a concrete wearing surface to the roadway, but where this is required the transverse expansion joints should be protected with suitable-sized steel angles on each corner (Fig. 22).

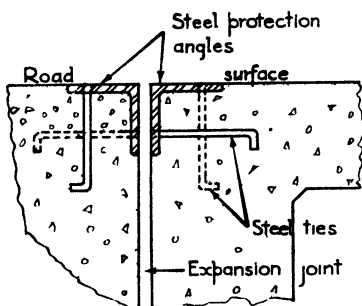
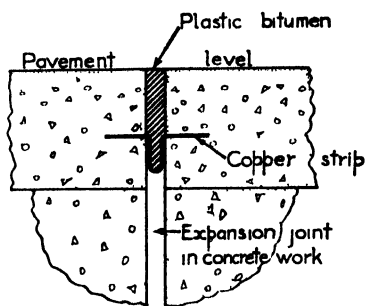



FIG. 22 —Expansion Joint in Roadway. FIG. 23 —Expansion Joint in Footpath.

There are obvious objections to expansion joints formed in the road surface, and to obviate these the method shown in Fig. 20 is usually adopted Provided that the material forming the road surface is reasonably elastic, this method is quite efficient, and has the merit of presenting an unbroken wearing surface to the roadway traffic. The lateral joints across the pavements are usually carried

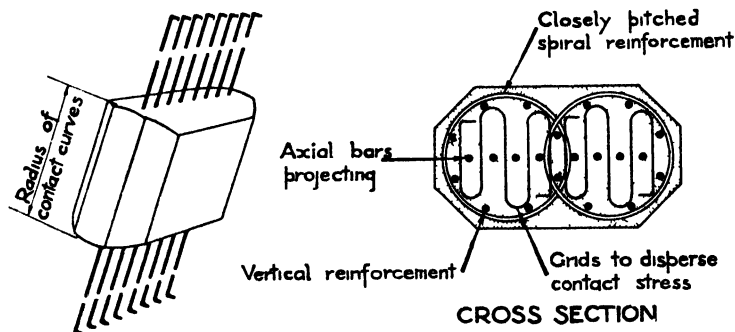



FIG. 24 —Reinforced Concrete Rocker Bearing

right through to the wearing surface. If desired, a strip of metal (usually copper) can be introduced (bent as shown in Fig. 23) and filled with pitch or plastic bitumen. Alternatively the joint may be dealt with as shown in Fig. 22.

33. Reinforced Concrete Rocker Bearings.—Fig. 24 shows a typical reinforced concrete rocker bearing. These members are

usually precast and held in the required position during construction by means of the projecting axial bars, which are partially provided for this purpose.

The height of the rocker is dependent upon the anticipated movement, and is usually about equal to the radius of curvature of the contact faces (Art. 126).

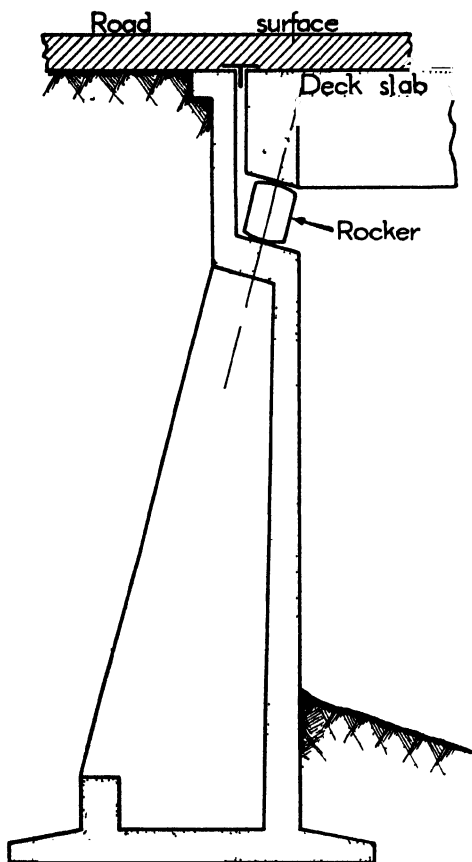



FIG. 25.—Abutment Support, showing Rocker in Position.

These rockers should preferably be constructed of a comparatively rich concrete, say a 1 : 1½ : 3 or a 1 : 1 : 2 mixture, and should be spirally reinforced either singly or of the interlocked type where the shape of the cross section permits this.

With this type of reinforcement the maximum stress upon the cross sectional area of the rocker should not exceed $\frac{2U}{3}$, where U = ultimate stress of concrete employed.

The compressive stress at the reduced areas in actual contact is much higher than that ordinarily employed. As explained in Art. 126, this is permissible, but provision must be made to disperse the thrust immediately above and below these points and also to restrain the material against undue lateral expansion.

To achieve the necessary dispersion of the load, several layers of reinforcement, placed normal to the direction of the thrust, are introduced. These may take the form of gridiron stirrups placed alternately at right angles to one another and arranged immediately above and below the contact surfaces.

Care must be taken to ensure that all these surfaces are in absolute contact, and that the rockers are in perfect horizontal alignment.

Bearings constructed in this material are usually cheaper than those of cast steel, and, of course, are free from possible corrosion.

They can be adapted to most cases, even being made continuous across the bridge width should the loading demand this. If continuous, however, the hinges are usually constructed *in situ* and the curved contact faces formed by means of plaster of paris or soft wood. Similar means are also used to facilitate the concreting of the beam surfaces immediately above the rockers when these latter are precast.

The reader will note that the bearing surfaces above and below the rockers are formed perfectly flat, the rocker moving about instantaneous centres usually situated at its theoretical points of contact with the supporting surfaces.

The inclination of abutment rockers is dependent upon the magnitude of the horizontal component of the inclined force transmitted by them necessary to support the abutment wall against the lateral pressure of the retained filling. The maximum permissible angle depends upon the load carried by the rocker and the maximum calculated horizontal movement of the supported surface. The practical limit is found to be from 20 degrees to 25 degrees with the vertical.

Fig. 26.—Type of Rocker Bearing for Large Span Bridge.

Fig. 25 shows a detailed section of a typical abutment support with rocker bearings in position. In this case the rocker is supporting the end of a long trestle bridge, the individual spans of which however, are not great.

Fig. 26 illustrates a type of concrete rocker required to support

a large span bridge, and which is consequently subjected to heavy loading. When this type of rocker bearing is designed to support the abutment retaining wall, its requisite inclination is relatively small, owing to the magnitude of the vertical load.

34. Metal Roller Bearings.—Bearings of this material are employed for the same purpose as the concrete rockers described above. They are usually made of cast steel, the contact faces being machined to ensure perfect contact. Great care must be taken in setting the base plates into position to obtain perfect alignment,

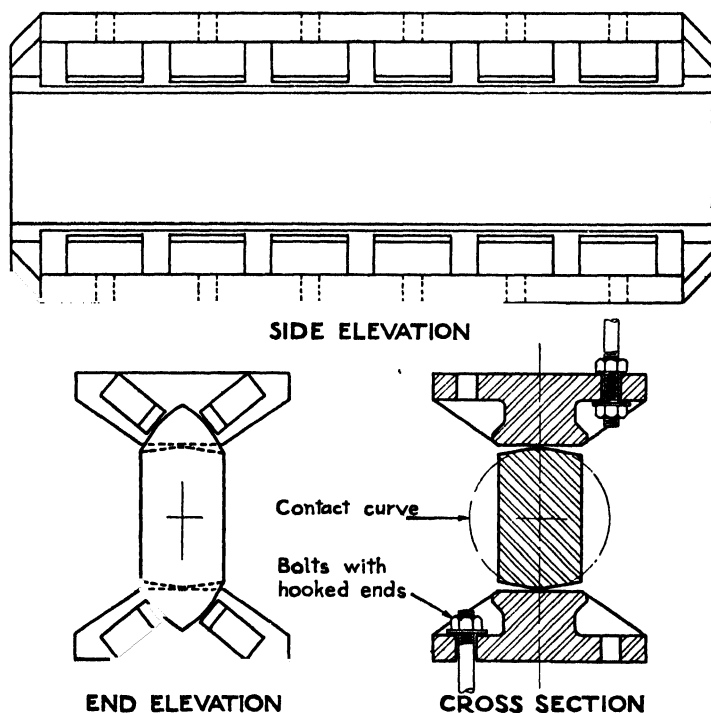



Fig. 27.—Cast Steel Roller Bearing.

especially where three or more bearings are used, as any appreciable difference in level might set up very serious and unprovided-for stresses.

Fig. 27 shows a good type of metal roller bearing. The end wings or lugs are shaped as shown and provided as a safety device in order to check any tendency to overturn or slide during or after construction.

With the arrangement shown, even should the bearing be slightly inclined and the load removed, the roller would remain correctly in position.

The bearing plates are attached to the reinforced concrete construction by means of the bolts shown. In the case of the base plates the bolts are either built into the work as it proceeds, or alternatively pockets may be provided, into which the ends of the bolts are grouted. This latter arrangement permits a slight adjustment to be made in their positions at the time of setting the metal base plates.

35. Construction of Sliding Joints.—Where the sliding joints mentioned in Art. 30 are adopted, they should be constructed as follows: Under each of the main longitudinal load concentrations slightly raised and prepared stools are formed. Upon these four thin rectangular plates of metal of suitable size are placed, the two outer plates usually being composed of steel and the two inner of copper.

They do not require to be attached either to the beam above, the base below, or in any way to one another.

The edges of the plates are usually wrapped in greased paper or other suitable material to protect the copper sliding faces from contamination, whilst the superimposed concrete is being poured.

The thickness of plates varies slightly with differing conditions, but an average thickness of $\frac{1}{8}$ inch for the copper and $\frac{5}{16}$ inch for the steel plates is customary (Fig. 95).

Expansion or contraction movement, taking place after completion of the structure, causes the two upper plates to slide over the two lower plates.

36. Provision for Lateral Expansion.—In bridges having widths of 75 feet and over, the question of lateral expansion arises. The deck platform of such bridges should always be provided with expansion joints at the abutments, in which case expansion and contraction of the deck construction, due to variation of temperature, will take place from a point situated at its centre of area and will act radially from this point.

If the supports are massive and not appreciably affected by temperature, the relative movement between them and the bridge platform will be radial, as explained above, and roller or rocker bearings introduced to accommodate this movement should be arranged so that their axes are normal to the direction of movement.

Reinforced concrete bridges having several spans and of a width requiring treatment for lateral expansion are infrequently met with, and usually necessitate a very exhaustive and expert study regarding the necessary provision to be made for temperature and other effects both from the engineering and architectural standpoints.

The question of lateral temperature movements, however, is not always confined to such structures. A bridge of 80 feet or so in width may possibly have a span of less than that amount, in which case some of the rocker bearings, may require to be placed at a considerable angle to, instead of approximately parallel with, the abutment walls.

CHAPTER IV

INFLUENCE LINES

37. Definition.—An influence line is a curve, which indicates the variation of a load function at a fixed section of a beam or other member, produced by the passage of a concentrated load along the member. For example, any ordinate of a bending moment influence line for any cross section of a beam, gives the value of the bending moment produced at this section by a concentrated load, when it is situated at the ordinate in question.

38. Utility of Influence Line Curves.—Whereas an ordinary bending moment diagram denotes the variation of the moment throughout a beam for a specific form of loading, an influence line enables the maximum value of the moment at one particular section to be found for any number of loads, however spaced. Influence line curves are not limited to bending moments, but can also be constructed for other functions of the loading, such as shear force, deflection, or for vertical and horizontal forces at arch supports. They may be drawn for simple or continuous beams, framed girders, and all types of arches.

39. Application of Influence Lines to Moving Point Loads.—Consider, for example, any particular section of a beam along which a train of two or more concentrated loads may travel. At one particular position of these loads during their passage the maximum possible bending moment (or other function) will be produced at the section under consideration. In order to design a beam for such loading, it is essential to ascertain the amount of this maximum moment, and in some cases other functions, such as the shear force. The derivation of these quantities analytically becomes impracticable, excepting for cases having the simplest conditions.

A very convenient method, however, of finding these required quantities, and that generally employed, is by the use of the influence line curves defined above.

These curves are usually plotted for an arbitrary unit load, but they are, of course, applicable to loads of any magnitude, the measured ordinates being increased or reduced in direct proportion to the ratio between the actual and unit loads adopted. Any number of loads may be considered, and by summing the products of the magnitude of the loads and their corresponding ordinates, the total

moment at the section is found for the loads in their particular positions. Moreover, it is comparatively simple by manipulation of the load train to place it in the position giving the maximum possible bending moment at the required section. This position is naturally the one where the product referred to above is a maximum, and is found after a few trial positions of the train upon the beam.

40. Application of Influence Lines to Distributed Live Load.—In addition to any combination of point loads, the above curves can be used for varying or uniformly distributed loads covering the whole or part of a span. They show at a glance on which portion of a beam or arch loads will require to be placed so as to produce the greatest efforts at the section ; and the bending moments, etc., may then be quickly read from the curves. They thus provide the clearest and simplest method for determining the maximum stresses at any given section.

As an example, in continuous beams and arches differing positions of a load cause bending moments of opposite sign. By loading only those portions of spans giving bending moments of the same sign a maximum may be easily found.

In order to utilise influence line curves for distributed loading, the latter is divided into transverse strips of convenient width, and each strip considered as a separate concentrated load, having its point of application immediately above or below the centre of area of the enclosed portion of influence line curve intercepted. The smaller the strips into which the load is divided the more accurate will be the result.

Having constructed a bending moment influence line curve, the maximum moment upon the section for which the curve is drawn can be quickly and easily obtained for any form of distributed and any combination of concentrated loads whatever.

41. Use of Standard Influence Line Curves and Tables.—Tables of ordinates for influence lines for various combinations of spans, assuming the basic spans as unity, have been published,* and others are given in Arts. 49 and 50 and in the chapters upon arches.

By the use of such tables or curves, calculations are reduced to a multiplication of the magnitude of the curve ordinate with a constant factor, depending upon the magnitude of the load and the linear dimensions of the structure. In the case of arch design the adoption of the above standard curves results in considerable saving of labour, owing to the many factors usually contained in the formulæ for the bending moments, thrusts, etc.

42. Construction of Influence Line Diagrams.—The import-

* "Interpolierbare Tabellen für Einflusslinien," by Gustav Griot; "Concrete Engineer's Handbook," by Hool and Johnson.

ance of influence line diagrams as an aid to the calculation of reinforced concrete bridges renders it necessary for their use to be thoroughly understood. In the chapters on arch and girder bridges, these curves are used to a large extent, and a preliminary study of a simple case will serve to explain the method of plotting the more complex curves given in these chapters. The values of the various ordinates to these simple influence line curves are calculated by the ordinary principles of mechanics. The following typical examples will indicate clearly to the reader the manner in which the formulæ for these ordinates are derived and plotted.

For cases where standard tables are not applicable, analytical treatment is usually impracticable, and the graphical methods in Arts. 46, 47 and 48 are resorted to.

43. Influence Lines for Single-span Beam freely supported.

Influence Line for Reaction at Support A (Fig. 28a).—When a moving load P travels across a beam AB (Fig. 28) the reaction at support A varies inversely as the distance of the load from the support.

This can be expressed by formula, thus :—

$$Ra = P\frac{(l - xl)}{l} = P(1 - x) \quad . \quad (1)$$

(where x is a function of the span l).

FIG. 28.—Influence Lines for a freely Supported Beam.

Influence Line for Bending Moment at Section S, distance a from Support A (Fig. 28b).—When the load is travelling over the portion AS of the span the moment at S = Rb $(l - a)$

$$= P\frac{xl}{l}(l - a) = P.x(l - a) \quad . \quad . \quad . \quad . \quad (2)$$

When the load is travelling over the portion SB the moment at S = R$a.a$

$$= P\frac{(l - xl)}{l}a = P(1 - x)a \quad . \quad . \quad . \quad . \quad (3)$$

Influence Line for Shearing Force at Section S (Fig. 28c).—When the load is passing over the portion AS of the beam the shearing force at section S is equal to the reaction at B.

$$Rb = P.x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

After the load passes section S the shearing force changes sign and is then equal to the reaction at A,

$$\text{or} \qquad \text{S.F. at section S} = P(1 - x) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The variable x in equations (1) to (5) is not raised above the first power, and therefore the influence line curves are straight lines having maximum ordinates of the values indicated in Fig. 28 at the considered section.

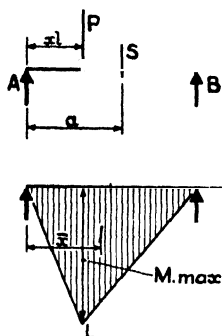



FIG. 29.

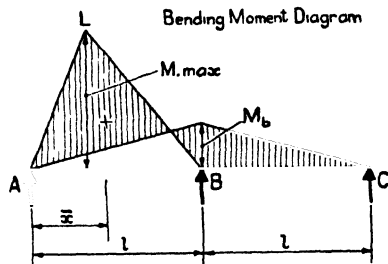
As single span beams having built-in (encastré) ends are rarely met with in bridge work, they will not be treated here; the method of calculating influence lines will now be applied to a more complicated example.

44. Influence Lines for Two-span Beam (Ends freely supported).—In Figs. 29 and 30 the more general case of a two-span beam is illustrated having equal spans and the outer ends "freely" supported. Using the same notation as for the first case, formulæ will be developed in the succeeding paragraphs for influence line ordinates of bending moments and shearing forces at any required section produced by a concentrated load P.

Bending Moments in a Continuous Beam.—In order to ascertain the bending moment at any section of a continuous beam, Clapeyron's theorem of three moments may be applied. The supports are assumed to be on the same level and their positions fixed.

FIG. 30.

The derivation of the general formula to this theorem is given in most text-books on applied mechanics. This formula can be simplified by assuming a constant moment of inertia and coefficient of elasticity, equal spans, and a concentrated load on one span only.

With the above assumptions it may be expressed thus :—

$$Ma + 4Mb + Mc = -\frac{6.Am.\bar{x}}{l^2} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where Ma, Mb and Mc are the moments at the supports A, B and C, and Am = area of *free* bending moment diagram (triangle ABL); $\bar{x}$ = distance of its centre of area from support A or C (Figs. 30 and 32).

Area of free bending moment diagram $= \text{M max}.\frac{l}{2}$ (from Fig. 29)

$$= \frac{P.xl.(l - xl)}{l}.\frac{l}{2} = \frac{P.x.l^2}{2}(1 - x) \quad . \quad . \quad . \quad (7)$$

The distance $\bar{x}$ from support A can be obtained graphically by construction, as shown in Fig. 31, or by the formula—

$$\bar{x} = xl + \frac{2}{3}\left(\frac{l}{2} - xl\right)$$

$$= l\left(x + \frac{1}{3} - \frac{2}{3}x\right)$$

$$= l\left(\frac{x}{3} + \frac{1}{3}\right)$$

$$= \frac{l}{3}(1 + x) \quad . \quad . \quad . \quad (8)$$

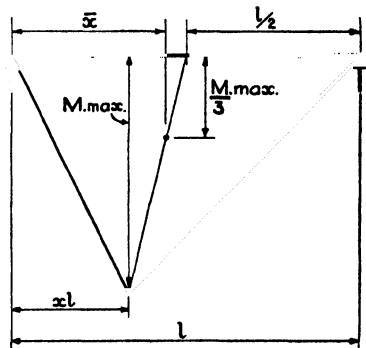



FIG. 31.

Equation for Moment Influence Line Ordinate at Centre Support.—When the load is on the span AB,

from (6) $\quad Ma + 4Mb + Mc = -\frac{6.Am.\bar{x}}{l^2}.$

(since Ma and $Mc = 0$) $\quad 4Mb = -\frac{6}{l^2}\frac{P.x.l^2}{2}(1 - x).\frac{l}{3}(1 + x)$

$$= -P.xl(1 - x)(1 + x)$$

$$\therefore Mb = -\frac{P.xl}{4}(1 - x)(1 + x)$$

$$= -\frac{P.xl}{4}(1 - x^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

When the load is on the span BC (Fig. 32),

$$x = l - \frac{l}{3}(1 + x)$$

$$= \frac{l}{3}(2 - x)$$

$$Ma + 4Mb + Mc = -\frac{6.Am.\bar{x}}{l^2} \text{ from (6)}$$

$$4Mb = -\frac{6.P.x.l^2}{l^2.2}(1-x).\frac{l}{3}(2-x)$$

$$= -P.xl(2-3x+x^2)$$

$$Mb = -\frac{P.xl}{4}(2-3x+x^2) \quad . \quad . \quad . \quad . \quad (9A)$$

Equations (9) and (9A) give the ordinates for the influence line curve for the moment at centre support. A table with various values of xl and a unit span is given in Art. 49.

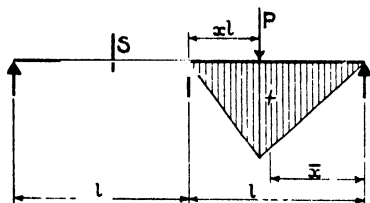



Fig. 32.

In order to simplify the calculation of the influence line ordinates for sections between supports AB and BC, an equation for the reaction at A will first be determined.

Ordinates to Influence Line for Reaction at A.—When the load is on span AB.

By taking moments about support B

$$Ra.l = P.l(1-x) + Mb$$

from (9)
$$= P.l(1-x) - \frac{P.xl}{4}(1-x)(1+x)$$

$$Ra = P(1-x)\left[1 - \frac{x}{4}(1+x)\right]$$

$$= \frac{P}{4}(1-x)(4-x-x^2)$$

$$= \frac{P}{4}(4-5x+x^3) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

When the load is on span BC.

$$Ra.l = Mb$$

from (9A)
$$Ra = -\frac{P.x}{4}(2-3x+x^2) \quad . \quad . \quad . \quad . \quad (10A)$$

The negative sign shows that the end support of the unloaded span tends to lift (the beam itself, of course, being assumed to have no weight).

Bending Moment Influence Line Ordinates for any Section S in either Span.—Three separate cases require to be considered as follows :—

(*a*) When the load is travelling between A and S.

Moment at section S $= Ra.a - P(a - xl)$

from (10) $$= \frac{Pa}{4}(4 - 5x + x^3) - P(a - xl)$$

$$= \frac{P}{4}(4a - 5xa + x^3a - 4a + 4xl)$$

$$= \frac{P.x}{4}(ax^2 - 5a + 4l) \quad . \quad . \quad . \quad . \quad (11)$$

This gives the ordinates to the portion of the curve between A and S.

(*b*) When the load is travelling between S and B.

Moment at section S $= Ra.a$,

and from (10) $$M = \frac{Pa}{4}.(4 - 5x + x^3) \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This gives the ordinates to the portion of the curve between S and B.

(*c*) When load is travelling between B and C.

Moment at section S $= Ra.a$

from (10A) $$M = -\frac{P.a.x}{4}(2 - 3x + x^2) \quad . \quad . \quad . \quad (13)$$

This gives the ordinates to the curve in the span opposite to which the section is situated.

Influence Line for the Shear Force at Intermediate Support.— The ordinates to the curve for shear (Fb) over the loaded span for a section very near to the internal support B, but on the same side as the load, are given by

$$Fb.l = P.xl - Mb$$

$$Fb = P.x + \frac{P.x}{4}(1 - x^2)$$

$$= \frac{P.x}{4}(4 + 1 - x^2)$$

$$= \frac{P.x}{4}(5 - x^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

After the load crosses the support, the shearing force at the same section is

$$Fb_1.l = -Mb$$

$$Fb_1 = +\frac{P.x}{4}(2 - 3x + x^2) \quad . \quad . \quad . \quad . \quad (14A)$$

Influence Line for Centre Reaction.

$$Rb = P - Ra - Rc$$

$$= P - \frac{P}{4}(4 - 5x + x^3) + \frac{P.x}{4}(1 - x^2)$$

$$= \frac{P.x}{2}(3 - x^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Influence line ordinates have been calculated for beams having three equal spans and are given in Art. 50.

45. Influence Lines for Continuous Beams having Unequal Spans.—It frequently occurs in practice that a beam or girder bridge, owing to some exigency of the site, is required to have unequal spans. Moreover, these spans are often of such a combination that they are not to be found in any of the published curves of influence lines or tables of ordinates (see Art. 41).

Where this occurs, the graphical application of the theorem of two moments by M. Maurice Levy may be employed with advantage.

46. Graphical Method for Moment Diagrams and Influence Lines.—The following graphical method of ascertaining bending moments is undoubtedly the one most suited for use in reinforced concrete design, and has been employed for this purpose for many years. Its advantages are now becoming more widely appreciated, and it is consequently being used to a correspondingly greater extent.

As the number and inequality of spans and complication of loading increase, so the advantages of this graphical method become greater.

The variation of spans is dealt with by ascertaining points situated at some fixed horizontal distance from the near supports and for the variations of loading a vertical distance from supports.

Making the usual assumptions regarding the condition of supports, constant moment of inertia, etc., the general equations for these distances are as follows :—

The horizontal distance of focus or fixed point from the support adjacent to the spans of which it is a function :

$$v_2 = \frac{(l_1 - v_1)\, l_2^2}{3\,(l_1 + l_2)\,(l_1 - v_1) - l_1^2} \quad . \quad . \quad . \qquad (16)$$

Note.—Should the moment of inertia not be constant, the values of v are altered, and may be ascertained mathematically. This is not difficult and, of course, does not alter the graphical application of the theory.

The vertical distance from the support (a function of the loading of the particular span when considered as being freely supported):

$$z = \frac{6.Am.\bar{x}}{l^2} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where Am = area of free bending moment diagram
$\bar{x}$ = distance from centre of area of Am to support.

It will be seen that the positions of the foci v, v_1, v_2 (Fig. 33), are affected only by adjacent spans or condition of end restraint, and these points can therefore be ascertained before any loading is imposed.

The vertical distance z is, on the contrary, independent of the number or influence of adjacent spans or form of end supports, and

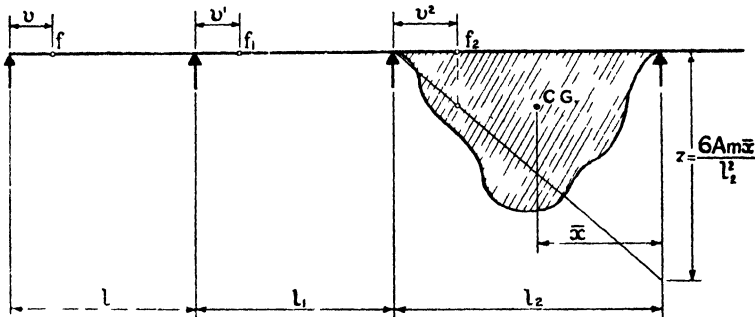



Fig. 33.

is dependent only upon the magnitude and position of the loading and the span under consideration.

With these functions determined at both supports, the true bending moment diagram can very quickly be drawn.

The above values of v may be found graphically for all combinations of spans having constant moments of inertia. The value of z may also be ascertained graphically for most cases of simple loading, which includes the single point load necessary for the construction of influence lines.

47. Positions of " Foci " or Fixed Points.—The positions of these points, which are situated upon the straight line representing the unstrained mean fibre or neutral layer of the member, are conditional upon the form of end restraint at the beam supports.

Each span has two foci, one near each support. The distance of any focus from its near support varies from zero for ends freely supported to one-third of the span for encastré beam ends, *i.e.*, ends fixed in such a manner that a tangent to the line representing

the strained mean fibre at the support is coincident with that line before deformation.

For any required degree of fixity between freely supported beams (where the focus coincides with the support), or rigidly fixed ends, the position of the focus can be adjusted accordingly.

The position of the foci for a beam having an infinite number of equal spans on either side is $0{\cdot}211l$ from the near supports.

To determine the position of a focus graphically the following construction is employed, and this may, if desired, very easily be checked with the general equation given previously.

Graphical Construction to obtain Foci.—Consider a beam comprised of a number of equal or unequal spans freely supported at its extreme ends.

Divide each span into three equal parts. Set off towards the

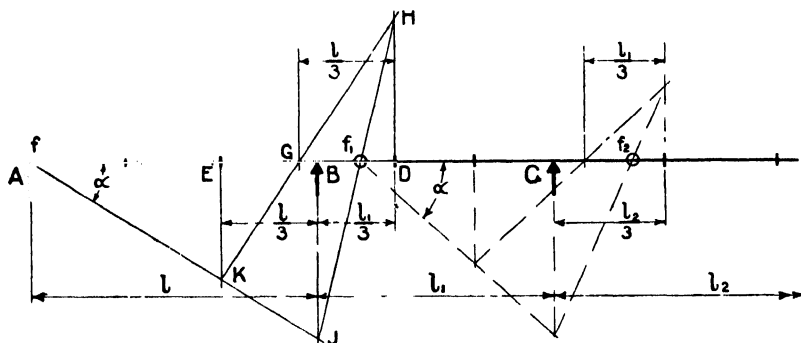



FIG. 34.—Construction to ascertain Fixed Points.

support from the third point near the desired focus a distance equal to one-third of the adjacent span; *i.e.*, from D set off a distance equal to EB giving point G (Fig. 34).

Draw any angle a from focus of end span (which in this case, owing to it being a "free" support, is at A), cutting perpendiculars through the third point E at K, and support B at J.

Join K to G and produce to intersection at H with a perpendicular through the third point D.

Join H with J, cutting BC at f_1.

f_1 is then the required left-hand focus for span l_1.

To find left-hand focus f_2 in span l_2 repeat above, in this case drawing any angle a from the focus f_1.

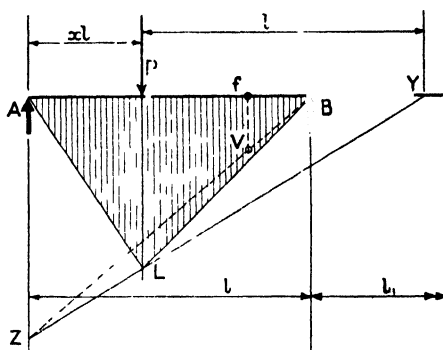
To ascertain the foci on the right-hand side of the supports the same procedure is adopted, the construction in this case commencing at the extreme right-hand support.

Bending Moment Diagrams for Point Load.—To obtain the bending moment diagram for a single concentrated or point load in any position, having located the fixed points or foci, it is necessary to construct the "free" bending moment diagram ALB and to determine the distance AZ (Fig. 35). This latter can be ascertained either by means of the equation $AZ = \frac{6.Am.\bar{x}}{l^2}$ or by the graphical method described below.

The maximum bending moment, considering the span as "free," occurs under the load P and equals $Pxl(1 - x)$. Having thus obtained the ordinate at L, the free bending moment diagram can be drawn to any convenient scale.

Fig. 35.

To ascertain distance AZ graphically.—Set off from position of load a length equal to the loaded span, viz., set off from P the length $PY = l$.

Join Y to L and produce to a vertical through A at Z.

Having now ascertained the two variable functions AZ and Bf (position of focus), the "closing line" is found as follows:—

Join Z and B, cutting a perpendicular through f at V.

V is a point on the "closing line" for the span and position of load considered.

Fig. 36

Exactly similar construction is now made to ascertain the corresponding point on the vertical through the left-hand focus.

Assuming in this case that the beam in Figs. 35 and 36 is freely

supported at A, the focus is at the support, and the point sought will also be situated here, and the line AV produced to U is the required closing line (Fig. 36).

This "closing line" is then continued so as to pass through the right-hand focus of every span to the right (as shown in Fig. 36), and should there be spans to the left, similarly through each left focus of these spans.

The shaded area represents the bending moment diagram for load P in position shown. The areas below the closing line are positive and the portion above negative (positive bending considered as that producing tension at bottom of beam).

This construction with practice becomes very easy, so enabling bending moment influence line diagrams to be rapidly plotted. It is found most valuable in ascertaining the bending moments for cases in which analytical treatment necessitates the use of extensive formulæ.

FIG. 37.

Graphical Construction of Influence Line Curves for Bending Moments.—Consider, for example, the section S adjacent to the support B (Fig. 37). For simplicity, consider span AB to be at the end of a series of spans and that no moment exists at support A.

It is well known that the curve of maximum free bending moments for progressive positions of a single load along a span takes the form of a "square" parabola with a maximum centre ordinate of $\frac{P.l}{4}$.

This parabola may then be constructed, using some suitable scale and P as unity.

The span under consideration should be divided into six, eight, or ten parts, according to the scale employed and degree of accuracy required.

The bending moment at section S is then ascertained for the load at each of the points and set off at the ordinate under the load in the manner already explained.

For example, taking the position P shown in Fig. 37, ordinate PL = maximum free bending moment. Set off PY = AB.

Join YL and produce to Z. Join ZB. Join AV and produce to U.

Transfer ordinate BU to vertical under P, thus obtaining W.

W is a point on the influence line curve.

This, repeated for each position of the load on all spans, enables the complete curve to be drawn.

For intermediate spans the construction is similar, but, of course, both foci have to be employed in order to obtain the closing line.

It should be emphasised that the above construction, although perhaps occupying some space to describe, is very straightforward, and, with practice, may be executed very rapidly.

FIG. 38.

48. Graphical Construction of Influence Line for Shear Force.—To obtain the shearing force influence line for any section of a beam it is necessary to consider the tangents to the bending moment diagrams.

For example, to find the influence line curve for the shear very near the first support A (Fig. 38).

Having the free bending moment for load in position shown, produce AL and form polar diagram by setting up vertical of unit load TR. The horizontal dotted line completes the diagram, and gives the reactions TK and KR from the load P *when beam is freely*

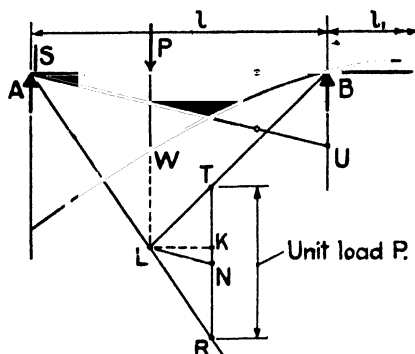
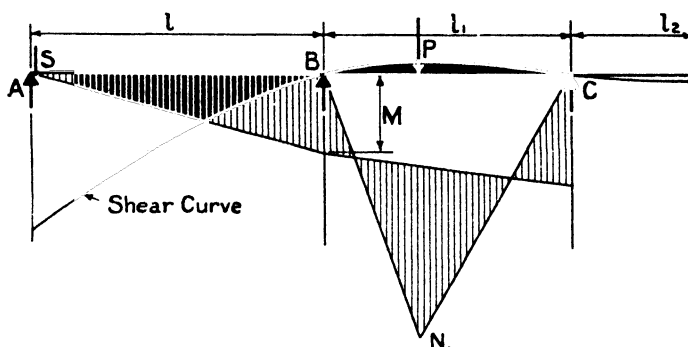



FIG. 39.

supported. For the case shown it is necessary to draw LN parallel to the closing line AU, when NR and TN give the shear force at A and B respectively.

The ordinate NR is accordingly set down under P; and W thus

found, is a point upon the required curve. This has to be repeated for all progressive positions of the load across the spans under consideration. The influence of loads on the adjacent spans is frequently ignored, but should the conditions of loading be such that this curve is required, it can be found by dividing the moment M at support B (produced by load in span BC, Fig. 39) by the span l.

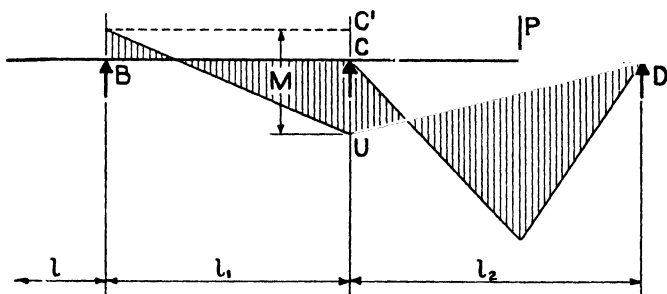



FIG. 40.

This gives the value of the shearing force at section S, and is the tangent to the angle made by the "closing line" and the horizontal.

The value so found should be, of course, plotted at the application point of the load P.

To facilitate the finding of these latter ordinates for differing load positions, the maximum free bending moment value of $\frac{P.l}{4}$ may

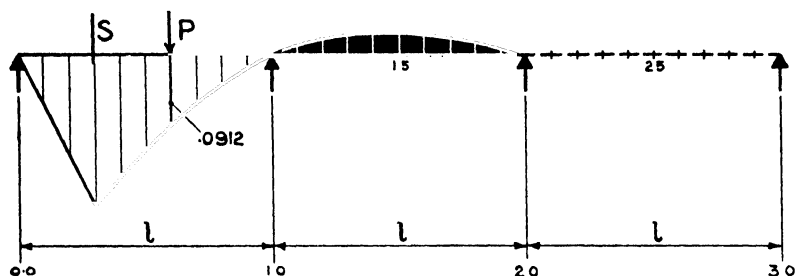



FIG. 41.

be regarded as 0·25 P, and a scale made for reading off direct the value of M for each load position.

In cases as shown in Fig. 40 the ordinate to the shearing force influence line at section B is found by dividing the arithmetical sum of the moments at supports B and C (represented by distance C_1 U) by the length of the span l_1.

TABLE A

49. Influence Line Ordinates for Two-span Beam.

Position of Load.	BENDING MOMENTS. Position of Section.						End Reaction and End Shear.	Centre Reaction.	Shear at Centre Support.
	$\frac{a}{l} = 0·3$	$\frac{a}{l} = 0·35$	$\frac{a}{l} = 0·4$	$\frac{a}{l} = 0·45$	$\frac{a}{l} = 0·5$	$\frac{a}{l} = 1·0$			
0·	0	0	0	0	0	0	1·0	0	0
0·1	·0625	·0563	·0500	·0437	·0375	—·0247	·875	·149	—·125
0·2	·1256	·1132	·1008	·0884	·0760	—·048	·752	·296	—·248
0·3	·1896	·1712	·1528	·1344	·116	—·0682	·632	·437	—·368
0·35	—	·2005	—	—	—	—	—	—	—
0·4	·1548	·1806	·2064	·1822	·158	—·084	·516	·568	—·484
0·45	—	—	—	·2070	—	—	—	—	—
0·5	·1218	·1421	·1624	·1827	·203	—·0937	·406	·687	—·594
0·6	·0912	·1064	·1216	·1368	·152	—·0960	·304	·792	—·696
0·7	·0633	·0739	·0844	·0949	·1055	—·0892	·211	·879	—·789
0·8	·0384	·0448	·0512	·0576	·064	—·072	·128	·944	—·872
0·9	·0171	·0199	·0228	·0257	·0285	—·0427	·057	·985	—·943
1·0	0	0	0	0	0	0	0	1·0	-1·0
1·1	—·0129	—·0151	—·0172	—·0193	—·0215	—·0427	—·0427	·985	—·0427
1·2	—·0216	—·0252	—·0288	—·0324	—·036	—·072	—·072	·944	—·072
1·3	—·0267	—·0311	—·0356	—·0401	—·0445	—·0892	—·0892	·879	—·0892
1·4	—·0288	—·0336	—·0384	—·0432	—·048	—·096	—·096	·792	—·096
1·5	—·0282	—·0329	—·0376	—·0423	—·047	—·0937	—·0937	·687	—·0937
1·6	—·0252	—·0294	—·0336	—·0378	—·042	—·084	—·084	·568	—·084
1·7	—·0204	—·0238	—·0272	—·0306	—·034	—·0682	—·0682	·437	—·0682
1·8	—·0144	—·0168	—·0192	—·0216	—·024	—·048	—·048	·296	—·048
1·9	—·0075	—·0087	—·01	—·0113	—·0125	—·0247	—·0247	·149	—·0247
2·0	0	0	0	0	0	0	0	0	0

Note.—See Fig. 41 for method of application.

TABLE B

50. Influence Line Ordinates for Three-span Beam.

Position of Load	BENDING MOMENTS. Position of Section.									End Reaction and End Shear.	Intermediate Reaction.	Shear in End Span at Intermediate Support.	Shear in Centre Span at Intermediate Support.
	$\frac{a}{l}=0\cdot3$	$\frac{a}{l}=0\cdot35$	$\frac{a}{l}=0\cdot4$	$\frac{a}{l}=0\cdot45$	$\frac{a}{l}=0\cdot5$	$\frac{a}{l}=1\cdot0$	$\frac{a}{l}=1\cdot4$	$\frac{a}{l}=1\cdot45$	$\frac{a}{l}=1\cdot5$				
0	0	0	0	0	0	0	0	0	0	1·0	0	0	0
0·1	·0622	·0559	·0496	·0433	·037	−·0264	−·0132	−·0115	−·0099	0·874	·159	−·126	·033
0·2	·1247	·1121	·0996	·0871	·0745	−·0512	−·0256	−·0224	−·0192	·749	·315	−·251	·064
0·3	·1881	·1695	·1508	·1321	·1135	−·0728	−·0364	−·0319	−·0273	·627	·464	−·373	·091
0·4	·1530	·1988 ·1785	·2040	·1795	·155	−·0896	−·0448	−·0392	−·0336	·51	·602	−·49	·112
0·5	·1200	·1400	·16	·2043 ·18	·2	−·1	−·05	−·0437	−·0375	·4	·725	−·6	·125
0·6	·0894	·1043	·1192	·1341	·149	−·1024	−·0512	−·0448	−·0384	·298	·83	−·702	·128
0·7	·0615	·0717	·0820	·0923	·1025	−·0952	−·0476	−·0417	−·0357	·205	·914	−·795	·119
0·8	·0369	·0431	·0492	·0553	·0615	−·0768	−·0384	−·0336	−·0288	·123	·973	−·877	·096
0·9	·0162	·0189	·0216	·0243	·0270	−·0456	−·0228	−·0199	−·0171	·054	1 003	−·946	·057
1 0	0	0	0	0	0	0	0	0	0	0	1·0	−1·0	1·0
1·1	−·0117	−·0137	−·0156	−·0175	−·0195	−·039	·0306	·0268	·0230	−·039	·963	−·039	·924
1·2	−·0192	−·0224	−·0256	−·0288	−·0320	−·064	·0688	·0604	·052	−·064	·896	−·064	·832
1·3	−·0231	−·0269	−·0308	−·0347	− 0385	−·077	·1142	·1006	·087	−·077	·805	−·077	·728
1·4	−·024	·0280	− 032	−·0360	−·04	−·08	·1664	·1472	·128	−·08	·696	−·08	·616
1·5	−·0225	−·0263	−·03	−·0337	−·0375	−·075	·125	·1731 ·1500	·175	−·075	·575	−·075	·5
1·6	−·0192	·0224	−·0256	−·0288	−·032	−·064	·0896	·1088	·128	−·064	·448	−·064	·384
1·7	0147	−·0171	−·0196	·0221	−·0245	−·049	·0598	·0734	·087	−·049	·321	−·049	·272
1·8	−·0096	−·0112	−·0128	−·0144	−·016	−·032	·0352	·0436	·052	−·032	·2	−·032	·168
1·9	−·0045	−·0053	−·006	−·0067	−·0075	−·015	·0154	·0192	·023	−·015	·091	−·015	·076
2·0	0	0	0	0	0	0	0	0	0	0	0	0	0
2·1	·0033	·0039	·0044	·0049	·0055	·0114	−·0114	−·0143	−·0171	·0114	−·068	·0114	−·057
2·2	·0057	·0067	·0076	·0085	·0095	·0192	−·0192	−·024	−·0288	·0192	−·115	·019	−·096
2·3	·0072	·0084	·0096	·0108	·012	·0238	−·0238	−·0297	−·0357	·0238	−·143	·024	−·119
2·4	·0078	·0091	·0104	·0117	·013	·0256	−·0256	−·0320	−·0384	·0256	−·154	·026	−·128
2·5	·0075	·0087	·01	·0113	·0125	·025	−·025	−·0313	−·0375	·025	−·15	·025	−·125
2·6	·0065	·0077	·0088	·0099	·011	·0224	−·0224	−·028	−·0336	·0224	−·134	·022	−·112
2·7	·0054	·0063	·0072	·0081	·009	·0182	−·0182	−·0227	−·0273	·0182	−·109	·018	−·091
2·8	·0039	·0045	·0052	·0050	·0065	·0128	−·0128	−·016	−·0192	·0128	−·077	·013	−·064
2·9	·0021	·0025	·0028	·0031	·0035	·0066	−·006	−·0083	−·0099	·0066	−·04	·007	−·033
3·0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note —See Fig. 41 for method of application

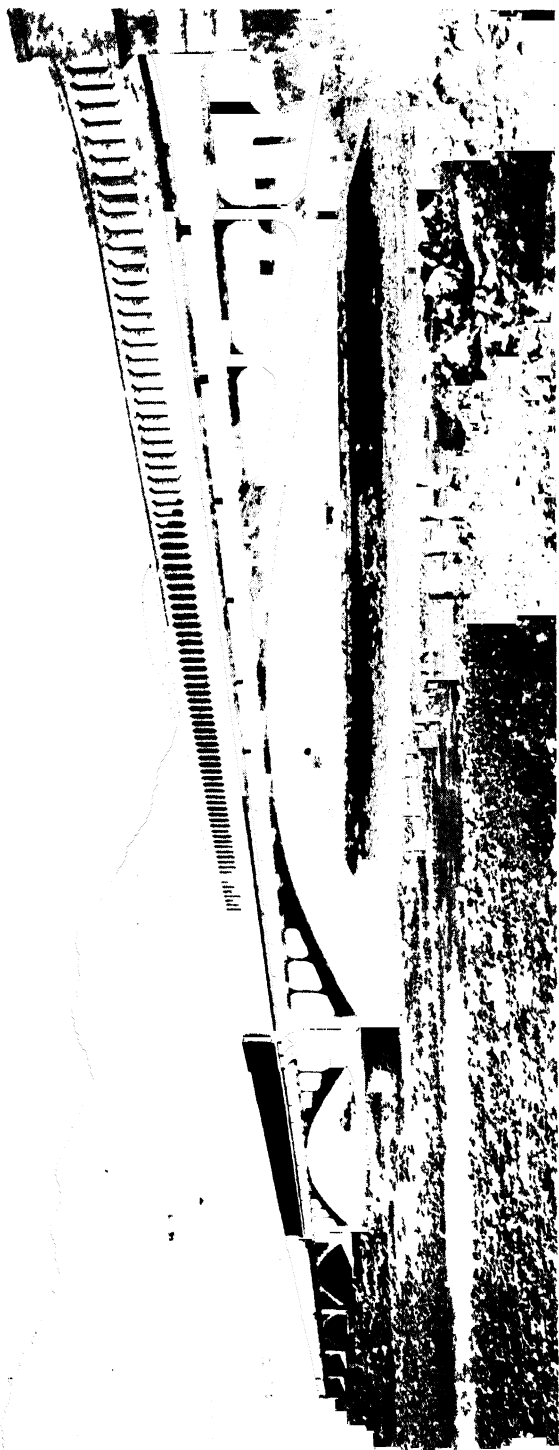



FIG. 42 CHARLES ALBERT BRIDGE SIX CONTINUOUS SPANS

PLATE III.

CHAPTER V

ARCH BRIDGES

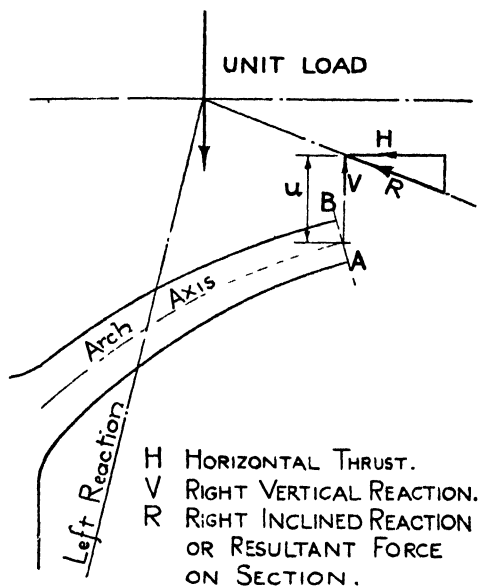
51. General.—It has already been mentioned that the essential difference between a beam and an arch is that the former has normal supporting reactions whilst the latter has inclined reactions. The effect of this difference upon any section of an arch rib or vault is to reduce the amount of bending upon it.

If the inclined reactions produced by a point load, at an arch support, are resolved vertically and horizontally, the bending moments produced by the vertical forces may be computed as for those in a horizontal beam.

In addition, a moment of opposite sense from the action of the horizontal component reduces the amount of bending produced by the vertical forces, and at the same time imposes a uniform thrust over the whole of each arch section equal in amount to itself.

In order to make the action of these component forces clear to the reader, it may be noted that they are equivalent to a single resultant force of some inclination situated either above or below the centroid of the section in question. Moreover, this force is one of the inclined reactions mentioned above, whether left or right hand, depending upon the position of the load in relation to the considered section. (See Fig. 43.)

Fig. 43.

Regarding the effect of this inclined force upon the section AB, it will be seen that its horizontal component represents the horizontal thrust upon the section. The distance which this force is

removed from the centroid of the section produces the bending moment upon it, while the vertical component represents the vertical shearing force acting upon the section.

When investigating the stresses produced upon a section by the above component forces, it is necessary to resolve these so that they are normal and parallel to the arch axis. This is explained more fully in subsequent pages.

In the following solutions the above horizontal component is ascertained for a single concentrated load in progressive positions along the arch. This force is then plotted, and an influence line for the horizontal thrust obtained.

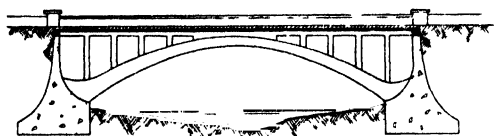



Fig. 44.

Fig. 45.

Fig. 46.

Fig. 47.

Sections at chosen distances apart are next considered, and influence lines for bending moments are plotted for the above unit load. The ease with which this may be done depends upon the type of arch, whether fixed or hinged. Referring to Fig. 43, the ordinate under the load to the bending moment influence line curve is the product of the horizontal thrust and the vertical distance from it to the centroid of the considered section (H.*u*).

Having plotted the above influence lines, the bending moment induced by any form of loading can be ascertained. With this information available, a complete investigation of the stresses in an arch member can be made with comparative ease.

The thrust and moments induced by temperature changes are calculated separately and then added to those produced by the live loading. Other effects, such as the rib shortening (due to shrinkage and loading), are also calculated separately, the resultant movement usually being added to the temperature movement. In most cases

these effects are provided for by the introduction of either temporary or permanent hinges.

52. Various Types of Arch Bridges.—Among the types of arch bridges employed in practice are the following :—

(1) Arched ribs with columns supporting deck construction (Fig. 44).
(2) Arched vault slab with cross walls (supporting deck construction) (Fig. 45).
(3) Arched vault slab with spandril walls and earth filling (Fig. 46).
(4) Combination of (1), narrow ribs, and (3), with thin vault (Fig. 47).

Any of the above types may be designed as either (*a*) Three-Hinged, (*b*) Two-Hinged, or (*c*) Fixed Arches, each being suitable for particular conditions.

The following solutions, therefore, will treat each of these kinds.

The One-Hinged Arch is unsuited to the majority of cases met with in practice, and is therefore not included.

These arches will be treated in the following order :—

(1) The Three-Hinged Arch, which is statically determinate and contains the greatest number of hinges or articulations possible for stability.
(2) The Hingeless Arch, from which will be derived the formulæ applicable to
(3) The Two-Hinged Arch.

53. Assumptions made in Design.—The solution of a statically indeterminate arch becomes somewhat involved and very laborious unless certain assumptions are made with regard to its form and condition of supports.

Such arches comprise all of those containing one or more statically indeterminate quantities essential for their solution, and embrace all arches possessing less than three definite articulations or hinges.

Subject to the assumptions mentioned above being made, the investigation of stresses taking place in an arched rib or vault can be made by analytical or graphical means in a straightforward manner.

Simplified methods as applied to three-hinged, two-hinged and fixed arches are fully explained in the following pages, and can be applied to the majority of cases met with in practice.

Where, owing to specified or natural conditions, the following assumptions cannot be admitted as approximately true, the investigation of stresses becomes more complicated, and requires for the

most part analytical treatment beyond the scope of this volume. For such cases the reader is referred to any of the works dealing with the mathematical treatment of elastic arches.

In addition to the assumptions regarding the form of arch and end conditions, there are the fundamental assumptions mentioned in Art. 3 essential to analytical or graphical calculations by the theory of elastic deformation, which can only be approximately true for a heterogeneous material such as reinforced concrete.

These latter assumptions, although somewhat inaccurate, do not appreciably affect the calculated results, and, it should be noted, are made in the design of other reinforced concrete structures in which the members are subjected to greater deformations than ever occur in an arch bridge.

Three types of arches, with the assumptions necessary for simplicity of treatment, will be considered as follows :—

(*a*) Three-Hinged Arch, being entirely solvable by statics. The rib will be assumed to be of parabolic form and symmetrical about a vertical centre line.

(*b*) Two-Hinged Arch, being of the first degree of static indetermination. The rib will be assumed to be of parabolic form and to be symmetrical about a vertical centre line. The supports will be considered as being mathematically fixed in regard to relative horizontal displacement, and the moment of inertia of any cross-section will be considered to vary proportionately to the secant of the angle formed by the tangent to the arch axis and the horizontal.

(*c*) The Hingeless Arch, being of the third degree of static indetermination. The rib will be assumed to be of parabolic form and to be symmetrical about a vertical centre line. The supports will be considered to be absolutely fixed and monolithic with the arch itself. The moment of inertia will be considered to vary proportionately to the secant of the angle formed by the tangent to the arch axis and the horizontal.

With these conditions it is now comparatively easy to investigate the stresses produced in the above types of arch under load.

The accuracy of the calculated results will approximate very closely with those actually produced in the work, if care is taken by the designer to ensure the above conditions being realised where these are arbitrary, and also to ascertain that they are reasonably true where they may not be under control.

54. Secondary Stresses.—Little definite information upon the subject of the elasticity or settlement of abutment supports is available, and the general assumption that such supports are mathematically rigid can only be approximately true, except where rock or similar foundation exists.

In addition to the stresses that may be set up by the above cause, little is known of the precise effects due to the shrinkage of concrete during setting and hardening or of the arch shortening under load. For these reasons it is expedient to adopt temporary hinges or other device (see Chapter IX.) for fixed or encastré arches having spans of 50 feet and upwards.

With the provision of temporary hinges, arched members are subject to static investigation for the whole or a portion of the dead or permanent loading, and the elastic theory only need be

FIG. 48.

employed for the rolling and superimposed distributed loads, which, it should be noted, are relatively small in all bridges excepting those of very moderate spans. Consequently any increase of stress that may be caused by slightly inaccurate assumptions made in the elastic theory is inappreciable and may be neglected.

The design of temporary hinges is dealt with in Art. 122. The design of an arched bridge in which these articulations are introduced is made in accordance with that given for the three-hinged arch for the loading coming upon the structure in that condition. For the loading added subsequently, which usually comprises the superimposed rolling loads, the method given for the hingeless arch is employed.

The greater known accuracy of calculations made possible by the introduction of temporary hinges results in the refinement in line and form so marked in many of the bridges in which they have been employed. For a number of large bridges, hydraulic jacks have been employed to nullify the effects of shrinkage. This device has several advantages, and is explained in Chapter IX., Arts. 128 and 129.

55. Statically Determinate and Indeterminate Arches.—Consider an arch ASB (Fig. 48) having a vertical unit load P situated at a distance a from support A. In a hingeless arch its ends are assumed to be rigidly fixed to the abutments, and consequently the deformation induced by the load P will cause bending moments to be produced at its springings.

The bending moments produced in a fixed arch by the load P can be solved statically when three unknown factors are determined. These may comprise the horizontal thrust common to both reactions, together with the vertical distances from A and B to its points of application.

Assuming this horizontal thrust to be applied at the points A and B, and the distances between these and the actual positions of H to be represented by q_a and q_b,

then $H.q_a$ and $H.q_b$ represent the fixing moments at A and B respectively.

The bending moment produced by the load P at the section S can now be expressed as follows :—

$$Ms = Vb.(l - x) - H\left\{y - q_b + \frac{l - x}{l}(q_b - q_a)\right\}$$

$$= Vb.(l - x) - H.y + H.q_b - H(q_b - q_a)\frac{l - x}{l}$$

$$= M_f - H.y + H.q_b - H(q_b - q_a)\frac{l - x}{l} \quad . \quad . \quad . \quad . \quad (1)$$

where M_f is the bending moment in a freely supported beam of span l.

It has already been stated that the moment at the arch section S is—

$$Ms = H.u,$$

and this formula can be identified with equation (1) as follows :—

From similar triangles

$$\frac{u + y - q_b + (q_b - q_a)\dfrac{l - x}{l}}{l - x} = \frac{Vb}{H}$$

$$\therefore \quad H.u + H\left\{y - q_b + (q_b - q_a)\frac{l - x}{l}\right\} = Vb.(l - x)$$

and $$\mathrm{H}.u = \mathrm{V}b\,(l - x) - \mathrm{H}\left\{y - q_b + (q_b - q_a)\frac{l - x}{l}\right\}$$

or $$\mathrm{H}.u = \mathrm{M}_f - \mathrm{H}.y + \mathrm{H}.q_b - \mathrm{H}\,(q_b - q_a)\frac{l - x}{l}$$

which proves the above formula.

V_1 and V_2 are the true reactions at A and B, these being

$$\mathrm{V}_1 = \mathrm{P}\left(\frac{l - a}{l}\right) - \frac{\mathrm{H}.q_a}{l} + \frac{\mathrm{H}.q_b}{l}$$

and $$\mathrm{V}_2 = \mathrm{P}.\frac{a}{l} + \frac{\mathrm{H}.q_a}{l} - \frac{\mathrm{H}.q_b}{l}$$

If hinges be introduced at A and B the moments $H.q_a$ and $H.q_b$ disappear, and the bending moment at any section S becomes

$$\begin{aligned} Ms &= \mathrm{V}b\,(l - x) - \mathrm{H}.y \\ &= \mathrm{M}_f - \mathrm{H}.y \quad . \quad . \quad . \quad . \quad . \quad . \quad (2) \end{aligned}$$

In this case it will be seen that there is only one unknown factor which cannot be ascertained statically. This may be the direction of either reaction or the magnitude of the horizontal thrust. This type is known as the *Two-Hinged Arch.*

If an additional hinge is placed at the crown of the arch, the directions of both reactions are known, and the bending moments and other functions of the loading can be statically determined. The horizontal and vertical components of the reactions R_a and R_b (Fig. 48) can be drawn in the ordinary way by the triangle of forces. The bending moment at any section is then

$$\begin{aligned} Ms &= \mathrm{V}b\,(l - x) - \mathrm{H}.y \\ &= \mathrm{M}_f - \mathrm{H}.y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3) \end{aligned}$$

56. Three-Hinged Arch.—Considering the case of a unit load on an arch, simple formulæ can be found for the bending moments at any section in the arch, and from these formulæ influence lines can be set up.

Referring to Fig. 49, let ACB be an arch carrying a unit load P, distance a from one end. To find the bending moment at any section S on the arch axis, the moments of all the external forces one side of the section are considered. Let y be the distance of the section above the arch chord AB.

(*a*) When the load is between support A and the section considered

$$\begin{aligned} Ms &= \mathrm{V}b.(l - x) - \mathrm{H}.y \\ &= \frac{\mathrm{P}.a}{l}(l - x) - \mathrm{H}.y \quad . \quad . \quad . \quad . \quad . \quad (4\mathrm{A}) \end{aligned}$$

(*b*) When the load is between section S and support B

$$Ms = Va.x - H.y$$

$$= P\frac{(l-a)}{l}.x - H.y \quad . \quad . \quad . \quad . \qquad (4B)$$

The formula for the horizontal thrust may be obtained by con sidering the moment at the centre (section C).

The hinges ensure that the line of pressure (or the inclined reactions for a single load) must pass through them, and therefore at the hinges there can be no moment.

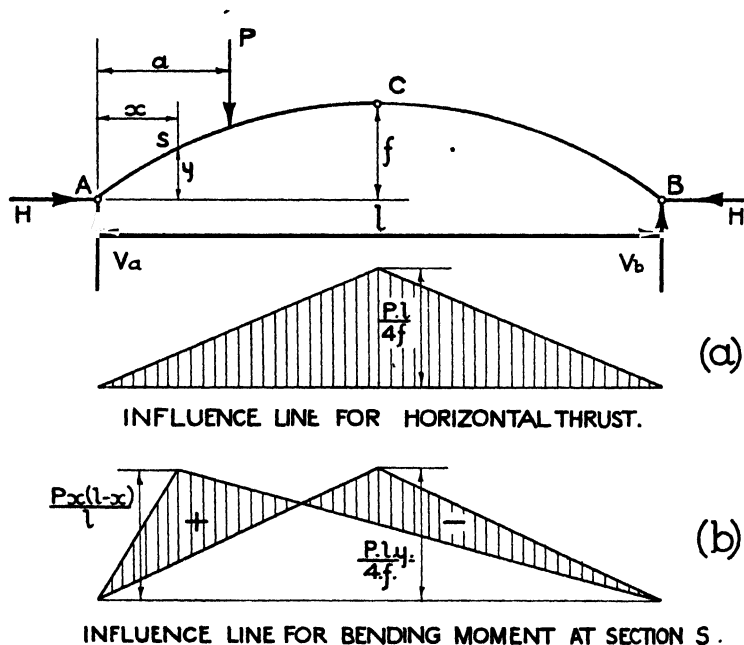



Fig. 49.

Consequently at the centre hinge Mc = 0, or from equation (4A) putting $x = \frac{l}{2}$ and $y = f$,

$$Ms = P.\frac{a}{l}.\frac{l}{2} - H.f = 0$$

$$= P.\frac{a}{2} - H.f = 0$$

$$\therefore \quad H = \frac{P.a}{2f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5A)$$

which is the formula for the horizontal thrust when $a < \frac{l}{2}$, and from equation (4B), putting $x = \frac{l}{2}$ and $y = f$,

$$Ms = P.\left(\frac{l-a}{l}\right).\frac{l}{2} - H.f = 0$$

$$= P.\left(\frac{l-a}{2}\right) \qquad - H.f = 0.$$

$$\therefore \qquad H = P.\left(\frac{l-a}{2f}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5B)$$

which is the formula for the horizontal thrust when $a > \frac{l}{2}$

57. Influence Line for Horizontal Thrust.—By putting values of a in equations (5A) and (5B) an influence line curve giving the horizontal thrust for any position of the point load (Fig. 49(a)) may be constructed.

TABLE C.

Position of load $\frac{a}{l}$ =	0	0·1	0·2	0·25	0·3	0·4	0·5	0·6	etc.
H (in terms of $\frac{P.l}{f}$) =	0	0·05	0·1	0·125	0·15	0·2	0·25	0·2	

The influence line is symmetrical, having a maximum ordinate in the centre equal to $\frac{P.l}{4f}$

58. Influence Line for Bending Moments.—The first terms in equations (4A) and (4B) are the moments in a freely supported beam of the same span. The influence lines for this are explained in Chapter IV.

To construct the influence line for the moment at S, draw the influence line curve for the bending moment on a freely supported beam having a maximum ordinate at section S of $\frac{P.x}{l}(l-x)$

Upon it superimpose the influence line for the horizontal thrust multiplied by the arch ordinate y at the section S. The algebraic sum of these two curves gives the influence line for the bending moment in the arch.

This is shown shaded in Fig. 49(b). As in beams, positive bending moments are taken as producing compression on upper face.

59. Uniform Load on Half-span of Three-Hinged Arch.—The bending moment at section S, distance x from support A (Fig. 50), in a freely supported beam of the same span, is

$$\text{Loaded half} \quad M = \frac{3}{8}p.l.x - \frac{p.x^2}{2}$$

$$= \frac{p.x}{8}(3l - 4x) \text{ when } x < \frac{l}{2}$$

$$\text{Unloaded half} \quad M = \frac{1}{8}p.l\,(l - x) \text{ when } x > \frac{l}{2}$$

The free moment when $x = \frac{l}{2}$ is

$$Mc = \frac{1}{16}p.l^2$$

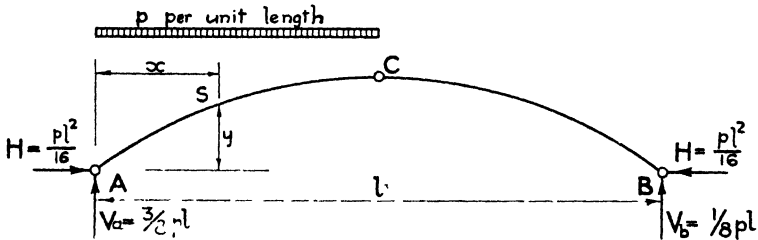



FIG. 50.

The bending moment in the arch at the crown $= Mc - H.f = 0$

$$\therefore \text{ the horizontal thrust } H = \frac{Mc}{f} = \frac{p.l^2}{16f}$$

In the *loaded half* $\left(\text{when } x < \frac{l}{2}\right)$ the bending moment at any section S in the arch is therefore

$$Ms = \frac{p.x}{8}(3l - 4x) - \frac{p.l^2}{16f}.y \quad . \quad . \quad . \quad . \quad (6\text{A})$$

and in the *unloaded half* $\left(\text{when } x > \frac{l}{2}\right)$

$$Ms = \frac{p.l}{8}(l - x) - \frac{p.l^2}{16f}.y \quad . \quad . \quad . \quad . \quad (6\text{B})$$

60. Parabolic Three-Hinged Arches.—The most economical curve of an arch and that requiring the minimum thickness is one whose axis coincides as nearly as possible with the mean of the lines

of resistance for the dead load and that for the worst positions of the live loads.

In practice, however, it is found that the principal load on an arch of moderate and large spans is the dead or permanent load, and the line of resistance for this condition is therefore usually adopted for the arch axis. For moderate spans the distribution of this load can be taken as approximately uniform.

The bending moment in any arch section, distance x from one support, is given by formula (3), Art. 55, and is

$$Ms = M_f - H.y$$

The free bending moment at any distance x when the loading is uniformly distributed is

$$\frac{p}{2}(l.x - x^2)$$

H can be found by equating the arch moment in the centre to zero;

$$\therefore \qquad Mc = \frac{p.l^2}{8} - H.f = 0$$

and

$$H = \frac{p.l^2}{8f}$$

It follows that under a uniformly distributed load on the arch, in order that there are no bending moments in the arch, the general formula for the moment must equal zero;

i.e.,

$$\frac{p}{2}(l.x - x^2) - \frac{p.l^2}{8f}.y = 0$$

and

$$y = \frac{4.f}{l^2}(l.x - x^2) \quad \ldots\ldots \quad (7)$$

which is the equation to a parabola of the second degree.

Thus in a three-hinged parabolic arch the line of resistance for a uniform load over the entire span falls on the axis of the arch and causes no bending moments or shearing forces to be developed in any section of the arch.

The only stress induced in the arch is uniform compression throughout its length.

Even in cases of spandril filled arches, provided the proportions and the spans are not unusual, the resistance line for the dead load approximates to a parabolic curve of the second degree.

This curve, in such cases, may therefore be adopted for the arch axis.

The following analysis gives formulæ for the moments on a para-

bolic arch produced by a single point load, and also for a uniform load covering one-half of the span.

Referring to Fig. 49,

As before, let a be the distance of the load and x be the distance of the section from the left-hand support.

The height of the arch axis above the chord AB of the arch is given by formula (7):

$$y = \frac{4f}{l^2}(l.x - x^2), \text{ as proved above.}$$

When load is between A and S.

From equations (4A) and (5A)

$$\text{M}s = \text{P}.\frac{a}{l}.(l - x) - \frac{\text{P}.a}{2f}.\frac{4f}{l^2}.x.(l - x)$$

$$= \text{P}.\frac{a}{l}.(l - x).\left(1 - \frac{2x}{l}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (8\text{A})$$

When load is between S and C.

From equations (4B) and (5A)

$$\text{M}s = \text{P}.x.\frac{(l - a)}{l} - \frac{\text{P}.a}{2f}.\frac{4f}{l^2}.x.(l - x)$$

$$= \text{P}.\frac{x}{l}\left[(l - a) - \frac{2a}{l}(l - x)\right] \quad . \quad . \quad . \quad . \quad (8\text{B})$$

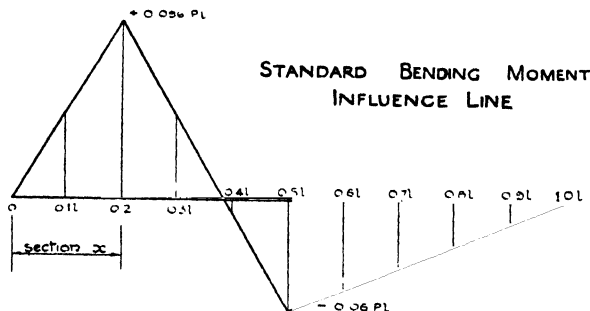



FIG. 51.

When load is between C and B.

From equations (4B) and (5B)

$$\text{M}s = \text{P}.\frac{(l - a)}{l}.x - \text{P}.\frac{(l - a)}{2f}.\frac{4f}{l^2}x.(l - x)$$

$$\text{M}s = \text{P}.\frac{(l - a)}{l}.x.\left[1 - \frac{2}{l}(l - x)\right] \quad . \quad . \quad . \quad . \quad (8\text{C})$$

From these results Table D has been calculated. This table

TABLE D

POSITION OF LOAD

POSITION OF SECTION	$\frac{a}{l}=0$	$\frac{a}{l}=0.1$	$\frac{a}{l}=0.2$	$\frac{a}{l}=0.25$	$\frac{a}{l}=0.3$	$\frac{a}{l}=0.4$	$\frac{a}{l}=0.5$	$\frac{a}{l}=0.6$	$\frac{a}{l}=0.7$	$\frac{a}{l}=0.75$	$\frac{a}{l}=0.8$	$\frac{a}{l}=0.9$	$\frac{a}{l}=1$
$\frac{x}{l}=0$	0	0	0	0	0	0	0	0	0	0	0	0	0
=0·1	0	·072	·044	·03	·016	—·012	—·04	—·032	—·024	—·02	—·016	—·008	0
=0·2	0	·048	·096	·09	·044	—·008	—·06	—·048	—·036	—·03	—·024	—·012	0
=0·25	0	·0375	·075	·094	·0625	0	—·0625	—·05	—·0375	—·0313	—·025	—·0125	0
=0·3	0	·028	·056	·07	·084	·012	—·06	—·048	—·036	—·03	—·024	—·012	0
=·04	0	·012	·024	·03	·036	·048	—·04	—·032	—·024	—·02	—·016	—·008	0
=0·5	0	0	0	0	0	0	0	0	0	0	0	0	0

Ordinates to Standard Influence Lines for Bending Moments in a Parabolic Three-hinged Arch.

Moments in terms of P.*l*.

Note.—The ordinates to the left of the heavy black line are positive and to the right negative.

gives the ordinates to the influence lines for values of $\frac{a}{l}$ and $\frac{x}{l}$ in terms of $P.l$, and from it standard influence line curves can be plotted for sections $x = 0{\cdot}1l,\ 0{\cdot}2l,\ 0{\cdot}25l,\ 0{\cdot}3l$ and $0{\cdot}4l$.

The bending moment influence line for section $x = 0{\cdot}2l$ is given in Fig. 51.

61. Uniform Load over Half-span of Parabolic Three-Hinged Arch.—As before, formula (7) gives the height of the arch axis above the springings.

$$y = \frac{4f}{l^2}(l.x - x^2)$$

Formula (6A) becomes when $x < \frac{l}{2}$

$$Ms = \frac{p.x}{8}(l - 2x). \qquad (9\text{A})$$

Formula (6B) becomes when $x > \frac{l}{2}$

$$Ms = \frac{p.(l - x)}{8}.(l - 2x) \qquad (9\text{B})$$

Table E gives the moments in progressive sections of the arch in terms of $p.l^2$.

TABLE E

	Loaded Half.							Unloaded Half.					
$\frac{x}{l}$	0	0·1	0·2	0·25	0·3	0·4	0·5	0·6	0·7	0·75	0·8	0·9	1
M	0	·01	·015	·0156	·015	·01	0	·01	·015	·0156	·015	− ·01	0

62. Linear Variations of a Three-Hinged Arch.—In a three-hinged arch the expansion and contraction of the material under variations of temperature, and the latter for shrinkage and arch shortening under thrust, do not produce any bending moments or thrusts. (See Art. 25.)

63. Hingeless Arches.—The type of arch most commonly used at the present time is the hingeless or encastré arch, where the ends are rigidly connected to the abutments. There are two methods of investigating the forces acting upon the arch sections of this type.

One method is to draw lines of resistance for the dead load in

combination with various positions of the live load, and to ascertain that these lie within the middle third of all sections. This method is approximate, and does not determine the true stresses in a concrete arch, although it may be regarded as a satisfactory test of the stability of mass concrete or masonry arches in which tension is undesirable.

This method, however, is not suitable for adoption in the case of modern reinforced concrete arch bridges where the rise span ratio may be relatively small.

A suitable solution, and the one now universally adopted for all monolithic arches, is based on the elastic deformation of the arch, and is known as the elastic theory. This is the most accurate method known, and the results given by applying the principle of elastic deformation agree closely with those that have been observed from time to time in actual structures.

Applied to the solution of the hingeless arch, however, the elastic theory demands the presumption of three conditions, which may not be exactly realised in the structure. The results for the hingeless type may therefore not be so accurate as for the static three-hinged arch or as those given by the elastic theory for the two-hinged arch, for which it is necessary to make only one of the above assumptions.

64. The Elastic Theory.—The three unknowns which have to be evaluated before the problem can be solved by means of the principles of statics consist of the horizontal thrust and the bending moment at each support.

The three presumed conditions mentioned above can be summarised thus :—

The effect of the total strains in the arch produced by a variable bending moment along the arch axis must be zero. Taken separately, the conditions—depending upon the rigidity of the abutments for their accuracy—are :

(*a*) The span of the arch remains unchanged.

(*b*) The levels of the supports A and B remain unchanged.

(*c*) The slope of the tangents to the arch axis at both supports must remain unchanged.

It can be proved that if one end of an arch subjected to bending were free to move the horizontal displacement of the end of the arch is represented by the formula $\frac{M.y}{E.I}.ds$, and, in the same way, that the vertical and angular displacements are represented by the expressions $\frac{M.x}{E.I}.ds$ and $\frac{M}{E.I}.ds$ respectively.

The above three conditions can now be expressed mathematically :—

$$(a) \int_A^B \frac{M.y}{E.I}.ds = 0 \quad \ldots \quad (10)$$

$$(b) \int_A^B \frac{M.x}{E.I}.ds = 0 \quad \ldots \quad (11)$$

$$(c) \int_A^B \frac{M}{E.I}.ds = 0 \quad \ldots \quad (12)$$

where M is the bending moment at any section in the arch;
- y is any vertical ordinate of the arch axis in respect to any origin of co-ordinates;
- x is the corresponding horizontal distance of the section in respect to the same origin;
- E is the coefficient of elasticity of the material;
- I is the moment of inertia of the cross section;
- ds is an element of the arch axis.

FIG. 52.

FIG. 53. FIG. 54.

The condition (*a*) expressed in equation (10) does not take into account the relatively small amount of elastic shortening of the arch span due to the horizontal thrust from the abutments. In order, therefore, to correct this equation it is necessary to add algebraically the cumulative deformation of each section of the

arch produced by a variable thrust N acting normally to the sections of the arch.

In Fig. 53 let α be the angle which the tangent to the curve of the arch makes with the horizontal. Therefore N, the normal thrust on the section, acts at an inclination α to the horizontal.

Now this thrust produces a uniform strain over the cross section (of area A) equal to $\frac{N}{A.E}$, and thus an element of the arch axis, whose length is ds, undergoes a shortening $\frac{N}{A.E}.ds$

The total strain along the arch axis is therefore $\int_A^B \frac{N}{A.E}.ds$

The normal thrust N can be resolved into a horizontal thrust H and a vertical force V (see Fig. 54), where V is the vertical shearing force, from which $N = H.\cos\alpha + V.\sin\alpha$.

∴ Total strain or shortening of arch axis =

$$\int_A^B \frac{(H.\cos\alpha + V.\sin\alpha)}{A.E}.ds$$

Now the summation of the vertical shearing force at all sections along the arch axis must equal zero for equilibrium.

$$\therefore \text{ Shortening of arch axis} = \int_A^B \frac{H.\cos\alpha}{A.E}.ds$$

$$\begin{array}{l}\text{and shortening of the span} = \\ \quad (\text{since } dx = ds\cos\alpha)\end{array} \int_A^B \frac{H}{A.E}.dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

This deformation is opposite in sign to the deformation produced by bending, and therefore the expression given in (13) must be subtracted from that given in equation (10);

Total horizontal displacement = 0,

$$\text{or} \qquad \int_A^B \frac{M.y}{E.I}.ds - \int_A^B \frac{H}{A.E}.dx = 0 \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Now the bending moment at any section of the arch is equal to

the moments of all the external forces on either side of it, and can be expressed by

$$\therefore \quad Ms = M_f - H.y + H.q_b - H.(q_b - q_a).\frac{(l-x)}{l} \quad . \quad (15)$$

See equation (1), Art. 55.

Substituting this expression for M*s* in equations (14), (11) and (12),

$$\int_A^B \frac{M.y}{E.I}.ds - \int_A^B \frac{H}{A.E}.dx = \int_A^B \frac{M_f.y}{E.I}.ds - H\int_A^B \frac{y^2}{E.I}.ds$$

$$+ H.q_b\int_A^B \frac{y}{E.I}.ds - H(q_b - q_a)\int_A^B \frac{(l-x).y}{l.E.I}.ds - H\int_A^B \frac{dx}{A.E} \quad . \quad (16)$$

$$\int_A^B \frac{M.x}{E.I}.ds = \int_A^B \frac{M_f.x}{E.I}.ds - H\int_A^B \frac{x.y}{E.I}.ds$$

$$+ H.q_b\int_A^B \frac{x}{E.I}.ds - H(q_b - q_a)\int_A^B \frac{(l.x - x^2)}{l.E.I}.ds \quad . \quad (17)$$

$$\int_A^B \frac{M}{E.I}.ds = \int_A^B \frac{M_f}{E.I}.ds - H\int_A^B \frac{y}{E.I}.ds$$

$$+ H.q_b\int_A^B \frac{ds}{E.I} - H(q_b - q_a)\int_A^B \frac{(l-x)}{l.E.I}.ds \quad . \quad . \quad . \quad (18)$$

65. Parabolic Hingeless Arch.—The following assumptions are now made to simplify the solution of the general equations (16), (17) and (18) :—

(*a*) That the axis of the arch is a parabola of the second degree;
(*b*) That the moment of inertia varies proportionately as the secant of the angle which the arch axis makes with the horizontal, *i.e.*, $I = Ic \sec a$, where I*c* is the moment of inertia at the crown.
(*c*) E is constant and disappears from the equations.

Assumption (*b*) enables the expression $\frac{ds}{I}$ to be in the form $\frac{dx}{Ic}$ (since $ds = dx.\sec a$).

From the above, equations (16), (17) and (18) can now be written.—

$$\int_A^B M.y.dx. - H.Ic\int_A^B \frac{dx}{A} = \int_A^B M_f.y.dx - H\int_A^B y^2.dx$$

$$+ H.q_b\int_A^B y.dx - H.(q_b - q_a)\int_A^B \frac{(l-x)}{l}.y.dx - H.Ic\int_A^B \frac{dx}{A} \quad . \quad (19)$$

$$\int_A^B M.x.dx = \int_A^B M_f.x.dx - H\int_A^B x.y.dx$$

$$+ H.q_b\int_A^B x.dx. - H.(q_b - q_a)\int_A^B \frac{(l.x - x^2)}{l}.dx \quad . \quad (20)$$

$$\int_A^B M.dx = \int_A^B M_f.dx - H\int_A^B y.dx$$

$$+ H.q_b\int_A^B dx - H(q_b - q_a)\int_A^B \frac{(l-x)}{l}.dx \quad . \quad (21)$$

The solution of these three equations will give the value of the unknown quantities H, and the bending moment at each support, where $Ma = H.q_a$ and $Mb = H.q_b$

66. Influence Line for Horizontal Thrust.—Let Fig. 55 represent a parabolic arch of span l and having a rise f. The equation for the ordinate at any distance x from the left-hand support is

See equation (7) $$y = \frac{4.f}{l^2}(l.x - x^2)$$

It is required to find the magnitude of the horizontal thrust produced by a load P situated at a distance a from support A.

When the load is in this position, the bending moment in a freely supported beam at any section x would be

(a) when $x < a$ $\quad M_f = P(l - a).\frac{x}{l}$

(b) when $x > a$ $\quad M_f = P(l - x).\frac{a}{l}$

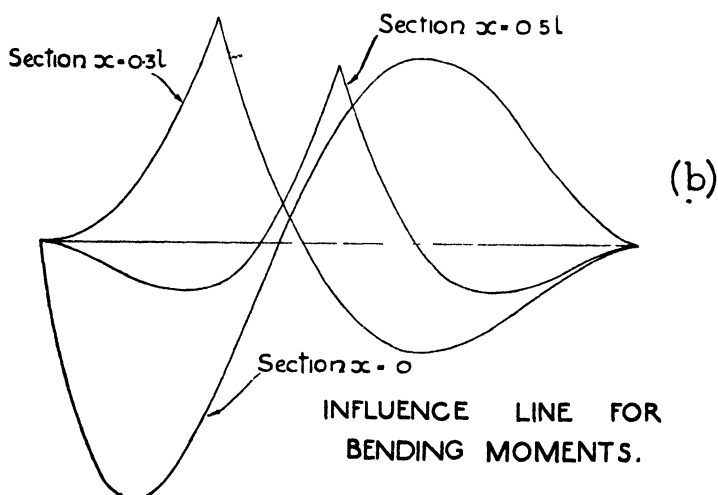
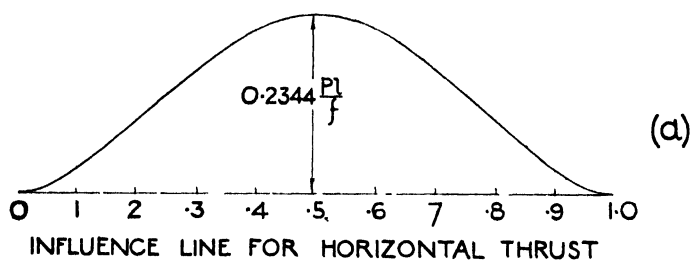
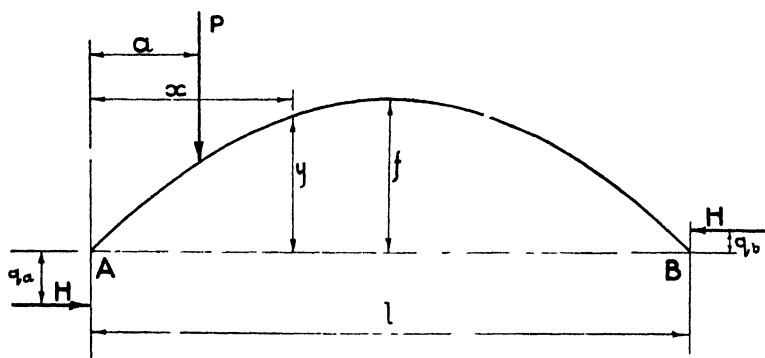



FIG. 55.—Influence Lines for Horizontal Thrust and Bending Moments. (Hingeless Arch.)

Introducing these values into equations (19), (20) and (21), the integrations of the separate terms produce the following results :—

$$\int_A^B M_f . y . dx = \frac{P . f}{3l^2} (a^4 + a . l^3 - 2a^3 . l)$$

$$\int_A^B M_f . x . dx = \frac{P . a}{6} (l^2 - a^2)$$

$$\int_A^B M_f . dx = \frac{P . a}{2} (l - a)$$

$$\int_A^B y^2 . dx = \frac{8}{15} f^2 . l$$

$$\int_A^B x . y . dx = \frac{f . l^2}{3}$$

$$\int_A^B y . dx = \frac{2}{3} f . l \qquad (22)$$

$$\int_A^B x . dx = \frac{l^2}{2}$$

$$\int_A^B \frac{(l - x)}{l} . y . dx = \frac{f . l}{3}$$

$$\int_A^B \frac{(l . x - x^2)}{l} . dx = \frac{l^2}{6}$$

$$\int_A^B \frac{(l - x)}{l} . dx = \frac{l}{2}$$

$$\int_A^B \frac{dx}{A} = \frac{l}{Av}$$

where Av is the average cross section of the arch.

Equations (19), (20) and (21) thus become :

$$\int_A^B M.y.dx - H.Ic\int_A^B \frac{dx}{A} = \frac{P.f}{3l^2}.(a^4 + a.l^3 - 2a^3.l) - H.\frac{8}{15}f^2.l +$$

$$H.q_b.\frac{2}{3}f.l - H.(q_b - q_a).\frac{1}{3}f.l - H.Ic.\frac{l}{Av} = 0 \quad . \quad (23)$$

$$\int_A^B M.x.dx = \frac{P.a}{6}.(l^2 - a^2) - H.\frac{f.l^2}{3} + H.q_b.\frac{l^2}{2} -$$

$$H.(q_b - q_a).\frac{l^2}{6} = 0 \quad . \quad . \quad . \quad (24)$$

$$\int_A^B M.dx = \frac{P.a}{2}(l - a) - H.\frac{2}{3}f.l + H.q_b.l -$$

$$H(q_b - q_a).\frac{l}{2} = 0 \quad . \quad . \quad . \quad . \quad (25)$$

By combining equations (24) and (25) the moments at the supports can be found :—

$$H.(q_b - q_a) = P.a.\left(1 + \frac{2a^2}{l^2} - \frac{3a}{l}\right) \quad . \quad . \quad . \quad . \quad . \quad (26)$$

$$H.q_a = P.a.\left(\frac{2a}{l} - \frac{a^2}{l^2} - 1\right) + H.\frac{2}{3}f. \quad . \quad . \quad (27)$$

$$H.q_b = P.a.\left(\frac{a^2}{l^2} - \frac{a}{l}\right) + H.\frac{2}{3}f. \quad . \quad . \quad . \quad . \quad (28)$$

and, finally, by introducing equations (26) and (28) into equation (23) the formula for the horizontal thrust is :—

$$H = \frac{15\,P}{4f.l^3}\left(a^4 - 2a^3.l + a^2.l^2\right) \frac{1}{1 + \frac{45\,Ic}{4\,Av.f^2}} \quad . \quad . \quad (29)$$

or $$H = \frac{15P}{4f.l^3.K}\left(a^4 - 2a^3.l + a^2.l^2\right) \quad . \quad . \quad . \quad . \quad . \quad (29A)$$

where $K = 1 + \frac{45\,Ic}{4\,Av.f^2}$.

To set up the standard influence line for the horizontal thrust, appropriate values of $\frac{a}{l}$ are introduced into formula (29A). These results are given in Table F.

TABLE F

Position of load $\frac{a}{l}$ =	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	0
Horizontal thrust $\frac{P.l}{f.K}$	0	0·0304	0·096	0·1654	0·216	0·2344	0·216	0·1654	0·096	0·0304	0

Horizontal Thrust for Parabolic Hingeless Arch in Terms of $\frac{P.l}{f.K}$

This curve is plotted in Fig. 55a.

67. Influence Lines for Bending Moments.—The formula for the bending moment at any section distance x produced by a load P at distance a from support A can easily be derived by inserting the formulæ (26), (28) and (29A) into equation (15),

$$i.e., \quad Ms = M_f - H.y + H.q_b - H.(q_b - q_a)\frac{(l - x)}{l}$$

When the section x is between support A and the load, *i.e.*, when $x < a$,

$$Ms = P\left(x - a + \frac{2a^2}{l} - \frac{a^3}{l^2} + \frac{2a^3.x}{l^3} - \frac{3a^2.x}{l^2}\right) - H.f.\left(\frac{4x}{l} - \frac{4x^2}{l^2} - \frac{2}{3}\right) \quad . \quad . \quad . \quad . \quad (30A)$$

and when the section x is between the load and support B, *i.e.*, when $x > a$,

$$Ms = P\left(\frac{2a^2}{l} - \frac{a^3}{l^2} + \frac{2a^3.x}{l^3} - \frac{3a^2.x}{l^2}\right) - H.f.\left(\frac{4x}{l} - \frac{4x^2}{l^2} - \frac{2}{3}\right) \quad . \quad . \quad . \quad . \quad (30B)$$

It should be noted that the first portions of formulæ (30A) and (30B) give the ordinates to the influence line for the bending moments in a beam having fixed ends.

The latter portions of formulæ (30A) and (30B) contain the constant K, which depends on the sectional quantities of the arch. As explained in Chapter IX., this constant can be assumed to equal unity where temporary hinges are employed.

Table G gives the bending moment influence line ordinates for eleven equally spaced sections. These ordinates are given in a form so that the constant value K can, where necessary, be evaluated and the numerical value of the ordinates for any specific case calculated. When these ordinates have been computed the resultant curves can be readily plotted in the usual manner.

By making the assumption that K = 1 it is possible to set

up standard bending moment influence lines which are then applicable to all parabolic hingeless arches having the characteristics mentioned in Art. 65 (see Table H).

Table G

POSITION OF SECTION

POSITION OF LOAD	$\frac{x}{l}=0$	$\frac{x}{l}=0{\cdot}1$	$\frac{x}{l}=0{\cdot}2$	$\frac{x}{l}=0{\cdot}3$	$\frac{x}{l}=0{\cdot}4$	$\frac{x}{l}=0{\cdot}5$
$\frac{a}{l}=0$	0	0	0	0	0	0
$=0{\cdot}1$	$-{\cdot}081+\frac{{\cdot}0203}{K}$	$+{\cdot}0162+\frac{{\cdot}0093}{K}$	$+{\cdot}0134+\frac{{\cdot}0008}{K}$	$+{\cdot}0106-\frac{{\cdot}0053}{K}$	$+{\cdot}0078-\frac{{\cdot}0089}{K}$	$+{\cdot}005-\frac{{\cdot}0101}{K}$
$=0{\cdot}2$	$-{\cdot}128+\frac{{\cdot}064}{K}$	$-{\cdot}0384+\frac{{\cdot}0294}{K}$	$+{\cdot}0512+\frac{{\cdot}0026}{K}$	$+{\cdot}0408-\frac{{\cdot}0166}{K}$	$+{\cdot}0304-\frac{{\cdot}0282}{K}$	$+{\cdot}02-\frac{{\cdot}032}{K}$
$=0{\cdot}3$	$-{\cdot}147+\frac{{\cdot}1103}{K}$	$-{\cdot}0686+\frac{{\cdot}0507}{K}$	$+{\cdot}0098+\frac{{\cdot}0044}{K}$	$+{\cdot}0882-\frac{{\cdot}0287}{K}$	$+{\cdot}0666-\frac{{\cdot}0485}{K}$	$+{\cdot}045-\frac{{\cdot}0551}{K}$
$=0{\cdot}4$	$-{\cdot}144+\frac{{\cdot}144}{K}$	$-{\cdot}0792+\frac{{\cdot}0663}{K}$	$-{\cdot}0144+\frac{{\cdot}0058}{K}$	$+{\cdot}0504-\frac{{\cdot}0374}{K}$	$+{\cdot}1152-\frac{{\cdot}0634}{K}$	$+{\cdot}08-\frac{{\cdot}072}{K}$
$=0{\cdot}5$	$-{\cdot}125+\frac{{\cdot}1563}{K}$	$-{\cdot}075+\frac{{\cdot}0719}{K}$	$-{\cdot}025+\frac{{\cdot}0063}{K}$	$+{\cdot}025-\frac{{\cdot}0406}{K}$	$+{\cdot}075-\frac{{\cdot}0688}{K}$	$+{\cdot}125-\frac{{\cdot}0781}{K}$
$=0{\cdot}6$	$-{\cdot}096+\frac{{\cdot}144}{K}$	$-{\cdot}0608+\frac{{\cdot}0663}{K}$	$-{\cdot}0256+\frac{{\cdot}0058}{K}$	$+{\cdot}0096-\frac{{\cdot}0374}{K}$	$+{\cdot}0448-\frac{{\cdot}0634}{K}$	$+{\cdot}08-\frac{{\cdot}072}{K}$
$=0{\cdot}7$	$-{\cdot}063+\frac{1103}{K}$	$-{\cdot}0414+\frac{{\cdot}0507}{K}$	$-{\cdot}0198+\frac{{\cdot}0044}{K}$	$+{\cdot}0018-\frac{{\cdot}0287}{K}$	$+{\cdot}0234-\frac{{\cdot}0485}{K}$	$+{\cdot}045-\frac{{\cdot}0551}{K}$
$=0{\cdot}8$	$-{\cdot}032+\frac{{\cdot}064}{K}$	$-{\cdot}0216+\frac{{\cdot}0294}{K}$	$-{\cdot}0112+\frac{{\cdot}0026}{K}$	$-{\cdot}0008-\frac{{\cdot}0166}{K}$	$+{\cdot}0096-\frac{{\cdot}0282}{K}$	$+{\cdot}02-\frac{{\cdot}032}{K}$
$=0{\cdot}9$	$-{\cdot}009+\frac{{\cdot}0203}{K}$	$-{\cdot}0062+\frac{{\cdot}0093}{K}$	$-{\cdot}0034+\frac{{\cdot}0008}{K}$	$-{\cdot}0006-\frac{{\cdot}0053}{K}$	$+{\cdot}0022-\frac{{\cdot}0089}{K}$	$+{\cdot}005-\frac{{\cdot}0101}{K}$
$=1{\cdot}0$	0	0	0	0	0	0

Hingeless Arch Bending Moment Influence Line Ordinates (in Terms of P.*l) at Section distance x when Load is at distance a from Support.*

Table H

POSITION OF LOAD

POSITION OF SECTION	$\frac{a}{l}=0$	$\frac{a}{l}=0{\cdot}1$	$\frac{a}{l}=0{\cdot}2$	$\frac{a}{l}=0{\cdot}3$	$\frac{a}{l}=0{\cdot}4$	$\frac{a}{l}=0{\cdot}5$	$\frac{a}{l}=0{\cdot}6$	$\frac{a}{l}=0{\cdot}7$	$\frac{a}{l}=0{\cdot}8$	$\frac{a}{l}=0{\cdot}9$	$\frac{a}{l}=1{\cdot}0$
$\frac{x}{l}=0$	0	−·0607	−·064	−·0367	0	+·0313	+·048	+·0473	+·032	+·0113	0
$=0{\cdot}1$	0	+·0255	−·009	−·0179	−·013	−·0031	+·0055	+·0093	+·0078	+·0031	0
$=0{\cdot}2$	0	+·0142	+·0538	+·0142	−·0086	−·0188	−·0198	−·0154	−·0086	−·0026	0
$=0{\cdot}3$	0	+·0053	+·0242	+·0595	+·013	−·0156	−·0278	−·0269	−·0174	−·0059	0
$=0{\cdot}4$	0	−·0011	+·0022	+·0181	+·0518	+·0062	−·0186	−·0251	−·0186	−·0067	0
$=0{\cdot}5$	0	−·0051	−·012	−·0101	+·008	+·0469	+·008	−·0101	−·012	−·0051	0

Ordinates to Standard Influence Lines for Bending Moments in a Parabolic Hingeless Arch.

Moments in terms of P.*l*.

The influence line curves for $\frac{x}{l} = 0, \frac{x}{l} = 0{\cdot}3$ and $\frac{x}{l} = 0{\cdot}5$ are given in Fig. 55b.

68. Influence Line for Vertical Reaction at Supports.—To find the reaction at support A take moments about B (Fig. 56).

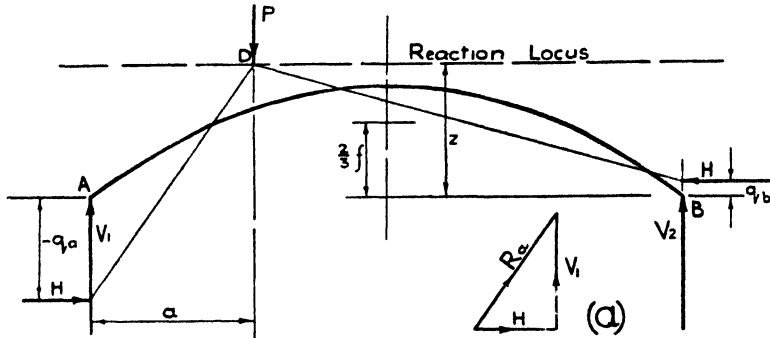



Fig. 56.

Then
$$V_1.l = P\,(l - a) - H.q_a + H.q_b.$$
$$V_1 = P.\frac{(l-a)}{l} + \frac{H.}{l}\,(q_b - q_a),$$

and with the aid of formula (26),
$$V_1 = \frac{P}{l}\,(l - a) + \frac{P.a}{l}\left(1 + \frac{2a^2}{l^2} - \frac{3a}{l}\right)$$
$$= P\left(1 + \frac{2a^3}{l^3} - \frac{3a^2}{l^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (31\text{A})$$
$$V_2 = (P - V_1)$$
$$= P.\,\frac{a^2}{l^2}\left(3 - 2\frac{a}{l}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31\text{B})$$

Table J given below enables the standard influence line for the vertical reaction at support A to be drawn.

TABLE J

Position of load	$\frac{a}{l} = 0$	$= 0{\cdot}1$	$= 0{\cdot}2$	$= 0\ 3$	$= 0\ 4$	$= 0\ 5$	$= 0{\cdot}6$	$= 0{\cdot}7$	$= 0{\cdot}8$	$= 0\ 9$	$= 1{\cdot}0$
V_1 in terms of P	1	·972	·896	·784	·648	·5	·352	·216	·104	·028	0

Influence Line for Vertical Support Reaction of a Parabolic Hingeless Arch.

69. Line of Reaction Intersections.—Considering a single point load on a parabolic arch, as shown in Fig. 56, it will be seen that the inclined reactions at the supports intersect on the load line at a height z above the springings. It is desired to calculate the height z for varying positions of the load, or, in other words, to find the position of the reaction locus. As before, the load is situated at distance a from the left-hand end. The inclined reaction at A can be resolved vertically and horizontally, as shown by the triangle of forces (Fig. 56a).

By similar triangles

$$\frac{z - q_a}{a} = \frac{V_1}{H}$$

$$\therefore \quad z = \frac{V_1 . a}{H} + q_a.$$

Introducing the formulæ for q_a, H, and V_1, given in formulæ (27), (29A) and (31A),

$$z = \frac{P.a}{H}\left(1 + \frac{2a^3}{l^3} - \frac{3a^2}{l^2}\right) + \frac{P.a}{H}\left(\frac{2a}{l} - \frac{a^2}{l^2} - 1\right) + \frac{2}{3}f$$

$$= \frac{P\left(\frac{2a^4}{l^3} - \frac{4a^3}{l^2} + \frac{2a^2}{l}\right)}{\frac{15P}{4f.K}\left(\frac{a^4}{l^3} - \frac{2a^3}{l^2} + \frac{a^2}{l}\right)} + \frac{2}{3}f = \frac{8}{15}f.K + \frac{2}{3}f \quad . \quad . \quad . \quad (32)$$

The distance $\frac{2}{3}f$ above the springing of a parabolic arch of the form assumed is known as the "elastic centre" of the arch axis.

The height of the reaction locus above the elastic centre is therefore

$$\frac{8}{15}f.K$$

or

$$\frac{8}{15}f.\left(1 + \frac{45\,Ic}{4\,Av.f^2}\right)$$

$$= \frac{8}{15}f + 6\frac{Ic}{Av.f}$$

The reaction locus is therefore a straight line.

70. Uniform Load over Whole Span of Parabolic Hingeless Arch.—Let p be the load per lineal foot, on the arch uniformly distributed (Fig. 57).

The three unknowns can readily be found from equations (19), (20) and (21), for any system of loading.

The formula for the *free* bending moment is

$$M_f = \frac{p}{2}(l.x - x^2)$$

The integrations of the quantities $M_f.y$, $M_f.x$ and M_f in equations (19), (20) and (21) for the loading under consideration become

$$\left.\begin{aligned}\int_A^B M_f.y.dx &= \frac{1}{15}p.f.l^3 \\ \int_A^B M_f.x.dx &= \frac{1}{24}p.l^4 \\ \int_A^B M_f.dx &= \frac{1}{12}p.l^3\end{aligned}\right\} \quad (33)$$

FIG. 57.—Line of Pressure for Uniform Load over Hingeless Arch.

Utilising these results, equations (19), (20) and (21) can now be written

$$\int_A^B M.y.dx - H.Ic\int_A^B \frac{dx}{A} = \frac{p.f.l^3}{15} - H.\frac{8}{15}f^2.l + H.q_b.\frac{2}{3}f.l$$

$$- H(q_b - q_a).\frac{1}{3}f.l - H.Ic.\frac{l}{Av} = 0 \quad (34)$$

$$\int_A^B M.x.dx = \frac{1}{24}p.l^4 - H.\frac{f.l^2}{3} + H.q_b.\frac{l^2}{2}$$

$$- H(q_b - q_a).\frac{l^2}{6} = 0 \quad (35)$$

$$\int_A^B M.dx = \frac{1}{12}p.l^3 - H.\frac{2}{3}f.l + H.q_b.l$$

$$- H(q_b - q_a).\frac{l}{2} = 0 \quad (36)$$

from which $\quad H.(q_b - q_a) = 0$

$$H.q_a = H.q_b = -\frac{p.l^2}{12} + H.\frac{2}{3}f \quad . \quad . \quad . \quad (37)$$

Putting in values of $H.(q_b - q_a)$ and $H.q_b$ in equation (34) the horizontal thrust is

$$H = \frac{p.l^2}{8f} \cdot \frac{1}{1 + \frac{45\ Ic}{4\ Av.f^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

or

$$H = \frac{p.l^2}{8f.K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38A)$$

Moment in any Section of the Arch.—By formula (15)

$$Ms = M_f - H.y + H.q_b - H(q_b - q_a)\frac{(l - x)}{l}$$

$$= \frac{p}{2}(l.x - x^2) - \frac{p.l^2}{8f.K} \cdot \frac{4f}{l^2}(l.x - x^2) - \frac{p.l^2}{12} + \frac{p.l^2}{8f.K} \cdot \frac{2}{3}f$$

$$= \left(1 - \frac{1}{K}\right)\left\{p\left(\frac{l.x}{2} - \frac{l^2}{12} - \frac{x^2}{2}\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

This formula shows that the only moments in a parabolic arch under a uniformly distributed load over the whole span are those due to the shortening of the arch from the normal compression. This shortening is represented by the factor K, as demonstrated in Art. 64. Where this factor is assumed to equal unity the moments disappear, and the only force acting upon any arch section is axial compression, producing a uniform compressive stress upon it.

If K is taken into consideration it is possible to find the deviation of the line of pressure from the arch axis.

Let u be the vertical eccentricity of the horizontal thrust. Then, since $M = H.u$ (see Arts. 51 and 55),

$$u = \frac{p\left(\frac{l.x}{2} - \frac{l^2}{12} - \frac{x^2}{2}\right)\left(1 - \frac{1}{K}\right)}{\frac{p.l^2}{8f.K}}$$

$$= \frac{45.Ic}{Av.f}\left(\frac{x}{l} - \frac{x^2}{l^2} - \frac{1}{6}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

The eccentricity of thrust at the crown $\left(\frac{x}{l} = 0{\cdot}5\right) = \frac{15\ Ic}{4\ Av.f}$ (above

the axis) and at the springing $\left(\frac{x}{l} = 0\right) = -\frac{15\,\mathrm{Ic}}{2\,\mathrm{A}v.f}$ (below the axis) as in Fig. 57.

71. Uniform Load over Half-span of Parabolic Hingeless Arch.—In Fig. 58 the arch is loaded over the left-hand half with a uniform load of p per unit length.

Fig. 58.—Line of Pressure for Uniform Load on Half Span. (Hingeless Arch.)

Proceeding in the same way as for the whole span load the three unknowns are first found.

The *free* bending moment at any section

(a) in the *loaded* half $\quad \mathrm{M}_f = \frac{p}{8}(3\,l.x - 4\,x^2)$

(b) in the *unloaded* half $\quad \mathrm{M}_f = \frac{p}{8}(l^2 - lx)$

$$\therefore \left.\begin{aligned} \int_A^B \mathrm{M}_f.y.dx &= \frac{1}{30}p.f.l^3 \\ \int_A^B \mathrm{M}_f.x.dx &= \frac{7}{384}p.l^4 \\ \int_A^B \mathrm{M}_f\,.\,dx &= \frac{1}{24}p.l^3 \end{aligned}\right\} \quad . \quad (41)$$

Equations (19), (20) and (21) can now be written in the form

$$\int_A^B \mathrm{M}.y.dx - \mathrm{H.Ic}\int_A^B \frac{dx}{\mathrm{A}} = \frac{1}{30}p.f.l^3 - \mathrm{H}.\frac{8}{15}.f^2.\,l\, +$$

$$\mathrm{H}.q_b\frac{2}{3}f.l - \mathrm{H}(q_b - q_a).\frac{1}{3}f.l - \mathrm{H.Ic}\frac{l}{\mathrm{A}v} = 0\,. \quad (42)$$

$$\int_A^B \mathrm{M}.x.dx = \frac{7}{384}p.l^4 - \mathrm{H}.\frac{f.l^2}{3} +$$

$$\mathrm{H}.q_b.\frac{l^2}{2} - \mathrm{H}(q_b - q_a).\frac{l^2}{6} = 0\,. \quad (43)$$

$$\int_A^B M.dx = \frac{1}{24} p.l^3 - H.\frac{2}{3}f.l +$$

$$H.q_b.l - H(q_b - q_a) \cdot \frac{l}{2} = 0 \quad . \quad (44)$$

The solution of these three equations gives the three unknowns.

$$H.q_b = H.\frac{2}{3}.f - \frac{5}{192}.p.l^2 \quad . \quad . \quad . \quad . \quad (45)$$

$$H.(q_b - q_a) = \frac{1}{32} p.l^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

$$H = \frac{p.l^2}{16f} \cdot \frac{1}{1 + \frac{45}{4} \frac{Ic}{Av.f^2}} \quad . \quad . \quad . \quad . \quad (47)$$

or

$$H = \frac{p.l^2}{16f.K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47A)$$

Moment in any Section of the Arch.—The unknown factors having thus been determined, the formula (15) for the moment in the arch gives

(*a*) In the *loaded* half—

$$Ms = \frac{p}{8}(3\,l.x - 4x^2) - H.\frac{4f}{l^2}(l.x - x^2) +$$

$$H.\frac{2}{3}f - \frac{5}{192}p.l^2 - \frac{p.l^2}{32} \cdot \frac{(l - x)}{l}$$

$$= \frac{p}{192}(78\,l.x - 96x^2 - 11\,l^2) - H.f\left(\frac{4x}{l} - \frac{4x^2}{l^2} - \frac{2}{3}\right)$$

$$= \frac{p}{192}(78\,l.x - 96\,x^2 - 11\,l^2) - \frac{p}{4K}\left(l.x - x^2 - \frac{l^2}{6}\right) \quad . \quad (48A)$$

(*b*) in the *unloaded* half—

$$Ms = \frac{p}{8}(l^2 - l.x) - H.\frac{4f}{l^2}(l.x - x^2) +$$

$$H.\frac{2}{3}f - \frac{5}{192}p.l^2 - \frac{p.l^2}{32}\frac{(l - x)}{l}$$

$$= \frac{p}{192}(13\,l^2 - 18\,l.x) - H.f\left(\frac{4x}{l} - \frac{4x^2}{l^2} - \frac{2}{3}\right)$$

$$= \frac{p}{192}(13\,l^2 - 18\,l.x) - \frac{p}{4K}\left(l.x - x^2 - \frac{l^2}{6}\right) \quad . \quad . \quad . \quad . \quad (48B)$$

Table K

Loaded Half by formula (48A).	Unloaded Half by formula (48B).
When $\frac{x}{l}=0 \quad M=p.l^2\left(-0·0573+\frac{0·04167}{K}\right)$	When $\frac{x}{l}=0·5\ M=p.l^2\left(+0·02083-\frac{0·02083}{K}\right)$
$=0·1\ M=p.l^2\left(-0·02167+\frac{0·01917}{K}\right)$	$=0·6\ M=p.l^2\left(+0·01146-\frac{0·01833}{K}\right)$
$=0·2\ M=p.l^2\left(+0·00396+\frac{0·00167}{K}\right)$	$=0·7\ M=p.l^2\left(+0·00208-\frac{0·01083}{K}\right)$
$=0·3\ M=p.l^2\left(+0·01958-\frac{0·01083}{K}\right)$	$=0·8\ M=p.l^2\left(-0·00729+\frac{0·00167}{K}\right)$
$=0·4\ M=p.l^2\left(+0·02521-\frac{0·01833}{K}\right)$	$=0·9\ M=p.l^2\left(-0·01667+\frac{0·01917}{K}\right)$
$=0·5\ M=p.l^2\left(+0·02083-\frac{0·02083}{K}\right)$	$=1·0\ M=p.l^2\left(-0·02604+\frac{0·04167}{K}\right)$

Bending Moments in Hingeless Parabolic Arch Sections produced by Uniform Load over Half Span.

Assuming K = 1 the moments become :—

Table L

	Loaded half					Unloaded half.					
	$\frac{x}{l}=0$	$\frac{x}{l}=0·1$	$\frac{x}{l}=0·2$	$\frac{x}{l}=0·3$	$\frac{x}{l}=0·4$	$\frac{x}{l}=0·5$	$\frac{x}{l}=0·6$	$\frac{x}{l}=0·7$	$\frac{x}{l}=0·8$	$\frac{x}{l}=0·9$	$\frac{x}{l}=1·0$
$M=p.l^2$	0·01563	−0·0025	+0·00563	+0·00875	+0·00688	0	0·00688	0·00875	−0·00563	+0·0025	+0·01563

Under the above assumption :—

the maximum positive bending moment $= +\frac{9}{1024}pl^2$ and

occurs at $x=\frac{5}{16}l$,

and the maximum negative bending moment $= -\frac{9}{1024}pl^2$, and

occurs at $x=\frac{11}{16}l$.

72. Linear Variations of a Hingeless Arch.—When an arch is subjected to a change in temperature it undergoes an alteration in length. If, as assumed, the abutments are immovable, the span between them remains unchanged. It therefore results that the arch exerts a thrust or a " pull " upon the abutments accordingly as the length of the arch is increased or decreased.

Actually, of course, the arch will never exert a pull on the abut-

ments, since the thrust due to the vertical loading will always be the greater force, causing an opposite thrust to be exerted upon the arch by the abutments. The "pull" mentioned above will effect a reduction of thrust, and the resultant moment induced in the arch needs to be added algebraically to those produced by the superimposed loading.

When the contraction of the arch, due to its shrinkage during setting and hardening, is not dealt with by the introduction of temporary hinges (as explained in Chapter IX.), it is necessary to include this resultant arch shortening in the calculations for the arch. This may be done by adding this latter shortening to that produced by the maximum drop in atmospheric temperature assumed.

The horizontal thrust and its point of application resultant from a change in length of an arch are found by making use of the conditions (*a*), (*b*) and (*c*), given in Art. 64.

Now the change in the length of the span, if the rib were free to expand or contract, would be

$$n.l$$

where n is the change in length per unit of length

l is the span of the arch.

As the span cannot change in length, because of the rigidity of the abutments, equation (14) will be written

$$\int_A^B \frac{M.y}{E.I}.ds \pm n.l - \int_A^B \frac{H}{A.E}.dx = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

and as there can be no change in the level of supports, from (11)

$$\int_A^B \frac{M.x}{E.I}.ds = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

and also no rotation of the end sections, from (12)

$$\int_A^B \frac{M}{E.I}.ds = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

In a symmetrical rib the bending moment in the arch sections produced by a linear variation is

$$M_t = -H.y + H.q_a$$

$H.q_a$ is the end fixing moment, and, since horizontal forces only are considered, q_a is the height of the point of application of the horizontal thrust above the level of both springings. As there are

only two unknowns to be evaluated (H and q_a), equation (51) is not required.

Expanding equations (49) and (50)

$$\int_A^B \frac{M.y}{E.I}.ds \pm n.l - \int_A^B \frac{H}{A.E}.dx = -\int_A^B \frac{H.y^2}{E.I}.ds + \int_A^B \frac{H.q_a.y}{E.I}.ds \pm n.l - \int_A^B \frac{H}{A.E}.dx = 0 \quad . \quad (52)$$

$$\int_A^B \frac{M.x}{E.I}.ds = -\int_A^B \frac{H.x.y}{E.I}.ds + \int_A^B \frac{H.q_a.x}{E.I}.ds = 0 \quad . \quad (53)$$

Horizontal Force due to a Linear Variation in a Parabolic Hingeless Arch.—Employing the assumptions with regard to the curve of the arch axis, variation of moment of inertia, and constant coefficient of elasticity, made in Art. 65, equations (52) and (53) become, by multiplying throughout by E and Ic

$$\int_A^B M.y.dx \pm E.Ic.n.l - H.Ic.\int_A^B \frac{dx}{A} = -H.\int_A^B y^2.dx + H.q_a\int_A^B y.dx \pm E.Ic.n.l - H.Ic.\int_A^B \frac{dx}{A} = 0 \quad . \quad (54)$$

$$\int_A^B M.x.dx = -H.\int_A^B x.y.dx + H.q_a\int_A^B x.dx = 0 \quad . \quad (55)$$

which, on further reduction, produce

$$\int_A^B M.y.dx \pm E.Ic.n.l - H.Ic.\int_A^B \frac{dx}{A} = -H.\frac{8}{15}f^2.l + H.q_a\frac{2}{3}f.l \pm E.Ic.n.l - H.\frac{Ic.l}{Av} = 0 \quad (56)$$

$$\int_A^B M.x.dx = -\frac{H.f.l^2}{3} + H.q_a.\frac{l^2}{2} = 0 \quad (57)$$

∴ Equation (57) gives $q_a = \frac{2}{3}.f$, which is the height of the elastic centre of the arch above the springings, and this result, combined with equation (56), determines the horizontal thrust

$$H.\left(\frac{4}{45}.f^2 + \frac{Ic}{Av}\right) \pm E.Ic.n = 0$$

$$\therefore H = \pm \frac{45.E.Ic.n}{4f^2 + 45\frac{Ic}{Av}} \quad . \quad . \quad . \quad . \quad . \quad (58)$$

The bending moments in the arch sections are evidently equal to the product of the thrust and the vertical distance between the arch axis at the section and the elastic centre.

The above horizontal force and resultant moments will be equal in amount but of opposite sign, depending on whether they are produced by a rise or a fall of temperature.

The following diagrams give the signs of the bending moments at the crown and the springing :

Rise of temperature produces an *increase* in thrust.

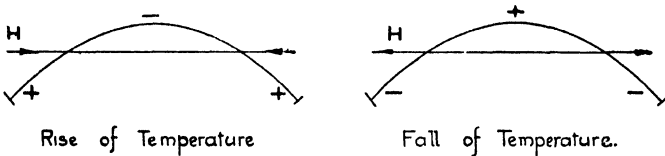
Fall of temperature produces a *decrease* in thrust.

BENDING MOMENTS IN A HINGELESS ARCH DUE TO VARIATIONS OF TEMPERATURE.

FIG. 59

TABLE M

Section .	$\frac{x}{l} = 0$	= 0·1	= 0·2	= 0·3	= 0·4	= 0·5	= 0·6 etc
Moment .	± ·667	± ·307	± ·027	∓ ·173	∓ ·293	∓ ·333	∓ ·293

Bending Moments in Terms of H.f.

Bending Moments in a Parabolic Hingeless Arch produced by Linear Variations.

73. Two-Hinged Arches.—The results developed for the hingeless arch can be employed in the investigation of the horizontal thrust and bending moments in two-hinged arches. The formulæ

are, however, simplified to a great extent by the elimination of the moments at the springings, due to the provision of hinges at these points. This reduces the problem to one of single static indetermination which lies in the evaluation of the horizontal thrust.

In order to ascertain this thrust the condition denoted "*a*" in Art. 64, is assumed as follows :—

"The span of the arch remains unchanged."

As before, this provides the general equation (14)—

i.e., $$\int_A^B \frac{M.y}{E.I}.ds - \int_A^B \frac{H}{A.E}.dx = 0.$$

The solution of this equation gives the formula for the horizontal thrust.

74. Parabolic Two-Hinged Arch.—The following assumptions which were employed for hingeless arches are made (see Art. 65) :—

(*a*) The axis of the arch is a parabola of the second degree.

(*b*) The moment of inertia of any cross section varies as the secant of the angle of inclination *a* of the rib with the horizontal, *i.e.*,

$$I - Ic.\sec a$$

(*c*) The coefficient of elasticity (E) is constant.

75. Influence Line for the Horizontal Thrust.—The moment in the arch sections distance x from support A is

$$M_s = M_f - H.y \quad \text{Equation (2), Art. 55.}$$

∴ Equation (14) becomes, for a parabolic arch with the assumptions made in Art. 74,

$$\int_A^B M.y.dx - H.Ic.\int_A^B \frac{dx}{A} = \int_A^B M_f.y.dx - H.\int_A^B y^2.dx - H.Ic\int_A^B \frac{dx}{A} = 0$$

$$\therefore \qquad H = \frac{\int_A^B M_f.y.dx.}{\int_A^B y^2.dx + Ic.\int_A^B \frac{dx}{A}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

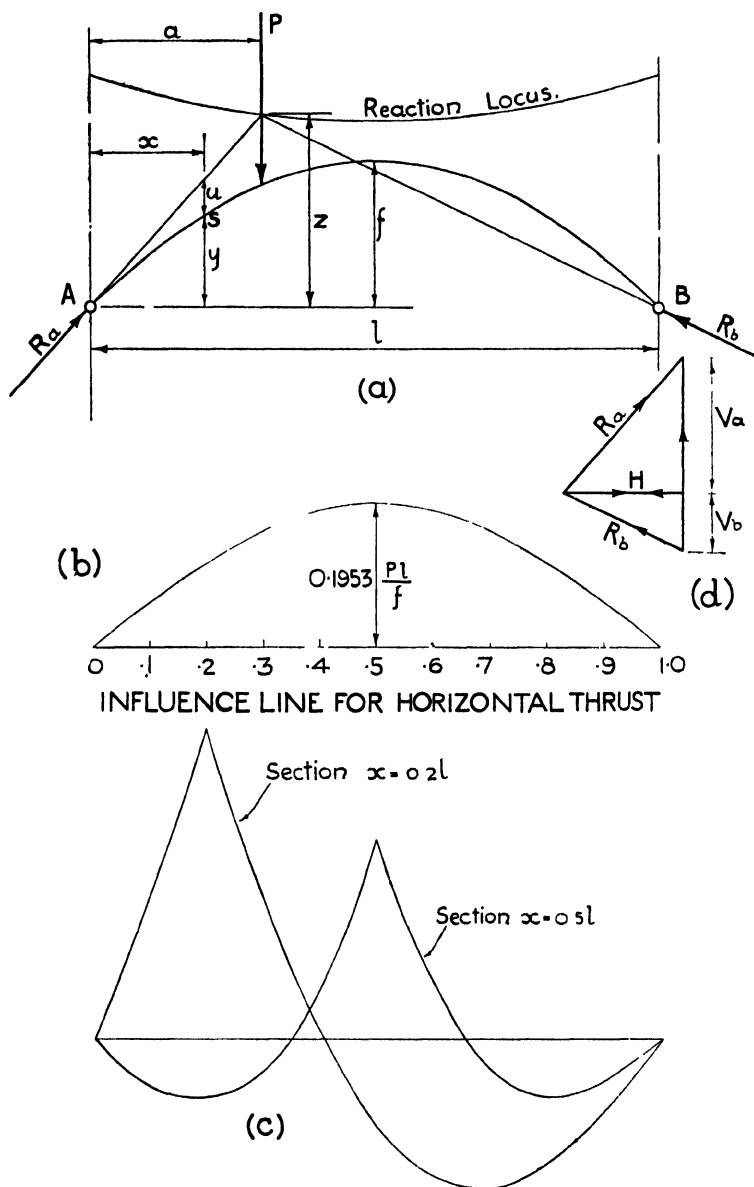



FIG 60.—Influence Lines for Horizontal Thrust and Bending Moments. (Two Hinged Arch.)

The formula for the free bending moment is (Fig. 60A)—

(a) When $x < a$ $\quad M_f = P.(l - a).\frac{x}{l}$

(b) ,, $x > a$ $\quad M_f = P.(l - x).\frac{a}{l}$

The integrations contained in formula (59) are given in equations (22), and with the aid of these the horizontal thrust produced by a single point load can be expressed as—

$$H = \frac{\frac{P.f}{3l^2}(a^4 + a.l^3 - 2\,a^3.l)}{\frac{8}{15}f^2.l + \frac{Ic.l}{Av}}$$

$$= \frac{5\,P}{8.l^3.f}(a^4 + a.l^3 - 2\,a^3.\,l)\frac{1}{1 + \frac{15\,Ic}{8\,Av.f^2}} \quad . \quad (60)$$

or $$H = \frac{5\,P}{8l^3.f.K}(a^4 + a.l^3 - 2\,a^3.l) \quad . \quad . \quad . \quad . \quad (60A)$$

The expression $1 + \frac{15\,Ic}{8\,Av.f^2}$ is the correction for the shortening of the arch under direct compression.

Denoting this latter expression by K, values can be computed for the horizontal thrust at progressive sections along the span of the arch. These values are tabulated as follows :—

TABLE N

Position of load $\frac{a}{l}$ =	0	0·1	0·2	0·25	0·3	0·4	0·5	0·6	etc.
Horizontal thrust $\frac{P.l}{f.K}$ =	0	0·0613	0·116	0·1392	0·1588	0·186	0·1953	0·186	etc.

Horizontal Thrust for Parabolic Two-Hinged Arch in terms of $\frac{P.l}{f.K}$

Table N.—Plotting these results, the standard influence line for the horizontal thrust for a two-hinged parabolic arch can be constructed. This is given in Fig. 60b.

76. Influence Lines for Bending Moments.—Having calculated the value of the horizontal thrust for any position of a point load, it is a simple matter to set up influence lines for the bending moment at any section of the arch by means of the formula

$$Ms = M_f - H.y$$

TABLE P

POSITION OF SECTION

POSITION OF LOAD	$\frac{x}{l} = 0.1$	$\frac{x}{l} = 0.2$	$\frac{x}{l} = 0.25$	$\frac{x}{l} = 0.3$	$\frac{x}{l} = 0.4$	$\frac{x}{l} = 0.5$
$\frac{a}{l} = 0$	0	0	0	0	0	0
= 0·1	·09 − ·0221/K	·08 − 0392/K	·075 − ·046/K	·07 − ·0515/K	·06 − ·0589/K	·05 − ·0613/K
= 0·2	·08 − ·0418/K	·16 − 0743/K	·15 − 087/K	·14 − ·0974/K	·12 − ·1114/K	·1 − ·116/K
= 0 25	·075 − ·0501/K	·15 − ·0891/K	·1875 − ·1044/K	·175 − ·1169/K	·15 − ·1336/K	·125 − ·1392/K
= 0 3	·07 − ·0572/K	·14 − ·1016/K	·175 − ·1191/K	·21 − 1334/K	·18 − ·1525/K	·15 − ·1588/K
= 0·4	·06 − ·067/K	·12 − ·119/K	·15 − ·1395/K	·18 − ·1562/K	·24 − ·1786/K	·2 − ·186/K
= 0·5	·05 − ·0703/K	·1 − ·125/K	·125 − ·1465/K	·15 − ·164/K	·2 − ·1875/K	·25 − ·1953/K
= 0·6	·04 − ·067/K	·08 − 119/K	·1 − ·1395/K	12 − ·1562/K	·16 − ·1786/K	·2 − ·186/K
= 0·7	·03 − ·0572/K	·06 − ·1016/K	·075 − ·1191/K	·09 − ·1334/K	·12 − ·1525/K	·15 − ·1588/K
= 0·75	·025 − ·0501/K	·05 − ·0891/K	·0625 − ·1044/K	·075 − ·1169/K	·1 − ·1336/K	·125 − ·1392/K
= 0·8	·02 − ·0418/K	·04 − ·0743/K	·05 − ·087/K	·06 − 0974/K	·08 − ·1114/K	·1 − ·116/K
= 0·9	·01 − ·0221/K	·02 − ·0392/K	·025 − ·046/K	·03 − ·0515/K	·04 − ·0589/K	·05 − ·0613/K
= 1·0	0	0	0	0	0	0

Two-hinged arch bending moment influence line ordinates (in terms of P.l.) at section distance x when load is at distance a from support.

Table Q

POSITION OF LOAD

POSITION OF SECTION	$\frac{a}{l}=0$	$\frac{a}{l}=0.1$	$\frac{a}{l}=0.2$	$\frac{a}{l}=0.25$	$\frac{a}{l}=0.3$	$\frac{a}{l}=0.4$	$\frac{a}{l}=0.5$	$\frac{a}{l}=0.6$	$\frac{a}{l}=0.7$	$\frac{a}{l}=0.75$	$\frac{a}{l}=0.8$	$\frac{a}{l}=0.9$	$\frac{a}{l}=1.0$
$\frac{x}{l}=0.1$	0	·0679	·0382	·0249	·0128	—·007	—·0203	—·027	—·0272	—·0251	—·0218	—·0121	0
$=0.2$	0	·0408	·0857	·0609	·0384	·001	—·025	—·039	—·0416	—·039	—·0343	—·0192	0
$=0.25$	0	·029	·063	·0831	·0559	·0105	—·0215	—·0395	—·044	—·0419	—·037	—·021	0
$=0.3$	0	·0185	·0426	·0581	·0766	·0238	—·0141	—·0362	—·0434	—·0419	—·0374	—·0215	0
$=0.4$	0	·0011	·0086	·0164	·0275	·0614	·0125	—·0186	—·0325	—·0336	—·0314	—·0189	0
$=0.5$	0	—·0113	—·016	—·0142	—·0088	·014	·0547	·014	·0088	—·0142	—·016	—·0113	0

Ordinates to Standard Influence Lines for Bending Moments in a Parabolic Two-Hinged Arch.

Moments in terms of P.l. (when K = 1).

The curves for sections $x = 0{\cdot}2l$ and $0{\cdot}5l$ are plotted in Fig. 60c.

The "free" bending moment at any section "S" (see Fig. 60a), for any position "a" of the load P, is as given previously;

i.e., when $\quad x < a \quad M_f = P.(l-a).\frac{x}{l}$

and when $\quad x > a \quad M_f = P.(l-x).\frac{a}{l}$

∴ when the section S is situated between support A and the load

$$Ms = P.(l-a).\frac{x}{l} - H.y$$

$$= P.(l-a).\frac{x}{l} - H.f.\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) \quad . \quad . \quad . \quad . \quad (61\text{A})$$

and when the section S lies between the position of the load and support B

$$Ms = P.(l-x).\frac{a}{l} - H.f\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) \quad . \quad . \quad . \quad . \quad (61\text{B})$$

These formulæ (61A) and (61B) enable Tables P and Q to be calculated (see pp. 96 and 97).

77. Line of Reaction Intersections.—The line of intersections of the reactions will be determined by means of the same method adopted for hingeless arches (see Art. 69).

By similar triangles (Figs. 60a and 60d)

$$\frac{z}{a} = \frac{Va}{H}$$

$$z = \frac{\frac{P.a.(l-a)}{l}}{\frac{5P}{8l^3.f.K}(a^4 + a.l^3 - 2a^3.l)}$$

$$= \frac{8f.K}{5} \cdot \frac{l^2}{l^2 + a.l - a^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

From which equation the intersections curve can be plotted. To facilitate this, the following table is given:—

TABLE R

Position of load $\frac{a}{l}$	0	0·1	0·2	0·25	0·3	0·4	0·5	0·6	etc.
Value of z in terms of f.K. .	1·6	1·468	1·38	1·348	1·323	1·28	1·29	1·29	etc.

Ordinates to Curve of Reaction Intersections for Two-Hinged Arch.

78. Uniform Load over Whole Span of Parabolic Two-Hinged Arch.—The horizontal thrust

$$H = \frac{\int_A^B M_f . y . dx}{\int_A^B y^2 . dx + Ic \int_A^B \frac{dx}{A}} \qquad \text{by formula (59).}$$

As before, M_f is the moment for a freely supported beam of same span, and is equal to $\frac{p.l}{2} x - \frac{p.x^2}{2}$, where p is the load per lineal foot of the span of the arch.

$$\int_A^B M_f . y . dx = \frac{p.f.l^3}{15} \text{ (from equation (33)),}$$

and by formula (59)

$$H = \frac{\frac{p.f.l^3}{15}}{\frac{8}{15} . f^2 . l + \frac{Ic\, l}{Av}} = \frac{p.l^2}{8f} . \frac{1}{1 + \frac{15}{8} \frac{Ic}{Av.f^2}} = \frac{p.l^2}{8f.K} \quad . \quad . \quad . \quad (63)$$

The moment at any point in the arch is therefore

$$M_f - H.y$$

$$= \frac{p.l.x}{2} - \frac{p.x^2}{2} - \frac{p.l^2}{8f.K} . \frac{4f.x}{l^2} . (l - x)$$

$$= \frac{p.x.(l-x)}{2}\left(1 - \frac{1}{K}\right) \quad . \quad . \quad . \quad . \quad . \quad (64)$$

To draw the line of pressure (Fig. 61) it is necessary to calculate the eccentricity of the thrust from the arch axis. This is given by the formula

$$M = H.u$$

$$\therefore \qquad u = \frac{M}{H} = \frac{\frac{p.x.(l-x)}{2}\left(1 - \frac{1}{K}\right)}{\frac{p.l^2}{8f.K}}$$

$$= \frac{4.x.(l-x)}{l^2} . \frac{15\, Ic}{8\, Av.f} \quad . \quad . \quad . \quad . \quad . \quad (65)$$

It will be seen that the eccentricity varies according to the ordinates

of a parabola. The maximum eccentricity is at the crown when $x = \frac{l}{2}$, and is then equal to $\frac{15}{8} \cdot \frac{Ic}{Av.f}$

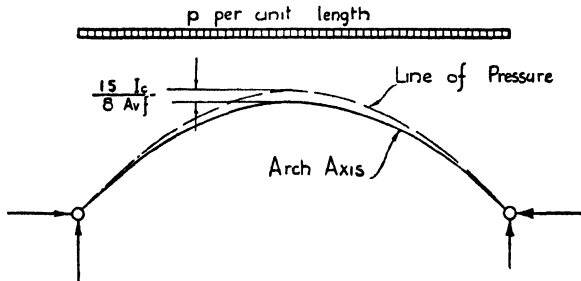



FIG. 61. -Line of Pressure for Uniform Load over Two-Hinged Arch.

Note.—If the constant factor K is assumed to equal unity, the moments at all sections in the arch ring are zero, because $u = 0$.

79. Uniform Load over Half-span of Parabolic Two-Hinged Arch.– Let p be the load per unit length on the left-hand half of the span (Fig. 62).

As before, the moments in a freely supported beam of the same span are—

In *loaded half* $M_f = \frac{p.x}{8}(3l - 4x)$

Unloaded half $M_f = \frac{p.l}{8}(l - x)$

$$\int_A^B M_f.y.\,dx = \frac{p.l^3.f}{30} \quad \text{(See equation (41).)}$$

By formula (59) $$H = \frac{p.l^2}{16f}\left(\frac{1}{1 + \frac{15}{8}\frac{Ic}{Av.f^2}}\right) = \frac{p.l^2}{16f.K} \quad . \quad . \quad (66)$$

Moment in any Section of the Arch.

Loaded half $$\frac{p.x}{8}(3l - 4x) - \frac{p.l^2}{16f.K} \cdot \frac{4x.(l - x).f}{l^2}$$

$$= \frac{p.x}{8}\left(3l - 4x - \frac{2(l - x)}{K}\right) \quad . \quad . \quad . \quad . \quad (67)$$

Unloaded half $$\frac{p.l}{8}(l - x) - \frac{p.x(l - x)}{4K}$$

$$= \frac{p.(l - x)}{8}\left(l - \frac{2x}{K}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

Table S gives the bending moment at progressive sections (distance x from support) in terms of load p and span l

TABLE S

Loaded Half		Unloaded Half	
$x = 0.1l$	$M = pl^2\left(.0325 - \frac{.0225}{K}\right)$	$x = 0.9l$	$M = pl^2\left(.0125 - \frac{.0225}{K}\right)$
$x = 0.2l$	$M = pl^2\left(.055 - \frac{.04}{K}\right)$	$x = 0.8l$	$M = pl^2\left(.025 - \frac{.04}{K}\right)$
$x = 0.25l$	$M = pl^2\left(.0625 - \frac{.046875}{K}\right)$	$x = 0.75l$	$M = pl^2\left(.03125 - \frac{.04688}{K}\right)$
$x = 0.3l$	$M = pl^2\left(.0675 - \frac{.0525}{K}\right)$	$x = 0.7l$	$M = pl^2\left(.0375 - \frac{.0525}{K}\right)$
$x = 0.4l$	$M = pl^2\left(.07 - \frac{.06}{K}\right)$	$x = 0.6l$	$M = pl^2\left(.05 - \frac{.06}{K}\right)$
$x = 0.5l$	$M = pl^2\left(.0625 - \frac{.0625}{K}\right)$	$x = 0.5l$	$M = pl^2\left(.0625 - \frac{.0625}{K}\right)$

Bending Moments in Two Hinged Parabolic Arch Sections produced by Uniform Load over Half span

Note Where the factor K = 1 the moments are identical with those in a three hinged arch The maximum moments occur at the quarter points, and are in the loaded half $+\frac{pl^2}{64}$ and in the unloaded half $-\frac{pl^2}{64}$

The line of pressure is obtained similarly to that for load over the whole span and is shown in Fig 62

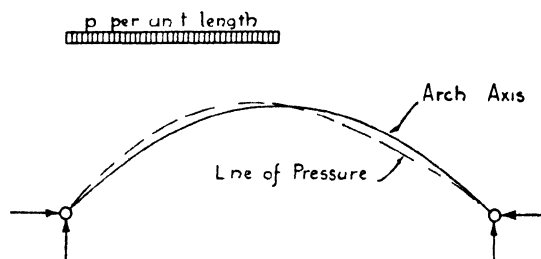



FIG 62 —Line of Pressure for Uniform Load over Half span (Two-Hinged Arch)

80. Linear Variations of a Two-Hinged Arch.—As explained in the case of a hingeless arch, the change in length of the span when free to expand or contract is expressed by

$$n\ l,$$

where n is the change in length per unit of length,
l is the span of the arch

With the single assumed condition that the span of the arch remains unchanged the equation (14) can be written

$$\int_A^B \frac{M\,y}{E.I}\,ds \pm n.l - \int_A^B \frac{H}{A.E}\,dx = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

or for a parabolic arch with the previous assumptions (Art. 65)

$$\int_A^B M.y.dx \pm E.Ic.n.l - H.Ic.\int_A^B \frac{dx}{A} = 0 \quad . \quad . \quad . \quad (70)$$

Because of the absence of any bending moment at the springings, due to the presence of the hinges, the resultant horizontal force acts through the hinges. Consequently the moment in the arch sections is the product of the horizontal thrust by the ordinate of the arch section ; *i.e.*,

$$Mt = -H.y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

Expanding equation (70),

$$\int_A^B M.y.dx \pm E.Ic.n.l - H.Ic.\int_A^B \frac{dx}{A} = -\int_A^B H.y^2.dx \pm E.Ic.n.l -$$

$$H.Ic.\int_A^B \frac{dx}{A} = -\frac{H.8.f^2.l}{15} \pm E.Ic.n.l - \frac{H.Ic.l}{Av}$$

$$\therefore \qquad H = \frac{\pm 15.E.Ic.n}{8.f^2 + \dfrac{15.Ic}{Av}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

When the value of the force H is ascertained the bending moments may be easily calculated by the aid of formula (71).

With a *rise* in the temperature or an increase in length of the arch the abutments exert a thrust upon the arch, inducing moments throughout its length which are *negative*.

Conversely, with a *fall* in temperature or a shortening of the arch the abutments exert a "pull," producing *positive* moments. Positive moments are considered to be those which tend to produce tension on the intrados of the arch.

81. Calculation of Maximum Stresses upon an Arch Section.—In designing an arch it is not possible to determine the requisite dimensions and reinforcement by direct calculations. As in the case of other reinforced concrete members, it is necessary to

assume the dimensions of the arch member and reinforcement, usually from the experience of other arches comparable in size and type. If this cannot be done, suitable proportions must be determined by trial. In the latter case, it may require several attempts before an economical solution is arrived at. This, however, is inevitable, owing to the large number of variables influencing the design of arch members.

Having adopted definite dimensions for the concrete arch and steel reinforcement, the maximum stresses produced at any cross section are determined as explained below.

It may be stated for guidance that for arches having normal proportions and loading the sections requiring investigation are those at the springing, haunches and crown.

Considering any of these sections, the maximum positive and negative bending moments due to the superimposed loading are computed from the appropriate influence lines. To these are added those moments induced by temperature variations (and shrinkage, if any). Let M represent the total maximum positive or negative moment so found.

The maximum horizontal thrust producing each of the above aggregate bending moments is also computed in a similar manner. To this must be added the horizontal thrust resultant from the dead weight of the structure. Let H represent this maximum total horizontal thrust.

The maximum and minimum stresses upon the concrete of the considered cross section are :—

$$S.\ max. = \frac{H}{Ae.\cos a} \pm \frac{M.y}{Ie}$$

where y = distance of extreme fibre from neutral axis of section ;
Ae = equivalent area of section in terms of the concrete ;
Ie = equivalent moment of inertia of section in terms of the concrete.

In determining the stress upon the longitudinal rods, the appropriate value for y must be employed and the stress increased by the modular ratio of the materials, *i.e.*,

$$S.\ max. = \frac{Es}{Ec}\left[\frac{H}{Ae.\cos a} \pm \frac{M.y}{Ie}\right]$$

In calculating the value of the cross sectional moment of inertia, the whole of the concrete is considered as contributing to it unless the tensile strength of the concrete is exceeded.

Where it is desired to ascertain the stress upon the tension rods

and the increased compressive stress upon the concrete, ignoring the concrete in tension, the eccentricity of the normal thrust $\frac{H}{\cos a}$ upon the section can be determined by dividing the latter quantity into the bending moment M and the maximum stresses investigated by any of the methods given for eccentric loading in the various text-books dealing with the subject.

It should also be noted by the reader, in calculating the value of the normal thrust $\frac{H}{\cos a}$ (Fig. 54) from the horizontal thrust, that the component of the latter is the vertical shearing force at the section under consideration. The resultant of the vertical and horizontal forces is the total inclined force acting upon the section.

The tangential shearing force upon arches of usual dimensions and with ordinary loading is usually unimportant, and may be neglected, but in special cases, such as an arch having relatively deep ribs connected together by a thin slab, the requisite amount of shearing reinforcement should be calculated.

The value of the actual shearing force upon any cross section of a parabolic arch may be obtained from the following formulæ :—

$Vt = V_1 \cos a - H \sin a$ (for loads to the right of section), and $Vt = (V_1 - 1) \cos a - H \sin a$ (for loads to the left of section), where V_1 is the vertical reaction at *left*-hand support.

Note.—Influence lines for the horizontal thrust are given in Table C (Art. 57) for the three-hinged arch, Table F (Art. 66) for the hingeless arch, Table N (Art. 75) for the two-hinged arch.

The influence lines for the vertical reaction at the left support for the three-hinged arch and two-hinged arch are the same as that for a freely supported beam (see Art. 43). For the hingeless arch the vertical reaction influence line is given in Table J (Art. 68).

82. Proportioning of Arches.—In the selection of the ratio of rise to span of an arch for any individual site, scope is generally limited by the physical conditions and the height of the proposed roadway. This latter point depends, in turn, on the gradients of the approaches and the architectural requirements.

The level of the springings may be controlled by the specified waterway or levels in the case of a river bridge, whilst for a bridge over a railway or roadway the minimum clearance is usually the governing factor.

Where, however, the engineer has a free hand in the selection of the rise and span of the bridge, it is advantageous, from an economical standpoint, to investigate the best rise-span ratio for different cases.

It has been found that this ratio varies from 1/4 to 1/6, depending

upon several factors, such as the loading, the type of arch, and the depth of foundations to the abutments.

A large rise-span ratio is economical in design, since an increase in rise gives a reduction in the thrust on the rib, as will be clearly seen by an inspection of the formulæ for the horizontal thrust.

A large rise has three beneficial effects in the design, as follows :—

(1) The cross sectional area of the arch can be reduced.
(2) The reduced horizontal thrust on the abutment in conjunction with
(3) The reduced overturning moment obtainable by the lower elevation of the springings above the foundation level provide a substantial economy in the cost of the abutments.

There is a limit to the ratio of the rise to the span of the arch beyond which any increase in rise is not accompanied by a proportionate economy. This can be explained by the following considerations: The bending moments on the arch are practically independent of the rise of the arch. In consequence the tensile and compressive stresses in the arch sections resultant from bending are far in excess of the compression resultant from the normal thrust on the sections when the arch has a large rise in relation to its span.

The final distribution of the stresses over the arch cross sections thus becomes very uneven, and the resultant compressive stress is practically constant above a certain limit of rise-span ratio.

Tensile stresses also become high, and these should be avoided where possible in order that no hair cracks shall appear on the intrados or extrados of the arch. It must be borne in mind that an arch rib is subjected to a greater algebraic range of stress (positive and negative) than a beam, since a change in the position of the loading from one half of the bridge to the other may reverse the sign of the moment at any particular section without greatly altering its magnitude. Consequently the extreme fibres of the sections in an arch having a large rise-span ratio may be subjected to alternating stresses, which, if the tensile stresses are high, are obviously undesirable.

On the other hand, the rise should never be less than about one-tenth or one-twelfth of the span, because the thrust under this condition becomes so great as to make the abutments expensive and difficult to design.

With abutments resting upon ground having a low bearing capacity the design becomes practically impossible. For truly monolithic hingeless arches the flattest arch suitable for construction is 1/10 (see Chapter III., Art. 29).

For flat arches having rise-span ratios of 1/10 to 1/8 with intermittent ribs, suitable depths for the ribs at the crown are found to be :—

For spans of	75 feet	about 24 inches.
,, ,,	100 ,,	,, 30 ,,
,, ,,	150 ,,	,, 36 ,,

With spans up to 150 feet, the method of calculation given in this chapter involves errors which are negligible. Especially is this the case where temporary hinges are introduced, since the statically indeterminate stresses are correspondingly reduced.

Independent ribs to an arch exceeding 100 feet span should be connected at suitable intervals by stiffeners in order to interest the whole bridge when the wind is acting on one side.

Care must also be taken to see that the bridge platform is supported in the lateral direction, so that it may act effectively as a horizontal girder against lateral wind pressure.

83. Example of Hingeless Arch Bridge Design.—The following example is for an open spandril arch road bridge of usual proportions in which the refinements in design mentioned in the chapter on temporary hinges have been introduced.

The principal dimensions of the bridge are :—

Clear span between abutments . . .	80 feet.
Rise of the axis of the arch . . .	12 ,,
Width between parapets	60 ,,

The axis of the arch is a parabola of the second degree, for the reasons given in Art. 60.

Loading.—The bridge will be designed to carry the Ministry of Transport's Standard Loading for Highway Bridges. The width of the roadway being 40 feet, it is possible to have four trains on the bridge at the same time.

The superimposed load on the footpaths will be one hundredweight (112 lbs.) per square foot.

Maximum Permissible Stresses.—The maximum permissible stresses in any part of the structure will be :—

Compressive stress on concrete .	600	lbs./square inch.
Tensile stress on steel . .	16,000	,,
Shearing stress on concrete. .	60	,,
Shearing stress on steel . .	12,000	,,

Design of the Deck Platform.—This portion of the work will be designed in the manner outlined for the beam type of bridge. The deck beams will run longitudinally and will be spaced at 8 feet 2 inch centres.

The calculations of the roadway and footpath slabs, beams and columns follow the general principles described in the example of a beam bridge (Art. 107).

Design of the Arch Ribs.—It has already been explained that it is not possible to arrive at the dimensions or reinforcement of an arch member by direct calculation. Trials are made based upon the experience of similar structures, and the caloulation adjusted, should this be found necessary, when the stresses for the first case have been determined.

In the following example the main arch ribs carrying the roadway will be assumed to have a cross section of 36 inches wide by 27 inches deep at the crown, increasing to 36 inches wide and 36 inches deep at the springings.

The longitudinal reinforcement will consist of 12 bars, 1 inch diameter at the top and bottom, at the crown, increasing to 12 top and bottom bars, $1\frac{1}{4}$ inches diameter, at the springings.

The arch ribs will have the same spacing as that adopted for the longitudinal beams, since the columns carrying the deck construction are directly supported by the ribs. It will be noted that the ribs have a rise-span ratio of

$$\frac{12}{80} = \frac{1}{6 \cdot 67}$$

Use of Temporary Hinges.—Temporary hinges designed and situated as explained in Arts. 121 to 124 will be introduced at the crown and at the springings of the arch.

The advantages of this type of hinge are enumerated in Art. 118, among which is the very appreciable advantage gained by the fact that under its own weight the bridge acts as a three-hinged arch.

This enables the resultant thrusts and bending moments (if any) to be accurately calculated by a statically determinate method. As the dead load usually comprises the greater proportion of the total load imposed upon the arch, the introduction of temporary hinges gives to the calculations a degree of known accuracy greater than that possible with a wholly monolithic arch, for which several assumptions require to be made.

It will be presumed that the hinge sections are to be concreted in upon, or immediately before, the completion of the structure, and it will therefore be necessary to employ the appropriate calculations for the arch in its hingeless or encastré state for the superimposed loading.

The dead load near the springings is brought upon the arch ribs through columns (see Fig. 68). These concentrated loads induce bending in the parabolic three-hinged arch, as may be seen by a

short calculation. The amount of this bending, however, is inappreciable and may be neglected.

The total dead load on the rib will be computed as follows, using the unit weights for road-filling and concrete as given in the example of a beam bridge (Art. 107):—

The weight of the arch is calculated from the presumed sizes given on p. 107.

Weight of road metal	80′ × 8·17′ × 0·75′ × 110 lbs.	= 53,840 lbs.
Weight of roadway slab . . .	80′ × 8·17′ × 0·67′ × 150 „	= 65,200 „
Weight of longitudinal beams . .	2′ × 27′ × 0·75 × 150 „	= 6,080 „
Weight of columns .	2′ × 8′ × 0·5 × 150 „	= 1,200 „
Weight of arch rib .	84·66′ × 7·5 × 150 „	= 95,250 „
		P = 221,570 lbs.

Horizontal thrust due to dead load $= \frac{P.l}{8f}$ (see Art. 60)

$$= \frac{221{,}570 \times 80}{8 \times 12} = 184{,}600 \text{ lbs.}$$

Horizontal Thrusts and Bending Moments produced by the Rolling Load.—The spacing of the ribs and deck beams is the same as given in the example of a beam bridge, and it follows that the proportion of wheel loads from adjacent trains of the specified loading is the same as given in Fig. 111 of the above example. The assumption is made that both trains are travelling in the same direction. The diagram of equivalent wheel loads to be carried by one of the main ribs is given in Fig. 63.

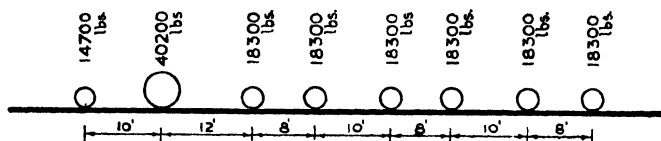



Fig. 63.

This load will now be applied to the standard influence line diagrams which have been calculated and constructed in Arts. 66 and 67. The correction factor for arch shortening entering into the formulæ for the ordinates of the influence lines, *i.e.*, $1 + \frac{45Ic}{4Av.f^2}$, need not be considered, because the elastic shortening of the arch,

due to the superload, is dealt with in the design of the temporary hinges. It should be noted that the standard influence lines are derived on the assumption that the moment of inertia of the arch section varies as the secant of the angle of the tangent to the arch axis at the section considered. Should this assumption not be fully realised, the resulting inaccuracy is comparatively small in arches of normal proportions and moderate spans, as in the present example.

It will be found that the sections subjected to the maximum stresses are situated at the crown, the haunches (about $0{\cdot}3l$ from either support) and at the springings.

Influence lines are given (Art. 67) for other sections, so that the reader may verify the stresses at these points.

Calculations will be given for the stresses at the following sections :—

(*a*) At the crown considering *positive* bending moments ;

(*b*) At the haunches, distance 24 feet from the left-hand end, considering *positive* bending moments ;

(*c*) At the springings considering *positive* bending moments ;

(*d*) At the springings considering *negative* bending moments.

At the *crown* the maximum positive bending moment will occur when the loads are in the positions indicated in Fig. 64.

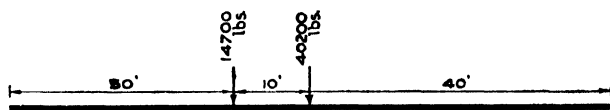



Fig. 64.

The value of this moment and the corresponding horizontal thrust will be obtained from the appropriate influence line for this section.

Positive Bending Moment.	Corresponding Thrust.
40,200 × 80 × 12 × 0·0469 = 1,810,000 inch-lbs.	40,200 × $\frac{80}{12}$ × 0·2344 = 62,800 lbs.
14,700 × 80 × 12 × 0·001 = 14,000 ,,	14,700 × $\frac{80}{12}$ × 0·207 = 20,300 ,,
1,824,000 inch-lbs.	83,100 lbs.

At the *haunches* the position of the load producing the maximum positive bending moments is given in Fig. 65.

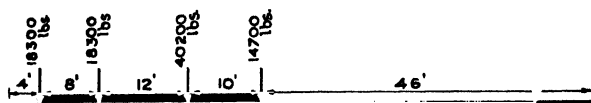



FIG. 65.

Positive Bending Moment.	Corresponding Thrust.
40,200 × 80 × 12 0 0595 = 2,296,000 inch lbs	40,200 $\frac{80}{12}$ 0 1654 = 44,300 lbs
14,700 × 80 × 12 × 0 005 = 71,000 „	14,700 × $\frac{80}{12}$ × 0 225 = 22,100 „
18,300 × 80 × 12 0 0135 = 237,000 „	18,300 × $\frac{80}{12}$ × 0 06 = 7,300 „
18,300 × 80 × 12 × 0 0012 = 21,000 „	18,300 × $\frac{80}{12}$ × 0 008 = 1,000 „
2,625,000 inch lbs	74,700 lbs.

At the *springings* the loads must be in the positions indicated in Fig. 66 to produce the maximum positive bending moments and in positions given in Fig 67 to produce maximum negative moments.

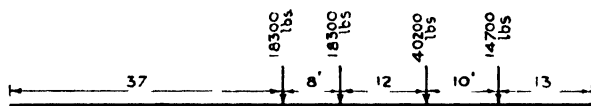



FIG 66.

Positive Bending Moment	Corresponding Thrust.
40,200 × 80 × 12 0 046 = 1,775,000 inch lbs	40,200 $\frac{80}{12}$ × 0 16 = 42,900 lbs.
14,700 80 × 12 × 0 0245 = 346,000 „	14,700 $\frac{80}{12}$ × 0 07 = 6,900 „
18,300 × 80 × 12 × 0 044 = 773,000 „	18,300 × $\frac{80}{12}$ × 0 228 = 27,800 „
18,300 × 80 × 12 0 021 = 369,000 „	18,300 × $\frac{80}{12}$ × 0 232 = 28,300 „
3,263,000 inch lbs	105,900 lbs.

FIG. 67.

Negative Bending Moment.	Corresponding Thrust.
$40{,}200 \times 80 \times 12 \times 0{\cdot}0607 = 2{,}343{,}000$ inch-lbs.	$40{,}200 \times \frac{80}{12} \times 0{\cdot}0304 = 8{,}200$ lbs.
$18{,}300 \times 80 \times 12 \times 0{\cdot}0527 = 926{,}000$ „	$18{,}300 \times \frac{80}{12} \times 0{\cdot}132 = 16{,}100$ „
$18{,}300 \times 80 \times 12 \times 0{\cdot}019 = 334{,}000$ „	$18{,}300 \times \frac{80}{12} \times 0{\cdot}194 = 23{,}700$ „
3,603,000 inch-lbs.	48,000 lbs.

Variation of Temperature.—When the temporary hinge sections of the arch ribs have been concreted, the arch has to be considered as hingeless or monolithic. This concreting is done at an assumed mean temperature, so that the maximum variations in the length of the rib will be approximately equal above and below the normal.

As explained previously in this chapter, a rise of temperature produces an increased thrust on the abutments and causes a negative bending moment above the level of the elastic centre and a positive moment below this level. (See Fig. 59, Art. 72.)

Conversely, a similar fall in temperature reduces the thrust upon the abutment and produces bending moments equal in amount, but of opposite sign, to those caused by a rise in temperature.

Formula (58) (Art. 72) gives the value of the horizontal thrust which acts through the elastic centre of the arch situated at a height of two-thirds the rise of the arch;

$$\therefore \qquad \text{Horizontal thrust} = \pm \frac{45 . n . \text{E} . \text{Ic}}{4f^2}.$$

The factor in Formula (58) $\left(45\frac{\text{Ic}}{\text{A}v}\right)$ is small in comparison with $4f^2$, and for this calculation is neglected.

The selected values in the above formula are :—

n = coefficient of expansion of concrete multiplied by the maximum variation from mean temperature. Taking this latter figure to be 30° F., $n = 0{\cdot}000006 \times 30 = 0{\cdot}00018$.

E = coefficient of elasticity of concrete = 2,000,000 lbs./square inch.

Ic = moment of inertia of the section (in terms of the concrete) at the crown.

$\therefore$ Moment of inertia of concrete $= \frac{bd^3}{12} = \frac{36 \times 27^3}{12} = 59{,}000$ inches^4.

Moment of inertia of steel $= (m - 1) \times 2 \times 9{\cdot}425 \times (13{\cdot}5 - 1{\cdot}5)^2$.

$= 14 \times 2 \times 9{\cdot}425 \times 12^2 = 38{,}000$ inches^4.

$\therefore$ Total equivalent moment of inertia of section $= 97{,}000$ inches^4.

f = rise = 144 inches.

Introducing these values in the above formula,

$$\text{Horizontal thrust} = \pm \frac{45 \times 0{\cdot}00018 \times 2{,}000{,}000 \times 97{,}000}{4 \times 144 \times 144}$$

$= \pm$ 19,000 lbs.

The bending moment at the three sections that are to be investigated is equal to $H\left(y - \frac{2}{3}f\right)$, where y is height of considered section above springings.

Bending moment at the crown

$$= 19{,}000 \times \left(144 - \frac{2}{3} \times 144\right) = 912{,}000 \text{ inch-lbs.}$$

Bending moment at the haunches

$$= 19{,}000 \times \left(121 - \frac{2}{3} \times 144\right) = 475{,}000 \text{ ,,}$$

Bending moment at the springings

$$= 19{,}000 \times \left(0 - \frac{2}{3} \times 144\right) = -1{,}824{,}000 \text{ ,,}$$

Summary of Bending Moments and Corresponding Horizontal Thrusts.—Collecting together the bending moments and thrusts produced by the dead load, live load and changes of temperature, the final results can be tabulated as follows :—

At the Crown.—

	Positive Bending Moment.	Corresponding Thrust.
Dead load . . .	0	184,600 lbs.
Live load . . .	1,824,000 inch-lbs.	83,100 ,,
Temperature. . .	912,000 ,,	– 19,000 ,,
Total . . .	2,736,000 inch-lbs.	248,700 lbs.

At the Haunches.—

	Positive Bending Moment.	Corresponding Thrust.
Dead load . . .	0	184,600 lbs.
Live load . . .	2,625,000 inch-lbs.	74,700 ,,
Temperature. . .	475,000 ,,	– 19,000 ,,
Total . . .	3,100,000 inch-lbs.	240,300 lbs.

At the Springings.—

	Positive Bending Moment	Corresponding Thrust.
Dead load . . .	0	184,600 lbs.
Live load . . .	3,263,000 inch-lbs	105,900 ,,
Temperature. . .	1,824,000 ,,	19,000 ,,
Total . . .	5,087,000 inch-lbs.	309,500 lbs.

At the Springings.—

	Negative Bending Moment	Corresponding Thrust.
Dead load . . .	0	184,600 lbs.
Live load . . .	3,603,000 inch-lbs.	48,000 ,,
Temperature. . .	1,824,000 ,,	– 19,000 ,,
Total . . .	5,427,000 inch-lbs.	213 600 lbs.

Maximum Stresses at the Sections.—The sections are subject to a normal thrust acting tangentially to the curve of the arch axis, and a bending moment.

The formula for the normal thrust on the rib is $N = \frac{H}{\cos a}$.

The maximum stresses upon a section under bending and direct thrust can be found by means of the well-known formula S max. $= \frac{N}{Ae} \pm \frac{M.y}{Ie}$ (see Art. 81), where

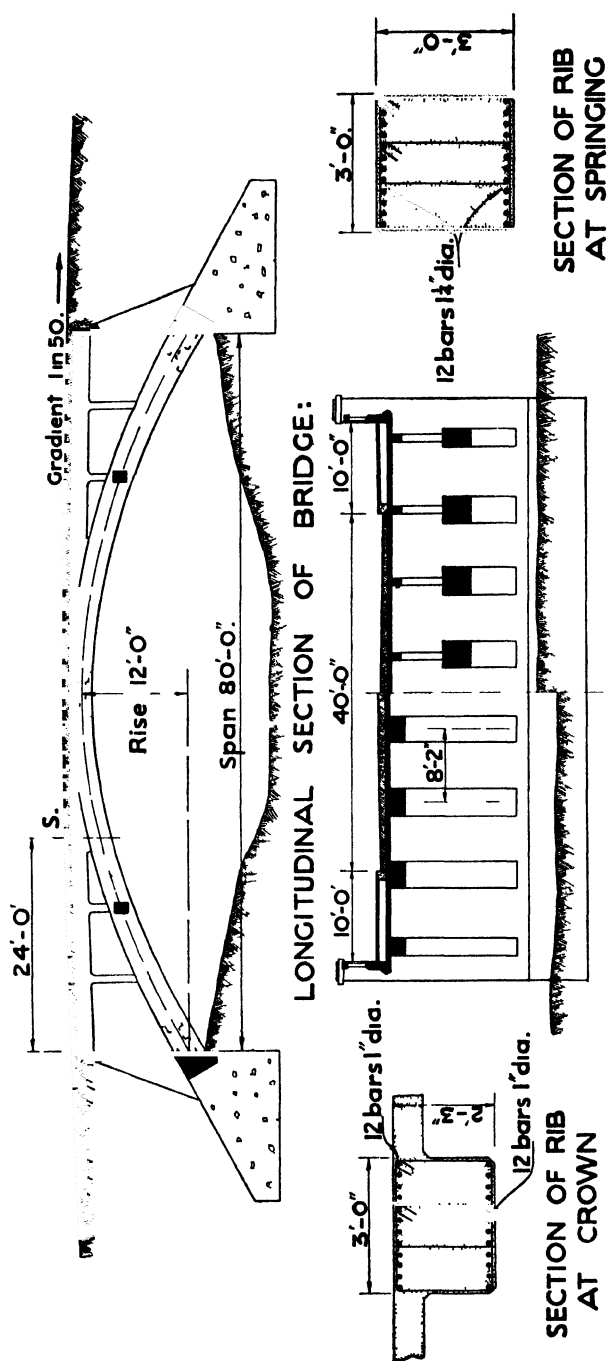



FIG 68 —Example of Hingeless Arch Bridge. (See Art. 83.)

M is the total bending moment,
Ae the equivalent cross sectional area of rib in terms of the concrete,
Ie the equivalent moment of inertia of rib in terms of the concrete,
and y the distance of the extreme fibre from the neutral axis of the section.

For the general theory of reinforced concrete sections under bending and direct thrust the reader is referred to any of the standard text-books dealing with this subject.

If the tensile stress on the concrete exceeds the breaking strength of the material (about 200 lbs. per square inch), the above formula will not be applicable. Resort must then be made to formulæ which neglect the resistance of concrete in tension.

The above formulæ will be applied to the various sections using the results contained in the summary of bending moments and horizontal thrusts, and adopting the dimensions of the arch rib allowed for, when computing the dead load and calculating the temperature effects.

If the final stresses are found to be too high or low, the whole of the calculations require to be amended and a fresh trial made.

At the Crown.—The section, as stated previously, will be 36 inches wide and 27 inches deep (see Fig. 68). It will be reinforced with 12 bars, 1 inch diameter on both faces, having a concrete cover of 1 inch.

Equivalent area of rib (in terms of the concrete):

Concrete $= 36 \times 27 = 972$ square inches.
Steel $= 2(15 - 1) \times 9{\cdot}425 = 264$,,

$Ae = 1{,}236$ square inches.

Equivalent moment of inertia of rib (in terms of the concrete) (see p. 112) $= 97{,}000$ inches^4.
The normal thrust $N = H = 248{,}700$ lbs.
Bending moment $M = 2{,}736{,}000$ inch-lbs.

Maximum Compressive Stress on Extrados.—

$$S \text{ max.} = \frac{N}{Ae} + \frac{M.y}{Ie}$$

$$= \frac{248{,}700}{1{,}236} + \frac{2{,}736{,}000 \times 13{\cdot}5}{97{,}000};$$

$$201 + 381 = 582 \text{ lbs. per square inch.}$$

Maximum Tensile Stress on Intrados.—

S max. $= 201 - 381 = 180$ lbs. per square inch.

At the Haunches.—Section 36 inches wide and 28½ inches deep. Reinforcement 12 bars 1 inch diameter on both faces.

Equivalent area of rib :

Concrete $= 36 \times 28\cdot5 \quad = 1{,}026$ square inches.

Steel $= 2\,(15 - 1) \times 9\cdot425 = 264$,,

$Ae = 1{,}290$ square inches.

Equivalent moment of inertia of rib :

Concrete $1{,}026 \times \frac{28\cdot5 \times 28\cdot5}{12} = 69{,}450$ inches^4.

Steel $264 \times (14\cdot25 - 1\cdot5)^2 = 42{,}930$,,

$Ie = 112{,}380$ inches.

The angle of inclination of the arch axis to the horizontal

$$a = 13^\circ\ 30'.$$

The normal thrust $N = \frac{H}{\cos a} = \frac{240{,}300}{0\cdot9724} = 247{,}100$ lbs., and the bending moment $M = 3{,}100{,}000$ inch-lbs.

Maximum Compressive Stress on Extrados.—

$$S\ max. = \frac{N}{Ae} + \frac{M.y}{Ie}$$

$$= \frac{247{,}100}{1{,}290} + \frac{3{,}100{,}000 \times 14\cdot25}{112{,}380};$$

$$192 + 393 = 585 \text{ lbs. per square inch.}$$

Maximum Tensile Stress on Intrados.—

$$S\ max. = 192 - 393 = 201 \text{ lbs. per square inch.}$$

At the Springings.—Section 36 inches wide and 36 inches deep. Reinforcement 12 bars 1¼ inches diameter on both faces.

Equivalent area of rib :

Concrete $= 36 \times 36 \quad = 1{,}296$ square inches.

Steel $= 2\,(15 - 1) \times 14\cdot724 = 412$,,

$Ae = 1{,}708$ square inches.

Equivalent moment of inertia of rib :

Concrete $= 1{,}296 \times \frac{36 \times 36}{12} = 140{,}000$ inches^4.

Steel $= 412 \times (18 - 1\cdot63)^2 = 110{,}000$,,

$Ie = 250{,}000$ inches.

The angle of inclination of the arch axis to the horizontal $a = 31^\circ$.

(*a*) *Positive Bending Moments.*—

The normal thrust $N = \dfrac{H}{\cos a} = \dfrac{309{,}500}{0{\cdot}8572} = 361{,}100$ lbs.,

and the bending moment M = 5,087,000 inch-lbs.

Maximum Compressive Stress on Extrados.—

$$S \text{ max.} = \frac{N}{Ae} + \frac{M.y}{Ie}$$

$$= \frac{361{,}100}{1{,}708} + \frac{5{,}087{,}000 \times 18}{250{,}000}$$

$$= 211 + 366 = 577 \text{ lbs. per square inch.}$$

Maximum Tensile Stress on Intrados.—

S max. = 211 − 366 = 155 lbs. per square inch.

(*b*) *Negative Bending Moments.*—

The normal thrust $N = \dfrac{H}{\cos a} = \dfrac{213{,}600}{0{\cdot}8572} = 249{,}200$ lbs.,

and the bending moment M = 5,427,000 inch-lbs.

Maximum Compressive Stress on Intrados.—

$$S \text{ max.} = \frac{N}{Ae} + \frac{M.y}{Ie}$$

$$= \frac{249{,}200}{1{,}708} + \frac{5{,}427{,}000 \times 18}{250{,}000}$$

$$= 146 + 391 = 537 \text{ lbs. per square inch.}$$

Maximum Tensile Stress on Extrados.—

S max. = 391 − 146 = 245 lbs. per square inch.

By applying the formula which neglects the tensile resistance of the concrete to the latter section the maximum compressive stress on the concrete is 560 lbs. per square inch, and the maximum tensile stress on the steel is 5,280 lbs. per square inch.

The stresses in the arch are thus within the permissible limits, and the sizes assumed could therefore be adopted.

CHAPTER VI

LONG-SPAN ARCHES

84. Introduction.—The method of calculation for hingeless arches given in the preceding chapter can be used for spans up to 150 feet without involving errors of any consequence.

For arches of greater span, more accurate assumptions and greater refinement of treatment is desirable. The method given in the following articles takes into account more precisely the actual conditions obtaining in most arches, and has been employed in the design of some of the largest spans yet constructed. The essential difference between this method and that already given is in the form of longitudinal arch axis and the inertia variation adopted.

The new method is applicable in principle to any span, but for small bridges of the open spandril type it may be unsuitable because, in the following articles, the superimposed load has been assumed to be uniformly distributed along the arch, which frequently, in small bridges, is not the case.

85. Longitudinal Arch Axis.—The method given in the previous chapter presumes for convenience that the arch axis curve coincides with the line of resistance from the total dead loading, and that this curve is a parabola of the second degree.

Such an assumption is only true when the dead load is uniform over the entire length of the arch, and, though this can seldom, if ever, be the case for an arch bridge, the error is unimportant for moderate spans. With large spans, however, and especially those with solid spandrils, the error caused by making the above assumption cannot be regarded as unimportant.

It is obvious that with all arch bridges the intensity of dead loading is greater at the springings than at the crown.

With arch bridges having solid spandrils this increase of dead weight towards the springings is greater than with the open spandril type, and is greater still where the spandrils are earth filled. The latter type, however, is unusual for large spans.

The gradual increase in dead weight of an arch bridge from the crown to the springings is approximately parabolic, and is taken to be so in practice. Any slight variation that the actual average loading increase may give is negligible.

Instead, therefore, of taking the curve of the arch axis to be a "square" parabola, as may be presumed with uniform loading, it

(To face page 118)

FIG. 69. PLOUGASTEL BRIDGE OVER ELORN RIVER, FRANCE. MAIN SPANS 613 FT.

is necessary to find the curve that will coincide with the line of pressure for loading, part of which is uniform and part variable.

There is yet a further refinement affecting the arch curve. For long-span arches it is usual to consider the superimposed loads (consisting of vehicular traffic) as being uniformly distributed along the arch span.

When the bridge is entirely covered by this distributed superload the line of pressure from all the loading, both dead and live, may deviate appreciably from the arch axis curve (drawn for the dead loading alone) and, by the eccentricity of thrust produced, cause considerable bending moments at certain arch sections. Should the curve of the arch axis be made exactly to coincide with the line of resistance, inclusive of the entire distributed superload, the arch will be subjected to similar but reversed bending moments when in the unloaded condition.

In the following method the arch axis is made to coincide with the line of pressure produced by the uneven dead load, plus *one-half* of the distributed superload.

When, therefore, no traffic is upon the bridge, there are unequal stresses at the intrados and extrados. When the full distributed superload is present, stresses are also unequal, but in this case reversed from the unloaded condition.

As the reader will see from the following articles, the above consideration in respect to the arch curve results in equal positive and negative maximum bending moments at all sections, and therefore ensures the minimum range of thrust eccentricity above and below the arch axis.

The uniformly distributed superload mentioned above will, in the case of the Ministry of Transport Standard Loading, comprise the weight of the trailers considered as distributed loads. The excess weights from the engine driving wheels are taken as concentrated loads at intervals of 75 feet, but are not taken into account in ascertaining the curve of the arch axis.

86. Inertia Variation.—The assumption made in the previous method (Art. 65) that $I = Ic. \sec a$ would, if produced in the work, result in an arch having a depth sensibly constant throughout its length. Experience shows that the most suitable depth for an arch at the springings is about 1·5 times that at the crown, causing the cross sectional moment of inertia at the springings to be about 3·4 times that at the crown. An inertia variation allowing for this form of arch is therefore adopted, viz :—

$$I = \frac{Ic}{\cos a \, . \, (1 - 0{\cdot}7m^4)} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Note.—*m* is in terms of half span (see Fig. 70).

87. General Formulæ for Influence Lines.—For convenience in the mathematical treatment the origin of the co-ordinates has been changed from the left-hand support (Art. 64, Fig. 52) to the elastic centre of the arch (Fig. 70).

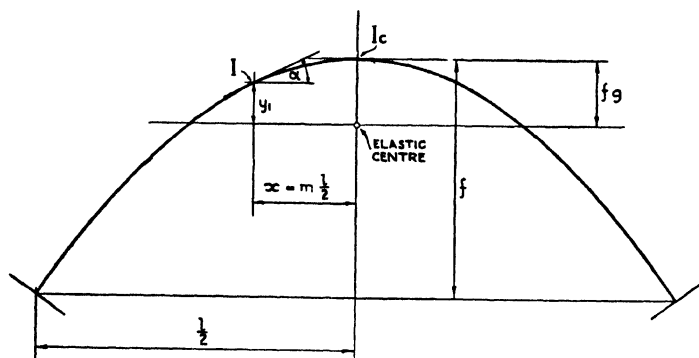



FIG. 70.

The new origin lies on the vertical centre line of the arch and at a distance $f \cdot g$ below the crown, where

$$f(1-g) = \frac{\int \frac{y \cdot ds}{E \cdot I}}{\int \frac{ds}{E \cdot I}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the integrations being taken over the half span.

The expression for the bending moment at any section (see formula 15, page 76), becomes

$$Ms = M_f - H \cdot y_1 - H\frac{(q_a + q_b)}{2} - H(q_a - q_b)\frac{x}{l} \quad . \quad . \quad (3)$$

The vertical ordinates (y_1) are measured from a horizontal line passing through the elastic centre and the horizontal distances x from the vertical centre line.

The three unknowns requiring to be evaluated (see Art. 64, first paragraph) are the horizontal thrust (H) and the bending moment functions

$$H \cdot \frac{(q_a + q_b)}{2}$$

and
$$H \cdot \frac{(q_a - q_b)}{l}$$

Let these quantities be denoted by U and V respectively; then

$$Ms = M_f - H \cdot y_1 - U - V \cdot x$$

The three unknowns are represented by the following expressions

$$H = \frac{\int_A^B \frac{M_f \cdot y_1 \cdot ds}{E \cdot I}}{\int_A^B \frac{y_1^2 \cdot ds}{E \cdot I} + \int_A^B \frac{dx}{E \cdot A}} \qquad U = \frac{\int_A^B \frac{M_f \cdot ds}{E.I.}}{\int_A^B \frac{ds}{E \cdot I}} \qquad V = \frac{\int_A^B \frac{M_f \cdot x \cdot ds}{E \cdot I}}{\int_A^B \frac{x^2 \cdot ds}{E \cdot I}}$$

The expression for the horizontal thrust (H) contains both the thrust produced by the vertical loading and also the secondary thrust resulting from distortion due to compression. In the following method this secondary thrust has been separated, as before, from the primary thrust, permitting the latter to be standardised. The secondary thrust and its effects can then be dealt with separately.

It has been necessary to presume a number of graduated load conditions and to calculate the ordinates to the bending moment influence lines, etc., for each of these conditions. Owing to the more involved inertia variation and curve of the arch axis, the calculation of the thrust and bending moment ordinates from the above formulæ is laborious and is therefore not given.

The general application of this method, however, follows that explained in the preceding chapter, and in the form given enables it to be applied in practice with a minimum of labour.

88. Loading.—For arches of considerable span the effect of a series of wheel loads may be represented by an equivalent uniformly distributed load, with additional point loads for the heavier wheels, if any. Taking, for example, the standard loading of the Ministry of Transport (page 7) a distributed load of 250 lbs. per square foot can be assumed with additional point loads at 75-feet intervals, representing the excess load from the engine driving wheels (see Fig. 71). These point loads amount to 2,700 lbs. on a longitudinal strip of roadway 1 foot wide. The figures given include 50 per cent. for impact.

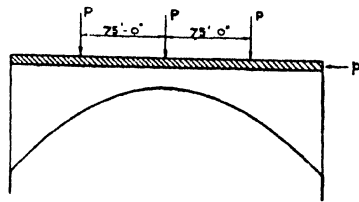



FIG. 71.

It will be observed that this assumption is conservative, since it presumes the spacing of the engine driving wheels to be the same as the trailer wheels, and also that the front axle load is equal to the trailer axle loads.

89. Determination of Arch Axis.—The curve of the arch axis

for the condition described above is found to approximate closely to a parabola of the fourth degree, viz :—

$$y = f\,[1 - (1 - Z)\,.\,m^2 - Z\,.\,m^4] \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where m is the horizontal distance from the centre line of the arch in terms of half span $\frac{l}{2}$, and Z a load function explained below.

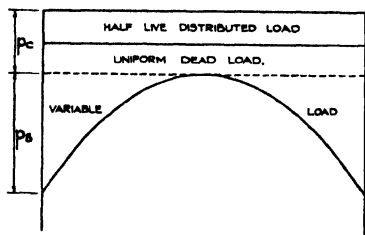



FIG. 72.

A close approximation of the value of Z can be obtained as follows (see Fig. 72) :—

$$Z = \frac{p_s}{6p_c + p_s} \quad . \quad . \quad (5)$$

Note.—In the above formula the variable portion of the dead loading is presumed to have a parabolic variation such as would be given were the spandrils filled solid.

90. Horizontal Thrust for Dead Load.—The horizontal thrust resultant from the dead load may be ascertained by dividing the arch into a convenient number of vertical strips and calculating the weight of each strip. These weights are then multiplied by the appropriate ordinates to the influence line diagram for horizontal thrust. The sum of these products for the whole arch will give the horizontal thrust from the loads in question.

The influence line diagram to be employed for this operation is the one calculated for the applicable value of Z. The influence line curves given can be employed for intermediate values of Z by direct proportion.

If temporary hinges are to be provided, the arch will be in a static three-hinged condition during construction, and the horizontal thrust may be calculated in the usual manner from polar and funicular diagrams. A close approximation to the dead load horizontal thrust for this latter condition is

$$Hd = \left(p + \frac{p_s}{6}\right).\frac{l^2}{8\,.f} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

p_c in this case will represent the uniform dead load per lineal foot.

91. Arch Sections to Investigate.—The sections requiring investigation are usually—

the crown	$m = 0$
the haunches	$m = 0{\cdot}4$
the springings	$m = 1{\cdot}0$

For very large spans, intermediate sections should be studied.

92. Maximum Moments and Thrusts.—For any cross-section the maximum bending moment due to the uniformly dis-

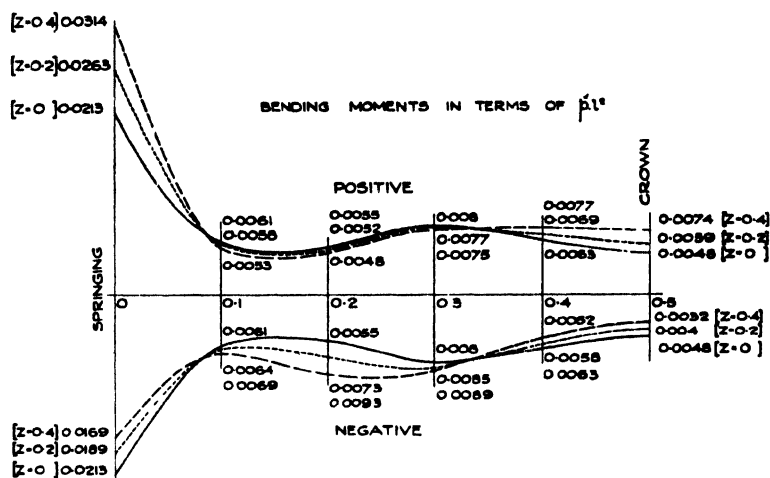



FIG. 73.—Envelope Curves of Maximum Bending Moments due to Distributed Loading.

tributed live loading occurs when certain lengths of the span are loaded.

For the maximum positive moments the loading must cover the appropriate influence line curve above the horizontal closing line,

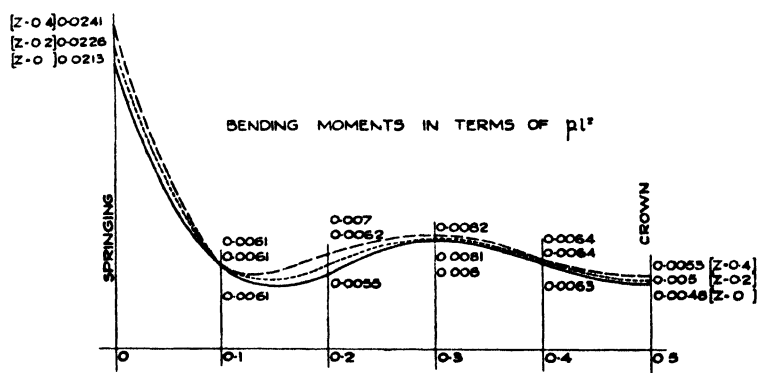



FIG. 74.—Envelope Curves for "Mean of Moments."

and below this closing line for maximum negative moments (see Figs. 75, 76 and 77).

Since the arch is already presumed to be loaded with one-half of the distributed live loading, it is necessary :—

Firstly to add one-half of the live load spread over the lengths to produce maximum moments.

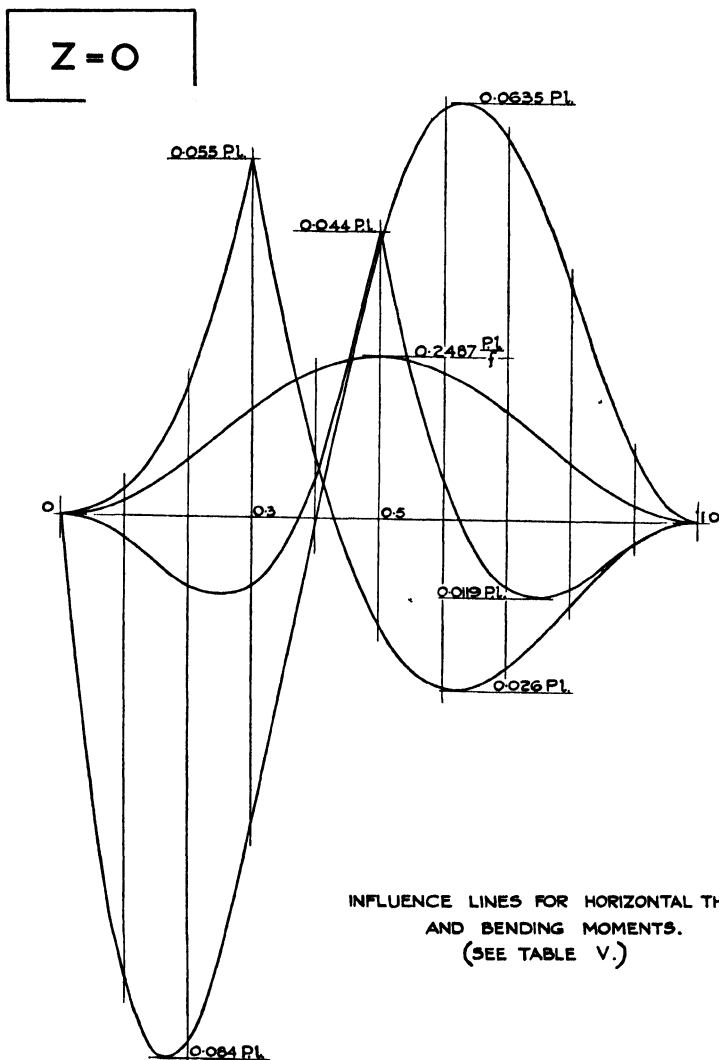



FIG. 75.

Secondly, to subtract one-half of the live load for the remaining lengths.

Referring to Fig. 73 (Envelope of Moments), it will be seen that curves and values are given for the maximum positive and negative moments (due to uniformly distributed loading) at eleven equally-

spaced sections. (The envelope curves are, of course, symmetrical about the mid-span section.)

If, therefore, one-half of the sum of the maximum positive and

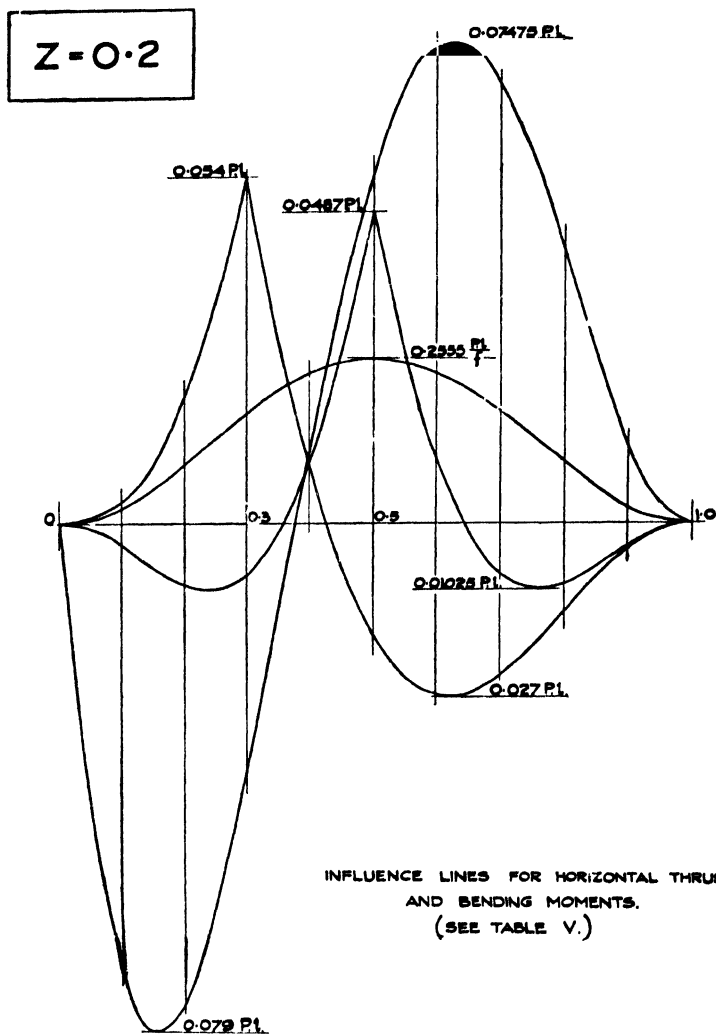



Fig. 76.

negative moment values is taken for any particular section, it gives the result required.

In Fig. 74 (Mean of Moments Envelope), these values have been plotted for each of the eleven sections. It is, therefore, only necessary

to multiply the appropriate values by $p \, . \, l^2$ to obtain the maximum moments due to various applications of the full superload (p = dis-

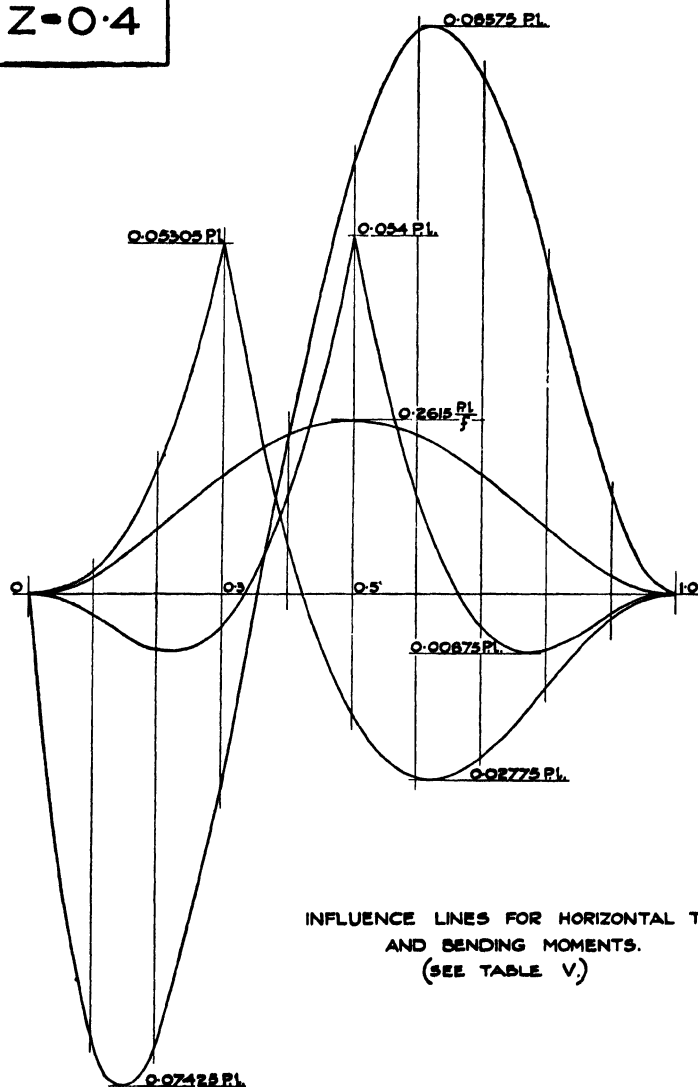



FIG. 77.

tributed live load per lineal foot). At each section the moments can be either positive or negative.

The accompanying values of the horizontal thrust are given in Table T (Horizontal Thrusts from Distributed Live Loads).

TABLE T

HORIZONTAL THRUSTS FROM DISTRIBUTED LIVE LOADS

		Springing	0·1	0·2	0·3	0·4	Crown
Z = 0	For Positive Moments.	·086	·069	·025	·045	·062	·059
	For Negative Moments.	·039	·056	·100	·080	·063	·066
Z = 0·2	For Positive Moments.	·097	·068	·023	·047	·069	·069
	For Negative Moments.	·035	·064	·109	·084	·063	·063
Z = 0·4	For Positive Moments.	·104	·066	·022	·049	074	·080
	For Negative Moments.	·032	·070	·114	·087	·062	·056

Thrusts in terms of $\frac{p \,.\, l^2}{f}$

The additional moments and thrusts due to the intermittent point loads are obtained from the standard influence lines, the loads being placed in positions causing maximum bending moments, as explained in previous articles. The bending moment and thrust ordinates so found for each section investigated have to be multiplied by $P \,.\, l$ and $\frac{P \,.\, l}{f}$ respectively, where P equals the applied concentrated load.

TABLE V

ORDINATE VALUES FOR INFLUENCE LINES IN FIGS 75, 76 AND 77

		0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
Z = 0	Horiz. Thrust .	0	·0229	·0875	·1656	2262	·2487	·2262	·1656	·0875	·0229	0
	M. at Crown .	0	−·0035	−·0103	−·0103	+·0059	+·044	+·0059	−·0103	−·0103	−·0035	0
	M at 0·3 l .	0	+ 0038	+ 0204	+ 055	+·0096	−·0163	−·0257	−·0226	−·0130	−·0035	0
	M. at Springing .	0	−·0715	−·0812	− 0476	−·0014	+·0427	+·0628	+·0583	+·0355	+·0103	0
Z = 0·2	Horiz Thrust .	0	·025	0934	·1729	·2334	·2556	·2334	·1729	·0934	·025	0
	M. at Crown .	0	−·0035	−·0095	−·0078	+·0100	+·0487	+·0100	−·0078	−·0095	−·0035	0
	M. at 0·3 . l .	0	+·0036	+·0196	+·054	+ 0087	−·0173	−·0267	−·0236	−·0137	−·0038	0
	M. at Springing .	0	− 0694	−·0745	−·0379	+·0127	+·0543	+·0741	+·068	+·0423	+·0124	0
Z = 0·4	Horiz. Thrust .	0	·0271	·0989	·1797	·2399	·2615	·2399	·1797	·0989	·0271	0
	M. at Crown .	0	− 0033	−·0083	− 0048	+·0147	+ 054	+·0147	−·0048	−·0083	−·0033	0
	M. at 0·3 . l .	0	+·0033	+·0169	+·0531	+·0078	− 0182	−·0276	−·0246	−·0144	−·004	0
	M. at Springing .	0	− 067	− 0677	+·0281	+ 0238	+·0655	+·0852	+·0779	+·049	+·0147	0

Thrusts in terms of $\frac{P \,.\, l}{f}$ *Moments in terms of* $P \,.\, l$

The horizontal thrust and bending moment influence lines have been calculated for values of Z = 0 (uniform dead load), Z = 0·2 and Z = 0·4, and in each case upon the inertia variation (1) and arch axis curve (4). The ordinates to these influence lines are given in Table V, and enable curves to be plotted to a convenient scale for use.

93. Arch Shortening Effects.—It is necessary to include the bending moments and thrusts from the various secondary effects described in Art. 28.

The effect of a change in the arch length can be represented by a horizontal force acting through the elastic centre of the arch (see Fig. 70).

For a shortening of the arch the induced forces act toward the abutments and cause a reduction in the primary thrust.

Sections above the elastic centre will be subjected to positive moments, and those below to negative moments.

Conversely, for a lengthening of the arch, the secondary force will increase the primary thrust; sections above the elastic centre having negative moments, and those below positive moments.

A close approximation to the value of the secondary force is

$$H = \pm \frac{(17 + 8 . Z) E . Ic . n}{f^2} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The maximum bending moments resulting from the secondary effects will be

$$\text{at the crown: } M = H . f . g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$
$$\text{at the springings: } M = H . f(1 - g) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where g is the level of the elastic centre (in terms of the rise) through which the resultant horizontal thrust is acting, the value of g may be determined as follows :—

$$g = f\,(0{\cdot}2716 - 0{\cdot}1292\ Z) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

94. Shearing Forces.—The shearing force V_t acting upon any arch section may be obtained from the formulæ given on page 104. It is not possible to standardise influence lines for shearing forces, since these vary with the curve of the arch, as will be seen from the formulæ referred to.

With arches having the properties considered in this chapter, the vertical reaction (V_1) at the left-hand support will be

$V_1 = Va. + V$ when the load is nearer the left support.
$V_1 = Va. - V$ when the load is nearer the right support.

Where Va is the reaction for a freely supported beam, and V is the function expressed in the formula on page 121.

The angle of inclination (α) of the rib at the section under consideration can be found as follows :—

$$\tan \alpha = \frac{4 . f}{l} [(1 - Z) . m + 2 . Z . m^3]$$

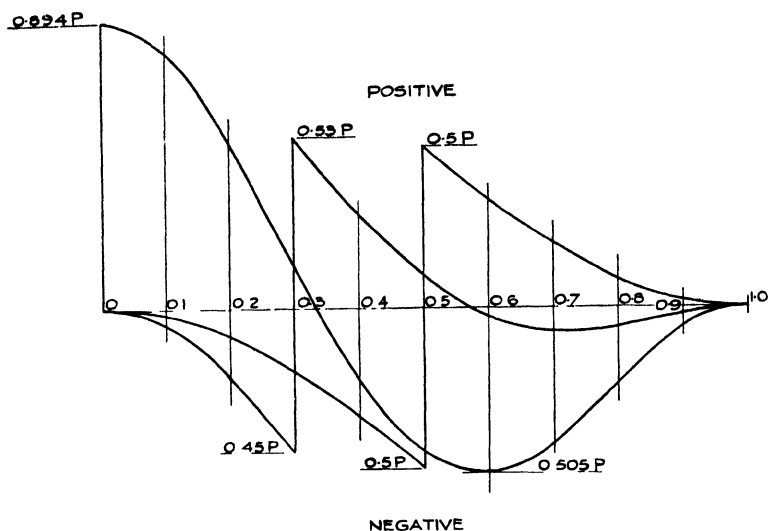



FIG. 78.—Influence Line Curves for Shearing Force Rise-span Ratio 1/8 (Z = 0).

Fig. 78 gives the influence line curves for the shearing forces on a parabolic arch of the second degree (Z = 0), and a rise-span ratio of one to eight.

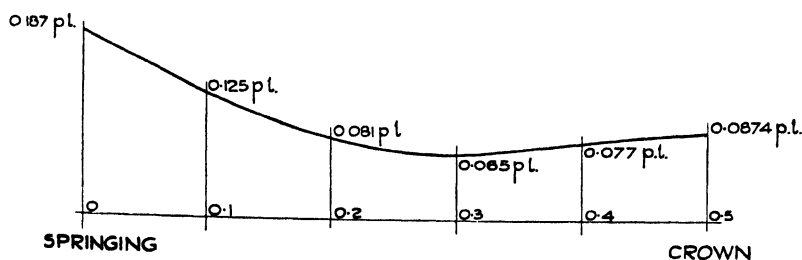



FIG. 79.—Envelope Curve for Maximum Shearing Force Rise-span Ratio 1/8 (Z = 0).

Fig. 79 gives the half span envelope curve for shearing forces obtained from Fig. 78.

95. Application of Method.—The procedure when employing the above method is as follows :—

Having decided generally on the form of arch likely to be suitable,

a diagram of the loading can be set up, and from this the appropriate value of Z determined. With this constant Z known, the exact curve of the arch axis can be drawn by employing equation (4).

If the arch may be considered to be in the three-hinged condition under dead load (*i.e.*, by employing temporary hinges), the value of the horizontal thrust (Hd) can be obtained from the formula (6).

Should the arch be constructed monolithic with the abutments, then Hd must be ascertained, as described in Art. 90.

It is not necessary to calculate the bending moments produced by the dead loading. By the use of the " Mean of Moments " curves the bending moments for the most severe condition of live loading, including those due to the dead loading, can be read direct.

The principal sections—*i.e.*, springing, haunches and crown (see page 122)—are investigated, and the maximum positive and negative bending moments found by reading the values direct from Fig. 74, giving the " Mean of Moments " envelope. The values ascertained from the applicable curves have to be multiplied by $p \cdot l^2$ (as explained on page 126). The corresponding values of the horizontal thrust due to the distributed loading are ascertained by employing Table T.

If there are, in addition, intermittent point loads to be considered (see Fig. 71), then the maximum bending moments produced at each of the above sections are obtained from the standard influence lines, using either Fig. 75, 76 or 77, depending, as before, on the value of Z. These bending moment ordinates, as measured, have in every case to be multiplied by $P \cdot l$.

Values for the horizontal thrust resulting from the point loads can be obtained in the same manner and at the same time as those for the bending moment, each having to be multiplied by $\frac{P \cdot l}{f}$.

The addition of distributed and concentrated live load moments (both positive and negative), together with the sum of the horizontal thrusts accompanying these, plus that calculated for the dead load (Hd), comprise the total moments and thrust directly due to the vertical loading.

Actual or equivalent variations in the length of the arch, due to the several causes dealt with in Art. 28, have next to be considered. If these are not compensated in some way it is necessary to add to the above bending moment and thrust totals, the supplementary values due to all shortening effects. By introducing the appropriate value of n (Art. 29) into formula (7), the horizontal force induced can be ascertained. With this force known, the accompanying bending moments can be calculated from formulæ (8) and (9).

For large spans it is desirable to eliminate the secondary stresses in order to prevent the final stresses becoming very unequal, with

the probability of some sections being overstressed. Suitable devices and their application in practice are explained in Chapter IX.

The reader should note that temperature variations may also cause an increase of length, and this must be considered when obtaining the final stresses upon any arch section. The same method is employed as for shortening; the forces and moments, however, being of opposite sign.

96. General.—It is not practicable to attempt any guidance as to the type most suitable for any particular arch of long span. There are so many factors likely to influence its form, and it is very rare that the conditions pertaining to any one bridge permit of a previous type being adopted without appreciable change.

Lateral rigidity is frequently an important factor in long-span arches. Although the span of the bridge may be considerable, it may only be required to carry a second-class road, and therefore, relatively narrow in consequence. Such arches should be made rigid transversely, the ribs either being connected by a vault slab at the intrados or extrados. The vault slabs may not necessarily extend along the whole length of the arch, it sometimes being sufficient to provide vault slabs at the intrados near the springings and at the extrados in the neighbourhood of the crown. In some cases, both top and bottom vault slabs are provided, resulting in a cellular construction. Such an arrangement has several advantages, and enables a graceful profile with increased headroom under the bridge to be provided.

Where the arch ribs are exposed on elevation, the depth should increase from the crown to the springings proportionately to the ordinates of a parabola. For arch ribs not exposed, and where economy in weight is of importance, the central portion, amounting to approximately 60 per cent. of the span, may be of constant depth, and thence increasing for the remaining 15 per cent. of the span at each end to the requisite thickness at the springings.

CHAPTER VII

GIRDER BRIDGES

97. General Notes.—There are two principal classes of girder or beam bridges so far as the cross sectional arrangement is concerned. These comprise bridges supported by a number of longitudinal beams placed under the deck slab, as in Fig. 80, no cross transverse supporting members being introduced in this case, and, alternatively, bridges of relatively narrow width, the main supporting members being formed by the parapet girders, with thick

Fig. 80.

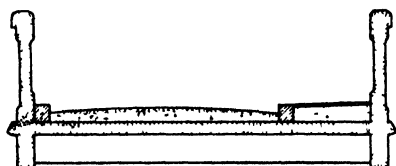



Fig. 81.

slab or transverse beams and thinner slab, forming the deck construction, as in Fig. 81. The suitability of these alternative types depends upon the local and specified conditions.

For spans up to 20 feet an ordinary flat slab type of bridge is usually suitable and frequently more economical than other types, having regard to the cost of construction in this country, especially the cost of joinery work.

Due regard should be paid to appearance in designing girder bridges. It is not necessary to provide elaborate ornamentation to obtain a pleasing finish, but care should be taken to see that the general lines are pleasing, and, if possible, to avoid the appearance of heaviness so often noticeable in this class of structure.

In many cases this is difficult, especially when large areas of wing walls are necessary, and where limitations regarding the spans and general profile exist.

With care, however, the worst of these structures can usually be made inoffensive to the eye, and as far as possible every effort should be directed to this aspect of the problem, since the architectural treatment of a bridge is constantly and permanently apparent, while its other virtues or faults are likely to pass unnoticed.

The question of the appearance of reinforced concrete construction as applied to bridges is relative, and with steelwork construction as the criterion, its superiority is indisputable.

In ordinary country districts, for many reasons, reinforced concrete should be the only material ever used for bridge construction, and as its efficiency, permanency, economy and general suitability become more widely appreciated by the local responsible authorities, its adoption will, if it has not already done so, become the general rule.

FIG. 82.—Elevation of the Madeleine Bridge, Nantes.

Beam bridges of the ordinary type do not often contain very great spans, 50 or 60 feet being the maximum single span usually adopted. The total length of this class of bridge, however, is frequently very great, comprising a number of moderate spans with intermediate supports, generally of the trestle type.

In special cases it is found necessary to construct bridges with greater spans than mentioned above, having main supporting members of the beam type.

For example, the Madeleine Bridge at Nantes comprises double cantilevers connected by a concealed central span. The ends of the side spans are partially supported on roller bearings. An elevation of this bridge is given in Fig. 82, the central span being 230 feet. The design of a bridge of this form, whilst presenting no particular difficulty, is somewhat laborious where the spans are continuous, owing to the constantly varying transversal inertia of the main longitudinal beams.

Another example is the bridge at Rue de La Fayette, spanning the railway at the Gard de l'Est, Paris. This bridge is of the parapet lattice girder type, and has a total length of 486 feet. It has one intermediate support, the longest girder having a span of 254 feet, and an overall depth of 34 feet.

98. Arrangement with Beams beneath Deck Slabs.—This arrangement is best suited for most bridges, and it forms a very simple and economical solution. The lateral spacing of the longitudinals varies between 6 and 10 feet, the exact amount depending upon the width of the roadway and the loading to be carried.

As a general rule, the lateral spacing of the main longitudinal beams is fixed by the economical thickness of the roadway slab

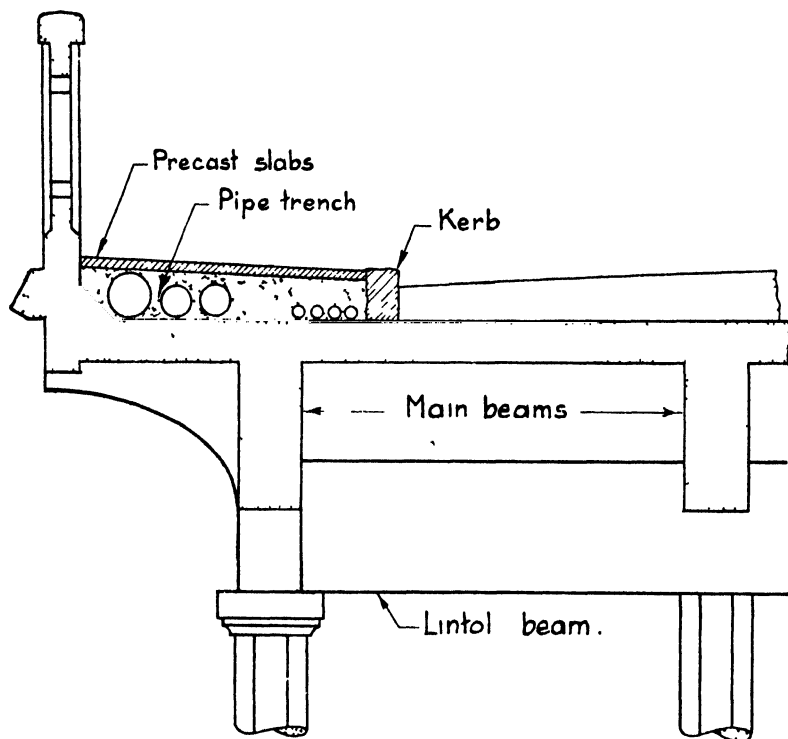



FIG 83 —Arrangement for carrying Pipe Mains under Footpath.

required to carry the specified loading. Subject to a reasonably economical slab, say up to 10 inches in thickness, being suitable, roadway beams should be spaced as far apart as possible, since the maximum wheel loads to be carried by them are practically unaltered, however closely they may be spaced.

Fig. 83 indicates this type of girder bridge, and also shows the arrangement for carrying pipe mains, cables, etc , under one or both of the footpaths, in such a manner that they are easy of access and protected from the weather. If required, these pipes and cables may be carried under the deck slab between the longitudinals, but in this case they are exposed to extreme variations of temperature with

the consequent liability of trouble from this cause, should the bridge be situated in an exposed part of the country. With the arrangement described above, the parapets are not self-supporting, and are designed accordingly. It is not good design to introduce parapet girders where the remainder of the main supporting longitudinals are placed beneath the deck slab, as such an arrangement may produce very serious stresses in the portion of deck slab between the parapet girder and the adjacent longitudinal. This is due to the fact that the longitudinal stress in the slab at the latter member is of opposite sign to that portion adjacent to the parapet.

In designing the main beams for bridges where a train of loads has been specified, it is necessary to place the wheel loads in the position giving the maximum bending moment for the section being investigated.

In the case of a freely supported beam this can be done by placing as many wheel loads as possible on the beam, with the

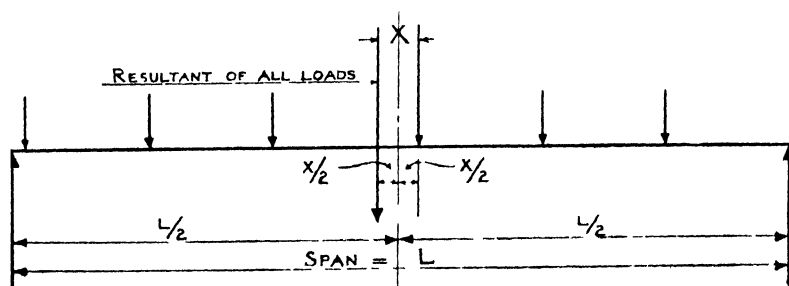



FIG. 84.

heaviest loads near the centre, in such a position that the centre of the span falls midway between the resultant of all loads and the nearest wheel (see Fig. 84). The maximum bending moment will then be found under the wheel nearest to the centre of the span.

For continuous beams, however, the maximum bending moments and shearing forces for the different sections are found by means of influence lines. The example given in Art. 107 illustrates the manner of ascertaining these maximum bending moments and shearing forces in a three-span girder bridge.

99. Arrangement with Parapet Girders as Main Supporting Members.—This arrangement is most economical when the width of the bridge is such that the slab can span transversely between the parapets without the introduction of any secondary beams. This usually means that the bridge is required only for pedestrian traffic or for a light rolling load on a private road, and is shown in Fig. 85.

For bridges up to 20 or 25 feet in width the parapets can be

arranged as main longitudinal girders and cross beams introduced to support the deck slab.

Some objections to this type of bridge are :—

(*a*) The whole of the load would be carried twice before it reached the vertical supports ;

(*b*) No continuity in the transverse beams, with consequent loss of economy and difficulty of obtaining satisfactory support for these members ;

(*c*) Concentration of the load from the bridge at the piers under the parapet beams.

As in the case of the longitudinal deck beams, the cross beams have to be designed to carry the maximum number of wheel loads that can come upon them under any possible condition of loading. This is realised by placing the greatest number of the heaviest axles along the beam, in the manner already explained for main longitudinal beams.

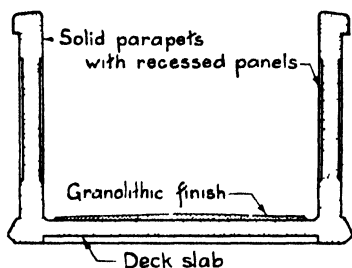



Fig. 85.—Cross Section of Parapet Girder Footbridge.

Fig. 86 shows this arrangement. Unless footpaths are provided on both sides, a protection kerb must be introduced against the parapet. This kerb may be of reinforced concrete or stone. If of the former, it should be protected on its exposed corner by an angle iron.

Parapet girders, where they form the supporting members, are usually constructed *in situ*, and are made solid, *i.e.*, without perforation, although in some cases, where the bridge width and the loading are small, the balustrades may be of precast units suitably incorporated with the plinth or string courses and copings which are formed *in situ*.

A longitudinal camber over the entire length of a bridge is useful in facilitating draining, and also usually assists in obtaining a pleasing appearance.

With the arrangement illustrated in Fig. 87 no special provision for expansion and contraction need be introduced for trestle bridges up to 200 feet long, provided the supports are relatively flexible in the longitudinal direction of the bridge. If desired, the ends of the cantilevers may be recessed into a sleeper beam and sliding plates arranged under each of the load reactions. This is a necessary provision where heavy rolling loads have to be carried and where the span of the cantilever exceeds one-half of the adjacent spans.

100. Trestle Supports.—In designing the trestles themselves,

appearance is important as well as the essential need of transverse stability. Diagonal bracing should, wherever possible, be avoided, as this is both ugly and costly. Satisfactory bracing can be obtained by introducing deep horizontal members, if necessary with gussets, as shown in Fig. 88. Where longitudinal beams are provided to sup-

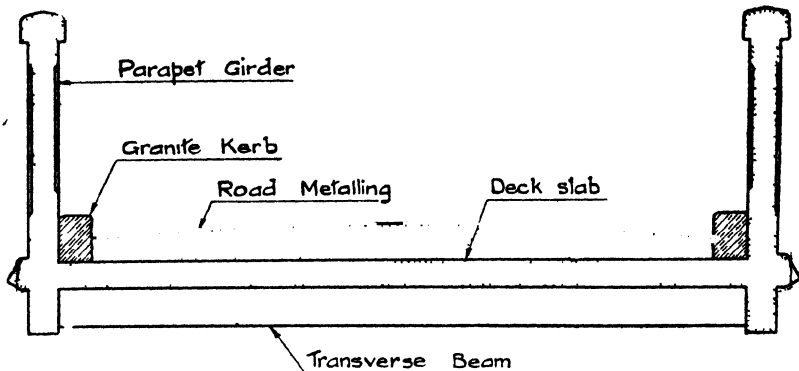



FIG. 86.—Cross Section of Parapet Girder Road Bridge.

port the deck slab, the top transverse members of the trestles are kept down a few inches from the underside of the slab. This will be found to be economical both in design and construction

The outside verticals of trestles, which are relatively narrow in respect to their heights, are usually inclined in the transverse direction in order to increase the stability against lateral wind pressure,

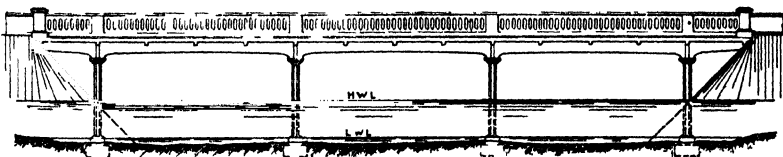



FIG. 87.—Elevation of Trestle Bridge.

or, as is sometimes the case, to give them the appearance of possessing this increased stability.

In considering wind pressure upon a trestle bridge, the whole exposed elevation of the windward face is taken into account and considered to be transmitted to the vertical supports through the deck construction acting in this respect as a horizontal girder. The stresses set up in the deck from this cause, unless the width-span ratio is low, are very small and often neglected.

101. Stability of Trestle Supports.—The stability of the trestle is found in the usual manner, the maximum wind pressure

being applied in combination with the minimum total weight from the structure (see Fig. 88).

102. Design of Trestles under Lateral Pressure.—The bending moments in the different members of a trestle without diagonal bracing may be found in the following manner* :—

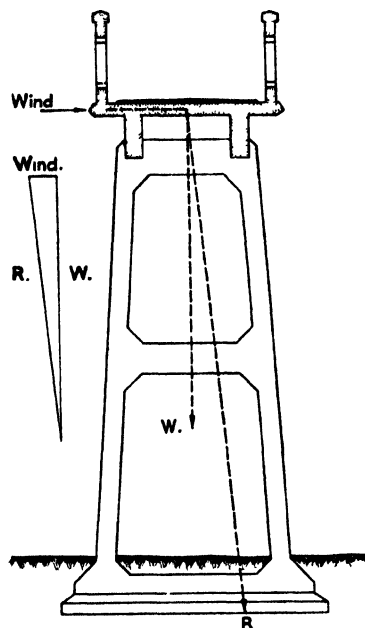
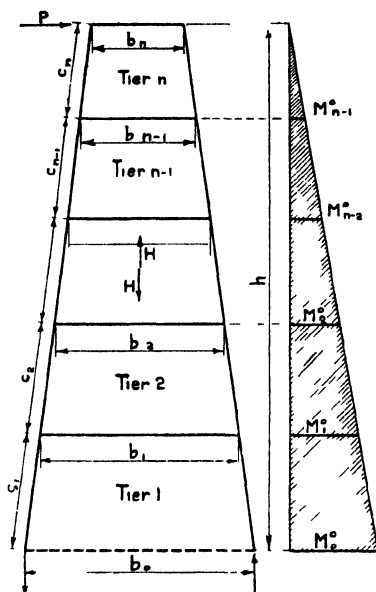



FIG. 88. FIG 89.

In a trestle with n horizontal bracings (see Fig. 89) and a horizontal force P acting at the top, the bending moment diagram, where the moment at foundation level $M_0^0 = P \times h$, is first ascertained.

The quantities $H_1, H_2 \ldots H_{n-1}$ and H_n for the tiers 1 to n are then obtained from the following equations :—

$$\text{Tier 1:}\quad \left[1 + \frac{1}{12}\left(\frac{\Delta b_1}{b_{m1}}\right)^2\right] H_1 + \frac{1}{6}\left(\frac{b_1}{b_{m1}}\right)^2 k_1' (H_1 - H_2) = \frac{1}{b_{m1}}\left[M_{m1}^0 + \frac{1}{12}\frac{\Delta b_1}{b_{m1}}\Delta M_1^0\right]$$

$$\text{Tier 2:}\quad -\frac{1}{6}\left(\frac{b_1}{b_{m2}}\right)^2 k_1'' (H_1 - H_2) + \left[1 + \frac{1}{12}\left(\frac{\Delta b_2}{b_{m2}}\right)^2\right] H_2 + \frac{1}{6}\left(\frac{b_2}{b_{m2}}\right)^2 k_2' (H_2 - H_3) = \frac{1}{b_{m2}}\left[M_{m2}^0 + \frac{1}{12}\frac{\Delta b_2}{b_{m2}}\Delta M_2^0\right]$$

* Professor A. Ostenfeld, "Reinforced Concrete Bridges," Copenhagen, 1917.

and for the nth tier—

$$-\frac{1}{6}\left(\frac{b_{n-1}}{b_{mn}}\right)^2 k''_{n-1}(H_{n-1}-H_n)+\left[1+\frac{1}{12}\left(\frac{\Delta b_n}{b_{mn}}\right)^2\right]H_n$$
$$=\frac{1}{b_{mn}}\left[M^0_{mn}+\frac{1}{12}\frac{\Delta b_n}{b_{mn}}\Delta M^0_n\right]$$

where—

$$M^0_{m_1}=\frac{1}{2}(M^0_0+M^0_1) \text{ and } M^0_{m_2}=\frac{1}{2}(M^0_1+M^0_2)$$

$$b_{m_1}=\frac{1}{2}(b_0+b_1) \qquad b_{m_2}=\frac{1}{2}(b_1+b_2)$$

$$\Delta M^0_1=M^0_1-M^0_0 \qquad \Delta M^0_2=M^0_2-M^0$$
$$\Delta b_1=b_1-b_0 \qquad \Delta b_2=b_2-b_1.$$

I^c_1, I^c_2, etc., are the moments of inertia for the columns.

I^b_1, I^b_2, etc., are the moments of inertia for the horizontal members.

$$k_1'=\frac{b_1}{c_1}\frac{I^c_1}{I^b_1} \qquad k_1''=\frac{b_1}{c_2}\frac{I^c_2}{I^b_1}$$

Note.—M^0 are all negative quantities.

Having found the quantities H, the bending moments in the columns and horizontal members are easily obtained. Thus: Moment in column directly above horizontal $n-1$,

$$M=\mp\frac{1}{2}M^0_{n-1}\pm\frac{1}{2}b_{n-1}\,.\,H_n$$

Moment in column directly below horizontal $n-1$,

$$M=\mp\frac{1}{2}M^0_{n-1}\pm\frac{1}{2}b_{n-1}.H_{n-1}$$

Moments in horizontal $n-1$,

$$M=+\frac{1}{2}b_{n-1}(H_{n-1}-H_n) \qquad \text{(left-hand side);}$$

$$M=-\frac{1}{2}b_{n-1}(H_{n-1}-H_n) \qquad \text{(right-hand side).}$$

When the columns are vertical, all horizontal members are the same length, and consequently $\Delta b=0$. The equations then take the following form :—

$$\text{Tier 1}: \quad H_1+\frac{1}{6}(H_1-H_2)k_1'=\frac{1}{b}M^0_{m_1}$$

$$\text{Tier 2}: \quad -\frac{1}{6}k_1''(H_1-H_2)+H_2+\frac{1}{6}k_2'(H_2-H_3)=\frac{1}{b}M^0_{m_2}$$

$$\text{Tier } n: \quad -\frac{1}{6}k''_{n-1}(H_{n-1}-H_n)+H_n=\frac{1}{b}M^0_{mn}$$

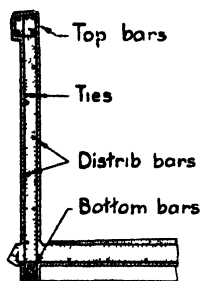



Fig. 90.

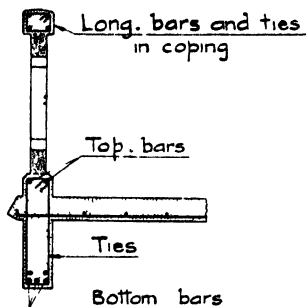



Fig. 91.—Arrangement of Reinforcement in Beam under Parapet.

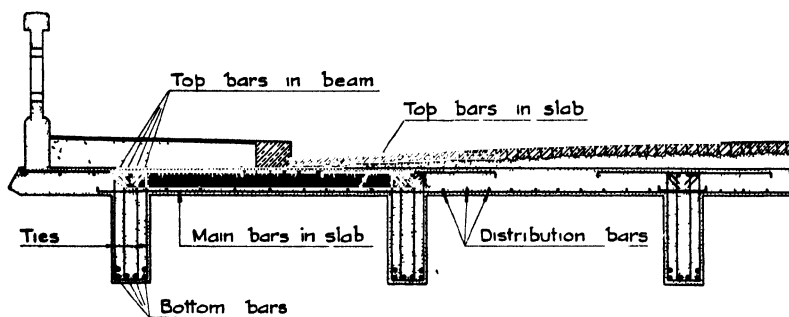



Fig. 92.—Arrangement of Reinforcement in Deck Slab and Longitudinal Beams.

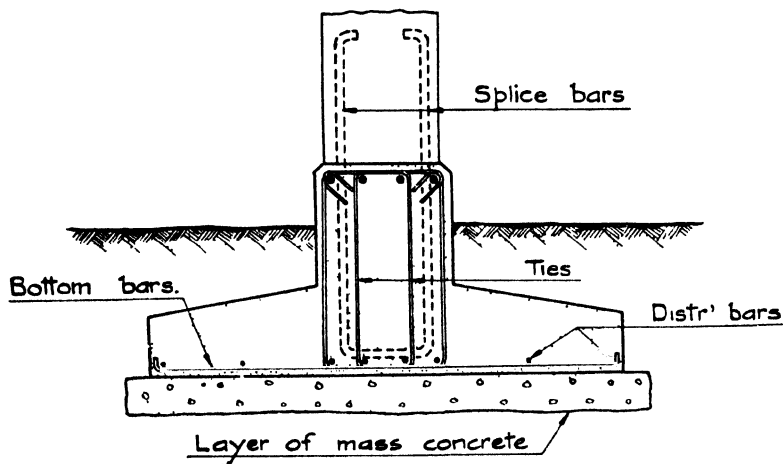



Fig. 93.

103. Arrangement of Reinforcement in Girder Bridges.—Figs. 90 to 95, and also Figs. 123 and 124, illustrate suitable arrangements for the reinforcement in the various members of girder bridges.

104. Design of Deck Slab.—It is desirable to keep the thickness of the deck slab of a bridge as small as possible. If this is not done, the design becomes uneconomical, owing to the increase in dead weight to be carried by the supporting members, causing these in turn to be heavier. The question of deck slab design is therefore important.

The calculation of bending moments, etc., due to the dead weight of slabs and surfacing materials is made in the usual manner as for floors. This, of course, also applies to any distributed live loading, such as occurs for footpaths, and also for wheel loads where a sufficient depth of fill exists to produce the requisite dispersion.

In the latter case the dispersion of the wheel loads should cover the entire transverse span and an equivalent uniformly distributed load adopted. Where, however, the load dispersion through the surfacing material and slab results in concentrated loading—acute or otherwise—it is necessary to allow for such conditions in the calculations.

Concentrated wheel loads provide the most serious loading. The two essential factors in this connection are :—

(1) The magnitude of the load ;

(2) The manner in which it is assumed to be carried by the slab.

The first is always known ; the second apparently varies, differing rules or methods producing different results (see Art. 106). So far there appears to be no accepted rule in this country, whilst most of those adopted in other countries are of an empirical nature.

A method which has the merit of theoretical as well as practical justification has been prepared by M. Pigeaud, and as it is in advance of any of those so far in use, it is explained in Art. 105 and adopted in the example given in Art. 107.

Reinforced concrete slabs designed on this principle are adopted by the French authorities, and have for a number of years been employed in France and in this country.

The essential difference between M. Pigeaud's method and other methods in use is in the amount of bending allowed for in the longitudinal direction.

The other methods provide an imaginary area over which the concentrated load is assumed to spread itself evenly. But, although a longitudinal spread is assumed, information regarding the provision of necessary reinforcement to take care of the longitudinal bending is either very meagre or often entirely absent.

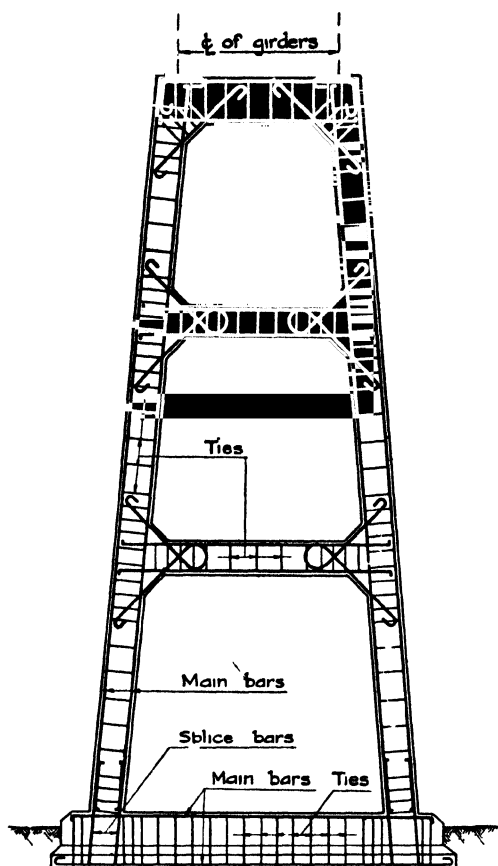



Fig 94 —Arrangement of Reinforcement in Trestle Support

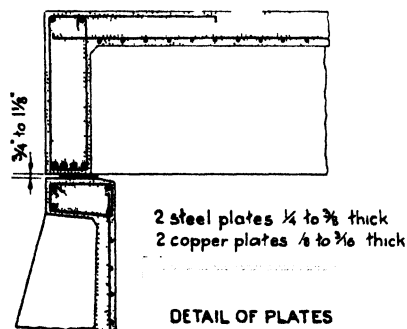



Fig. 95.

It is obvious that the strip of slab directly under this area of spread cannot deflect independently of the adjacent strips without fracture at $x - x$ and $y - y$ (see Fig. 96).

FIG. 96.

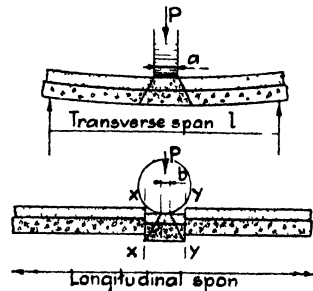
As this is not the case, or at least should not be the case in a properly designed slab, it follows that the adjacent strips must also deflect to a certain extent, and consequently become interested in supporting the load.

As this longitudinal deflection brings a wider part of the slab into action, the transverse bending moment per unit strip must necessarily be less than when it is assumed that the load is carried only by the strip $x - y$. But, on the other hand, a considerable longitudinal bending moment is developed which must be taken into account.

If the load is concentrated on a very small area the deformation at the point of contact produces circular " contours " of equal deflection, no matter what shape the panel may be. Should the contact area be increased, the " contours " are no longer circular; the concave surface of the rectangular slab taking the form of a partial ellipsoid.

Thus, with a highly concentrated central load, the longitudinal moment may approach in magnitude the transverse moment. This applies to all shapes of slab panels, including slabs with parallel supports in one direction only, and is a point of considerable importance which previously had been obscure. Further, for a given transverse span and load, the transverse and longitudinal moments remain sensibly constant for all panel ratios greater than 2 to 1, and even when the shorter edge supports are absent.

It will be seen, therefore, that to assume the load to be supported only by the strip over which it is spread, and to neglect the longitudinal bending entirely, not only causes the reinforcement in the transverse direction to be unnecessarily heavy, but also in many cases results in seriously overstressing the longitudinal reinforcement. Even where a generous longitudinal spread is allowed (and as a rule found adequate in practice), a more rational method is likely to be preferred by most engineers. While it is admitted that the total moments, and consequently the amount of slab reinforcement, found by M. Pigeaud's method, are less than is necessary according to the above method, there is no reduction in the safety of any structure so designed, for the following reasons :—

First, it is not necessary to provide for a greater transverse

moment than actually occurs; secondly, by allowing for a definite and calculated longitudinal moment, provision is made for the true condition of deformation; and, thirdly, there is a reserve of strength in reinforced concrete slabs not present in most other structural members of this material.

Admittedly, in the highly mathematical investigations required to solve this problem, the usual assumptions for elastic materials were made, and the results given are for freely supported slabs—a condition infrequently met with in ordinary practice. M. Pigeaud, in his treatise, draws attention to this fact, and points out that a constant moment of inertia has been assumed, whereas in reinforced concrete work this can vary, owing to change in the area of the reinforcement at different sections.

Notwithstanding this, by making suitable allowances for the effects of fixity or continuity at the supports, the design of the slab panels can be effected with ease and with a very reasonable degree of accuracy.

As an answer to possible criticism, it is contended that it is better to employ a method which, although involving the use of preliminary assumptions, is based on a correct mathematical analysis, than to accept rules having neither the support of theoretical proof nor of exhaustive tests.

Slabs designed by the method described below possess a much greater reserve of strength than beam members calculated in the customary manner, and, therefore, if any increase in the safety factor of a bridge be desired, the latter members should be increased in strength rather than the deck slab. In many designs material is wasted in the slab which could with advantage be devoted to other parts of the structure which are frequently in need of it.

105. M. Pigeaud's Method.—In his contributions to the *Annales des Ponts et Chaussées Français* (February, 1921), M. Pigeaud presented his thesis on the mathematical analysis of thin rectangular plates and slabs supported along their edges, and for the first time offered a theoretically correct investigation. The issue "Part II., 1929," contains a further contribution, and includes a more complete series of curves for slabs whose transverse and longitudinal spans bear the ratios (l/L) of 0·1 to 0·9.

From the curves the load coefficients are read, and the bending moment per unit strip of slab ascertained. These coefficients are expressed as 100 times the bending moment, and, therefore, if the unit length is feet and the unit in the bending moment is required in inches, the values of these curves must be multiplied by $\frac{12}{100} = 0{\cdot}12$.

For the convenience of readers, the curves included in the present

article have been compiled so that the final results of calculations are read direct in lbs.-inches.

The following panel shapes are dealt with :—

$L/l = 1$ (square panel) . .	Fig. 99
$L/l = \sqrt{2}$	Figs. 100 and 101
$L/l = 2$	Figs. 102 and 103
$L/l = \infty$ (parallel supports only) .	Figs. 104 and 105

The general procedure for ascertaining the maximum moments due to a concentrated load on slab panels is as follows :—

Where filling is present the load concentrated on the contact area $a \times b$ is assumed to spread through the filling and slab over a loaded area * $u \, . \, v$ (see Fig. 97),

where $u = \sqrt{(a + 2d)^2 + H^2}$

and $v = \sqrt{(b + 2d)^2 + H^2}$

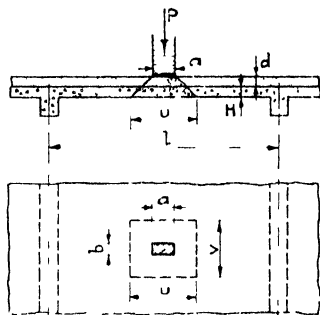
d = depth of filling,

H = thickness of slab.

Fig. 97.

It should be noted that the dimension a is not necessarily the width of wheel, but is the contact dimension in the direction of the shorter span. Thus, if the effect produced by the wheel of a vehicle in one position on a panel has been investigated, and it is desired to compare this with the results for the same vehicle turned at right angles, it is necessary to transpose the values of a and b in the foregoing equations.

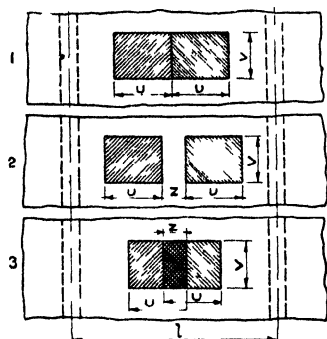
Having ascertained the values of u and v, the ratios of u/l and v/L can be obtained, all dimensions being in inches. Where there are two parallel supports only ($L/l = \infty$), l is used in both cases. With the ratios so found, the coefficients m_1 and m_2 are read from the appropriate graphs. The bending moments are then $M_1 = P \, . \, m_1$ in the transverse direction and $M_2 = P \, . \, m_2$ in the longitudinal direction, where P in pounds gives the moments in inch-pounds per foot width of slab for a concentrated load in the centre of the span.

The above moments are for freely supported spans, and for continuous spans they have to be reduced accordingly. It should be noted that the effect of continuity in one direction, by reducing the central deflection, causes a diminution of the positive moments in

* Note.—This load spread is not dealt with in M. Pigeaud's work, due to the fact that the general theory treats thin plates where the load spread is negligible. It will be observed that where surfacing material is present the load spread is smaller than given by most of the older rules.

both directions, although to what extent and in what proportion is obscure. Due to the absence of data, the reduction of the positive moments on account of continuity or fixity at the supports, whether partial or complete, must remain a question of discretion depending on the circumstances of the case. In practice, a reduction of 20 per cent. in both the transverse and longitudinal "free" positive moments may be made for all cases of continuity, and "negative" reinforcement, equal to that in the span, should be provided over such supports as may be continuous or "fixed."

FIG. 98.

It will generally be found in practice that a wheel placed centrally on a panel produces a more serious moment than would be the case if two or four wheels of the same vehicle were arranged symmetrically about the panel axes.*

It is necessary, however, to take into consideration the possibility of two vehicles passing or running parallel to one another, the distance between their respective wheels being less than the track spacing of either vehicle alone. The bending moments are found in much the same way as for one load.

The two loads will spread themselves through the filling and slab, as shown above, and there are three conditions to consider (see Fig. 98), where—

(1) The two areas of spread just touch each other;
(2) There is a clear space between them;
(3) They overlap.

In the first case, find the ratio $\frac{v}{L}$ as usual and substitute $\frac{2u}{l}$ for $\frac{u}{l}$, and read the new values m_1 and m_2 from the curves as before. The bending moments are $M_1 = 2P.m_1$ in the transverse direction and $M_2 = 2P.m_2$ in the longitudinal direction.

In the second case find the unit load $p = \frac{P}{u.v}$ and the coefficients m_1 and m_2 for the whole area $(2u + z).v$.

* NOTE.—This latter case and other symmetrical forms of loading are explained by the author in articles on this method of slab design published in the March, April, and May, 1930, issues of "Concrete and Constructional Engineering."

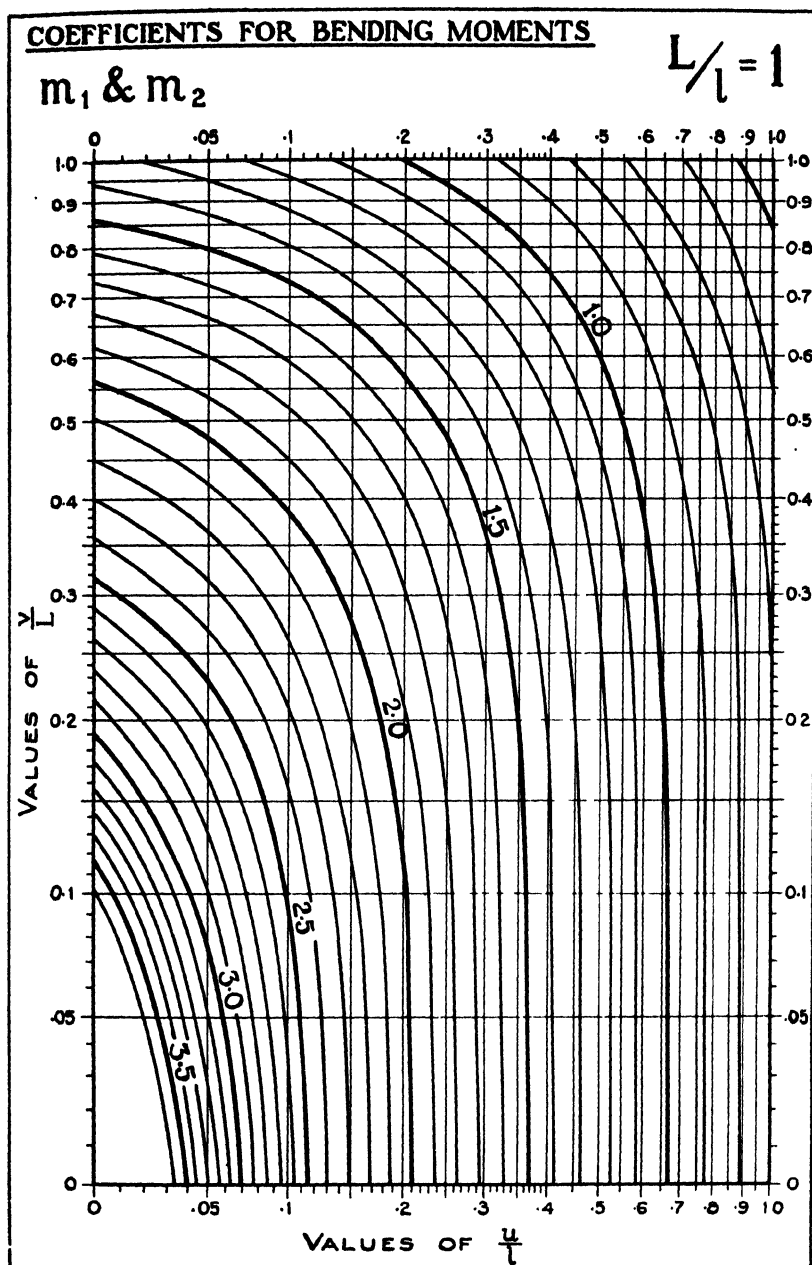



FIG. 99.—Coefficients for Slab Bending (Pigeaud's Method). (See Art. 105.)

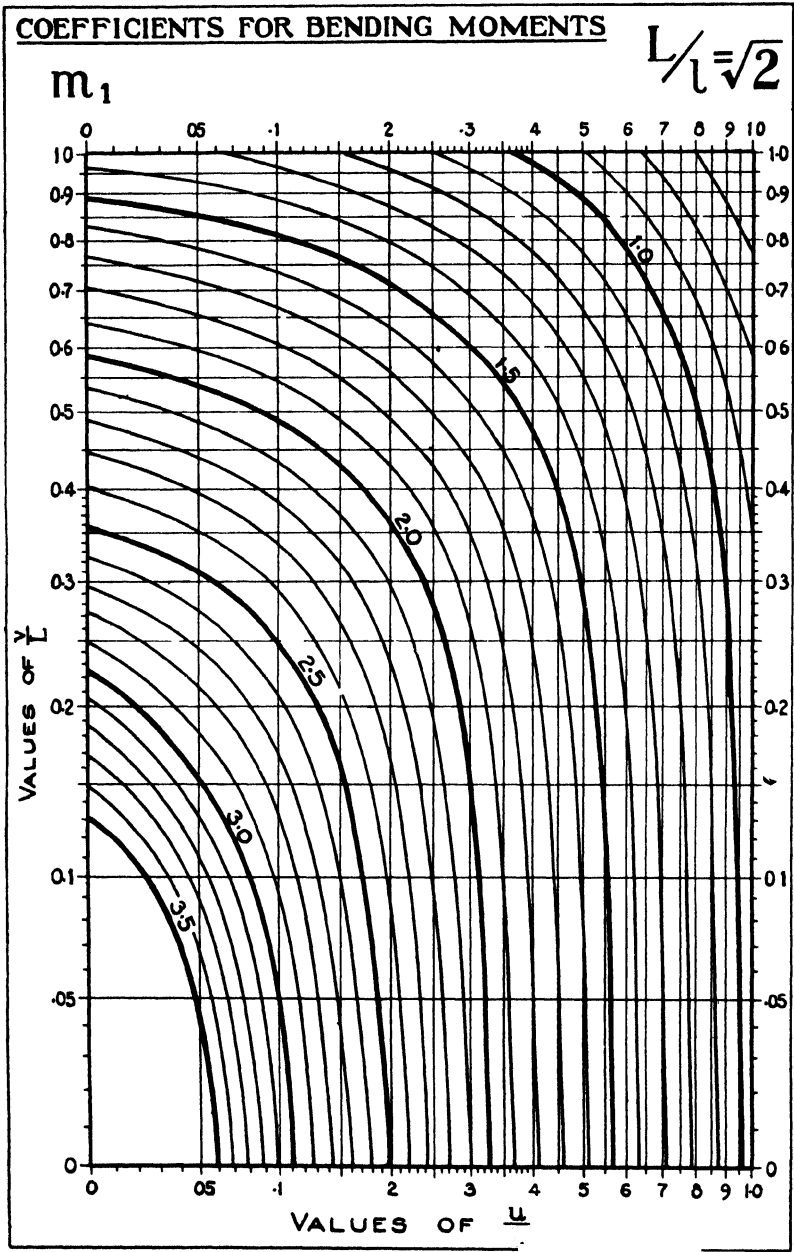



FIG. 100.—Coefficients for Slab Bending (Pigeaud's Method). (See Art. 105.)

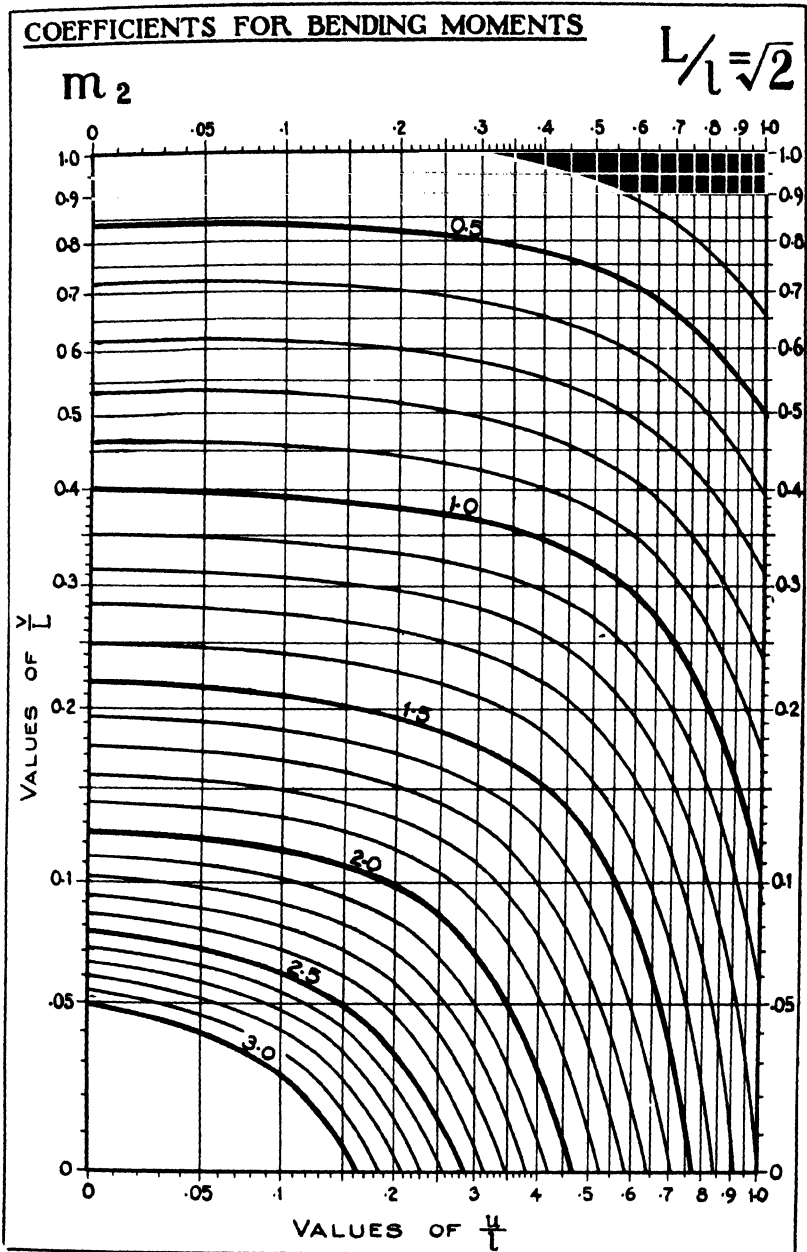



FIG. 101.—Coefficients for Slab Bending (Pigeaud's Method). (See Art. 105.)

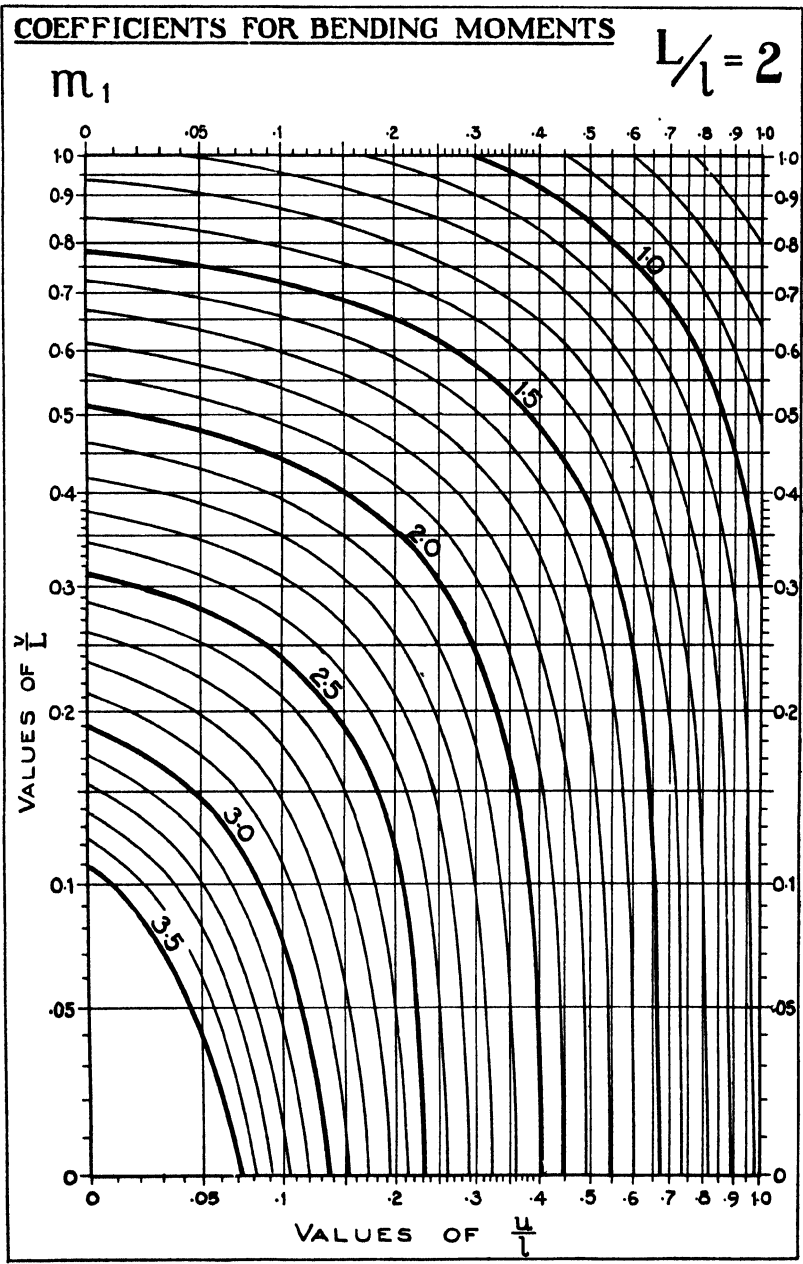



FIG. 102.—Coefficients for Slab Bending (Pigeaud's Method). (See Art. 105.)

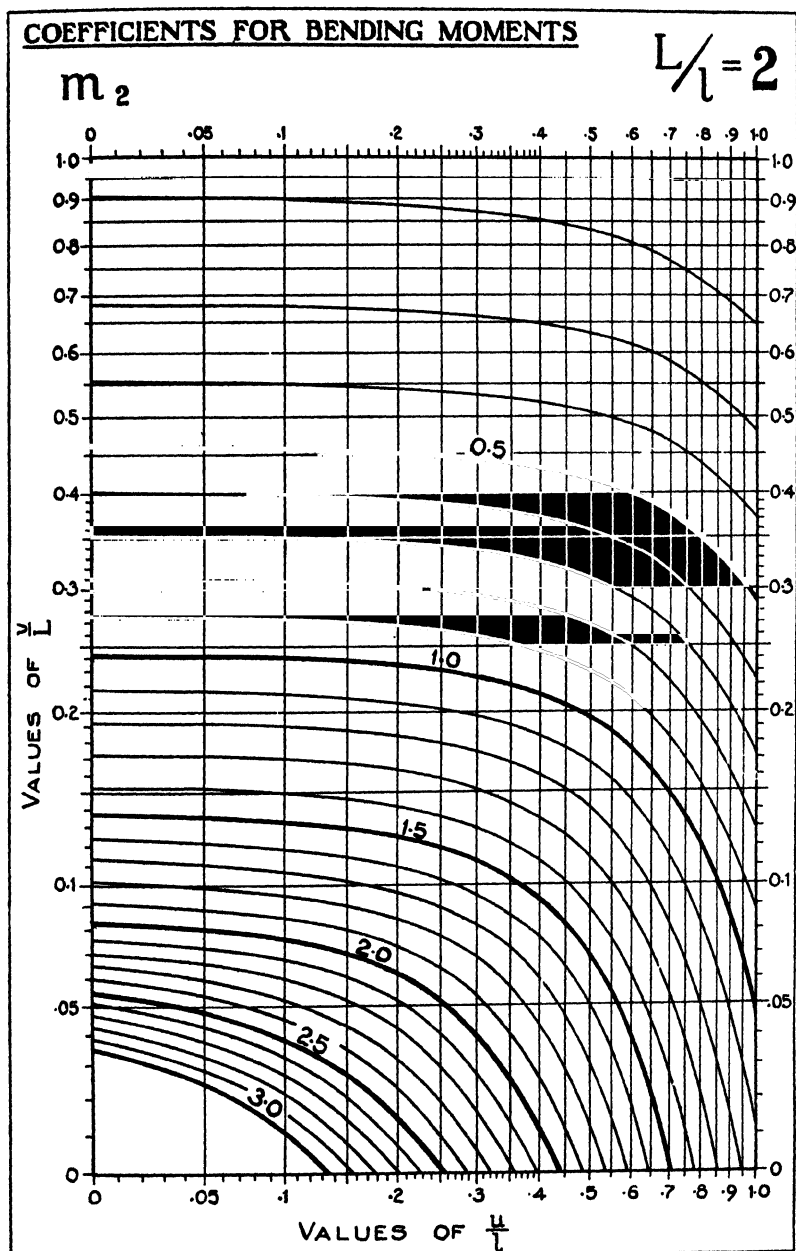



Fig. 103.—Coefficients for Slab Bending (Pigeaud's Method). (See Art. 105.)

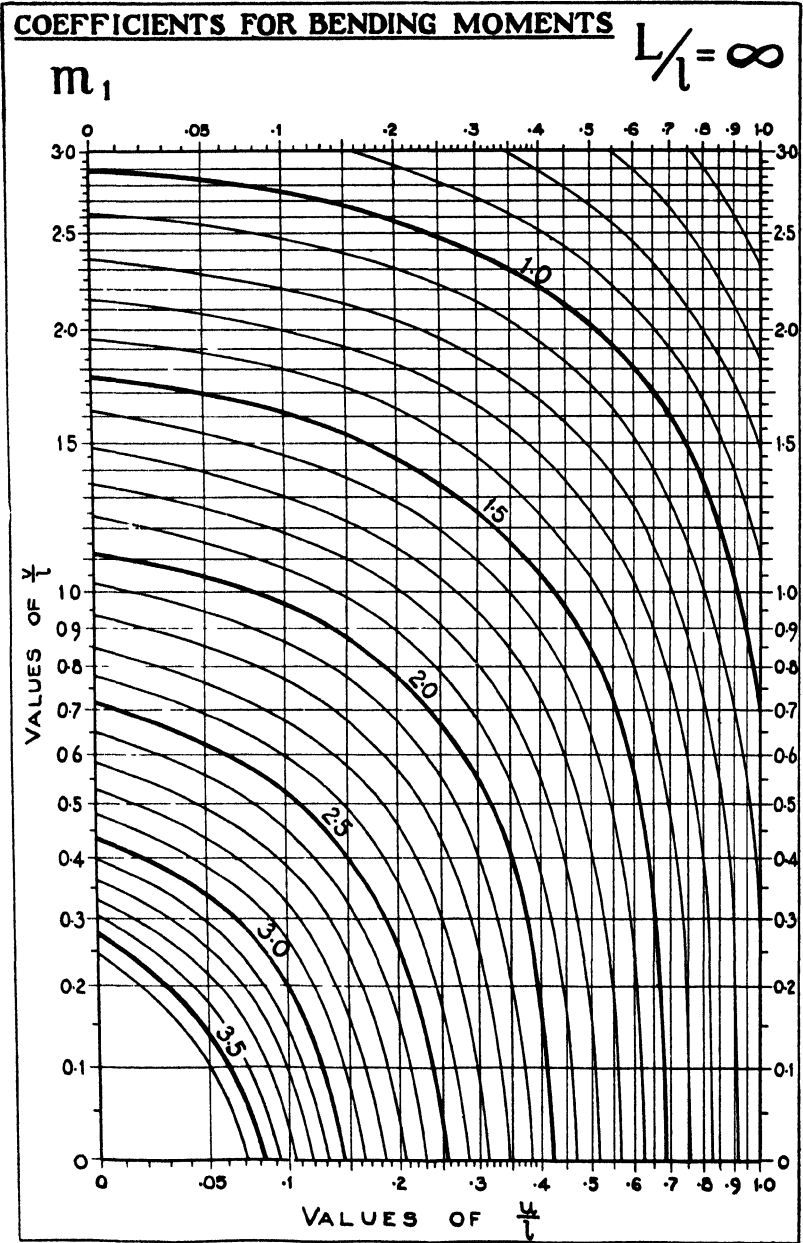



FIG. 104.—Coefficients for Slab Bending (Pigeaud's Method). (See Art. 105.)

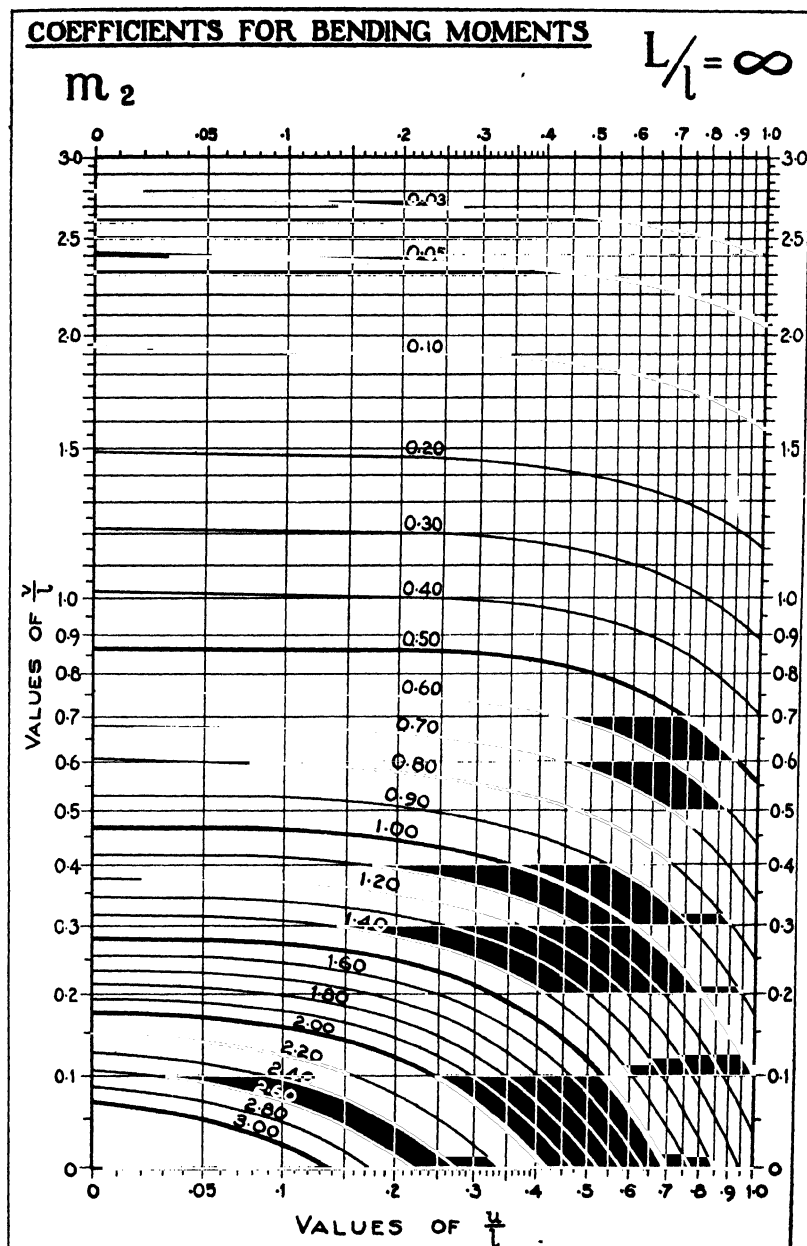



FIG. 105.—Coefficients for Slab Bending (Pigeaud's Method). (See Art. 105.)

The corresponding bending moments are, in the transverse direction $p.(2u + z).v.m_1$ and in the longitudinal direction $p.(2u + z).v.m_2$.

Next, find the coefficients m_1 and m_2 for the area $z.v$ and the corresponding bending moments :

in transverse direction $= p.z.v.m_1$ and
in longitudinal direction $= p.z.v.m_2$.

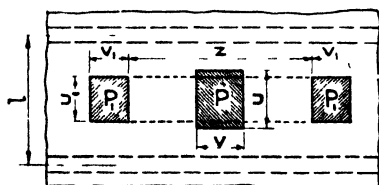
Subtract the last moments from those previously found, and the resultant bending moments M_1 and M_2 are obtained.

Similarly, in the third case find the two bending moments for the area $(2u - z).v$ and the two moments for the area $z.v$, and add them together, which gives the bending moments M_1 and M_2 for that case.

For most cases of this class it will be found sufficiently accurate to proceed as for a single wheel, but making the length of contact area equivalent to the widths of both wheels plus the distance between and using 2P as the load factor.

In the case of a train of loads moving parallel with the supports of the slab, as in Fig. 106, it will in many cases be necessary to find not only the bending moments caused by the heaviest wheel load but also the additional bending moments directly under P caused by the adjacent wheel loads P_1.

FIG. 106.

The moments due to the load P are found in the manner already explained. The additional bending moments due to the two loads P_1 are obtained as follows : Having found the unit load $p_1 = \frac{P_1}{u_1.v_1}$ the transverse and longitudinal bending moments for an imaginary load $p_1\ [2(u_1.v_1) + u_1.z]$ spread over an area $2\ (u_1.v_1) + u_1.z$ and the moments for a load $p_1.z.u_1$ spread over an area $z.u_1$ may be obtained.

The difference between the two transverse moments and the difference between the two longitudinal moments thus obtained represent the increase to the bending moments already found for the wheel load P.

For uniformly loaded slab panels, u/l and v/L are both considered as being equal to unity. The load coefficients m_1 and m_2 can easily be converted into " free " bending moment coefficients. Denoting these latter by α and β for transverse and longitudinal spans respectively, the moments for panels freely supported are :—

$$M_1 = \alpha . \frac{w . l^2}{8} \text{ and}$$

$$M_2 = \beta . \frac{w . L^2}{8} \text{ where } w = \text{unit load} \left(= \frac{P}{l . L}\right).$$

Table W gives values of α and β according to M. Pigeaud's method, for fully-loaded panels :—

TABLE W

BENDING MOMENT COEFFICIENTS FOR RECTANGULAR SLABS

L/l	α	β
1·0	0·296	0·296
1·1	0·363	0·236
1·2	0·425	0·190
1·3	0·480	0·154
1·4	0·534	0·122
1·5	0·581	0·101
1·6	0·625	0·084
1·7	0·665	0·068
1·8	0·702	0·056
1·9	0·736	0·045
2·0	0·768	0·036

The bending moment coefficients α and β given in the table above are directly comparable with those according to the Grashof and Rankine and old French Government Rules (see R.I.B.A. Joint Committee Report on Reinforced Concrete).

106. Comparison of Methods for Deck Slab Design.—It is interesting to make a comparison of the relative bending moments arrived at by different methods of design for the same central load P and a typical transversal " free " span and other factors, the slab in the longitudinal direction being assumed to be continuous and unsupported.

This comparison is given in Table X, and is based upon the following example (see Fig. 107) :—

Span = 8 feet. Depth of filling = 6 inches.

Thickness of slab = 8 inches.

Wheel load P = 11 tons. Width of wheel = 24 inches.

Length of contact = 3 inches.

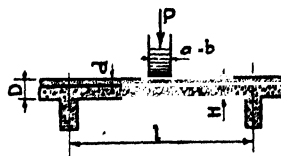



FIG. 107.

TABLE X

Authority	Assumed Width of Distribution	Assumed Length of Distribution.	Transverse Bending Moment per foot strip for above example	Longitudinal Bending Moment per foot strip for above example.
Ministry of Transport	$a + 2D$	$b + 2D$	167,000 inch-lbs. $\frac{Pl}{14\cdot2}$	None.
Swiss Government .	$a + 2d$	$b + 2d + \frac{l}{3}$	123,000 inch-lbs. $\frac{Pl}{19\cdot3}$	None.
French Government (old rule)	$a + D$	$b + D + \frac{l}{3}$	116,000 inch-lbs. $\frac{Pl}{20\cdot4}$	None.
American rule (Conference Committee Report, 1929)	a	$0\cdot7\ (l + a)$	74,000 inch-lbs. $\frac{Pl}{31\cdot9}$	None.
M. Pigeaud's rule .	$\sqrt{(a+2d)^2+H^2}$	$\sqrt{(b+2d)^2+H^2}$	50,000 inch-lbs. $\frac{Pl}{47\cdot3}$	37,000 inch lbs $\frac{Pl}{64}$

107. Example of a Three-span Girder Bridge.—*General Conditions.*—In the example which follows, it is assumed that the girder type of bridge has been definitely adopted, either because this type has been specified or because it conforms best to the natural conditions of the site.

The main dimensions of the proposed bridge are as follows :—

Total length	128 feet.
Width between parapets . .	60 ,,
Width of roadway . . .	40 ,,
Width of footpaths . . .	10 ,,
Thickness of road metal . .	9 inches at crown and 6 inches at kerb.

Loading.—The footpaths are to be designed to carry a superload of one hundredweight (112 lbs.) per square foot.

The roadway is to be designed to carry the standard loading for highway bridges laid down by the Ministry of Transport. The diagram of this loading is given in Fig. 3, p. 7.

Maximum Stresses.—The quality of concrete is presumed to be

1 : 2 : 4, and the maximum permissible stresses in any part of the structure are as follows :—

Compressive stress on concrete .	600 lbs.	per square inch.
Tensile stress on steel . .	16,000	,, ,,
Shearing stress on concrete .	60	,, ,,
Shearing stress on steel . .	12,000	,, ,,

It is necessary in the first case to consider the spacing of the main supports and the arrangement of the deck construction from the point of view of economy. The problem will be taken up at this point.

Spacing of the Supports.—Where the work consists in the reconstruction of an existing bridge, it is sometimes economical to support the new bridge platform wholly or partially upon the existing supports, in which case the various spans of the new main supporting members are fixed. The positions of the bridge supports are also occasionally fixed, as, for example, by particular accommodation being required for passage under the bridge. In such cases the design has to be prepared with this limitation.

Where, however, the overall length is given, and no restrictions are necessary regarding the position of the intermediate supports, a considerable economy can be gained by a suitable arrangement of the spans.

With ordinary conditions equal spans are the more economical, and in the present example the length of 128 feet can be divided up in the following different ways :

(*a*) Two equal spans of 64 feet ;
(*b*) Three equal spans of 42 feet 8 inches ;
(*c*) Four equal spans of 32 feet ;
(*d*) Five equal spans of 25 feet 7 inches.

A two-span continuous beam is usually unsuitable and unsightly, as well as uneconomical because of the very high negative moments induced over the central support.

Furthermore, spans of about 25 feet are usually uneconomical for the length of bridge under consideration. This is more marked should the supports be high and the foundations difficult to execute.

The division of the length of the bridge therefore lies between a three-span beam and one having four spans.

The central support in a four-span beam is frequently an objection, and therefore a compromise can be made by having three spans of 36 feet and a cantilever of 10 feet at each end. With this arrangement the dead load moments in the end spans and central supports are reduced, and the necessity for abutment retaining walls

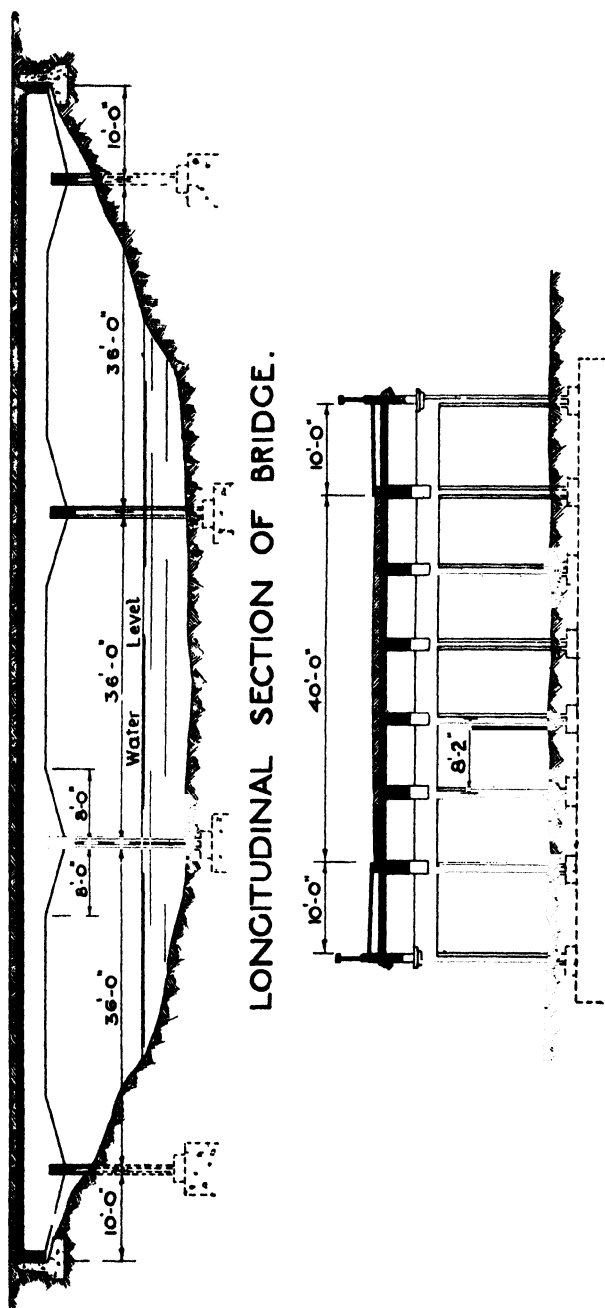



Fig. 108.—Arrangement of Members in Three-span Bridge Example. (See Art 107)

at the extremities of the bridge obviated. The earth filling can be allowed to take its natural slope, commencing from the toe of the mass concrete sleeper beam (see Fig. 108).

Where cantilever spans are not suitable, an economical arrangement would be to have the side spans about 0·8 that of the central span or spans. This arrangement prevents excessive moments in the end spans.

Arrangement of Deck Construction.—A brief study of the plan of the proposed bridge shows it to be uneconomical to support the deck slab by beams arranged transversely, which in turn would have to be carried by girders under or incorporating the parapets.

The technical reasons against this arrangement are given in Art. 99.

The best arrangement will consist of a series of longitudinal beams spaced at 8 feet 2 inch centres. This dimension is governed by the economical design of the deck slab, as already explained in this chapter.

Calculation of the Deck Slab.—The effective span of the slab is 7 feet 6 inches.

The *dead load* per square foot is calculated, using the following weight of materials :—

Weight of road metal . . .	110 lbs. per cubic foot.
Weight of reinforced concrete .	150 ,, ,,

∴ Dead load per square foot of roadway area is as follows :—

Road metal	(9 inches thick) = 0·75 × 110 lbs. =	83 lbs.
Deck slab	(8 inches thick) = 0·67 × 150 lbs. =	100 ,,
		183 lbs.

To calculate the dead load bending moment per foot strip of slab, the formulæ for uniformly distributed continuous slabs are used as follows :—

$$\text{In span} \quad + \frac{183 \times 7{\cdot}5' \times 7{\cdot}5' \times 12}{24} = + \ 5{,}150 \text{ inch-lbs.}$$

$$\text{At support} - \frac{183 \times 7{\cdot}5' \times 7{\cdot}5' \times 12}{12} = - \ 10{,}300 \text{ inch-lbs.}$$

The live load producing the maximum bending moment in the slab will consist of two of the maximum wheel loads in the Ministry of Transport's standard loading passing along the centre of the slab. The distance between the wheels will be assumed to be not less than 12 inches.

For the calculation of the transverse and longitudinal bending

moments produced by this loading use is made of M. Pigeaud's method, explained in Art. 105.

Below are given the requisite data :—

Maximum wheel load	= 11 tons	=	24,640 lbs.
Width of wheel	$= a$	=	24 inches.
Width of contact between wheel and filling	$= b$	=	3 ,,
Depth of filling	$= d$	=	9 ,,
Thickness of slab	$=$ H	=	8 ,,
Minimum clearance between driving wheels of adjacent tractors . .	=		12 ,,

By the formulæ in Art. 105—

$$u = \sqrt{(24 + 2 \times 9)^2 + 8^2} = 42{\cdot}8 \text{ inches};$$

$$v = \sqrt{(3 + 2 \times 9)^2 + 8^2} = 22{\cdot}5 \quad ,,$$

The case under investigation is the third case mentioned in the explanation of this method of calculation.

The overlapping portion denoted z on Fig. 98 is therefore equal to 6·8 inches. As this distance is small in comparison with the overall distance $(2u - z)$ 78·8 inches, the two wheels are considered as a single load of magnitude 2 × 24,640 lbs. = 49,280 lbs. and having a width a of 24 inches + 12 inches + 24 inches = 60 inches.

$$\therefore \quad u = \sqrt{(60 + 2 \times 9)^2 + 8^2} = 78{\cdot}4 \text{ inches}.$$

Thus $$\frac{u}{l} = \frac{78{\cdot}4}{90} = 0{\cdot}871 \text{ and } \frac{v}{l} = \frac{22{\cdot}5}{90} = 0{\cdot}25.$$

With these ratios the moment coefficients obtained from the curves (Figs. 104 and 105) are $m_1 = 1{\cdot}24$ and $m_2 = 0{\cdot}89$.

The spans are continuous over the supports, and the free bending moments derived by the above method are multiplied by 0·8 to allow for this continuity.

Total positive bending moment at centre of slab in a transverse direction is :—

From dead load	5,150 inch-lbs.
From live load = 49,280 × 1·24 × 0·8 =	48,890 ,,
Total bending moment per foot strip .	54,040 inch-lbs.

Total depth of slab . .	= 8 inches.
Effective depth of slab .	= 7·25 inches.
Lever arm . . .	= 6·38 ,,

Area of steel per foot strip. $= \dfrac{54{,}040}{16{,}000 \times 6{\cdot}38} = 0{\cdot}53$ sq. ins.

½-inch dia. bars at 4-inch pitch placed in bottom of slab transversely.

Note.—The same area of steel is provided in the top of the slab over the supporting beams to take the negative bending moments produced by the above loading. The maximum value of these latter moments is less than that presumed for the positive bending moments in the span, so that the provision of equal transverse reinforcement is conservative.

Total positive bending moment in a longitudinal direction :—

From live load $= 49{,}280 \times 0 \cdot 89 \times 0 \cdot 8 = 35{,}000$ inch-lbs.

Total depth of slab . . . = 8 inches.
Effective depth of slab . . = 6·75 inches.
Lever arm . . . . . = 5·95 „

$$\text{Area of steel per foot strip} = \frac{35{,}000}{16{,}000 \times 5 \cdot 95} = 0 \cdot 368 \text{ square inches.}$$

½-inch dia. bars at 6-inch pitch placed in bottom of slab longitudinally.

(Note—on top of transverse rods.)

Calculation of Roadway Beams.—The *dead load* per lineal foot of bridge, taking a width equal to the spacing of the beams, is given below :—

Load on beam produced by road metal .	8·17 × 0·75′ × 110 lbs.	= 673 lbs./lin. ft.
Load on beam produced by deck slab.	8·17′ × 0·67′ × 150 lbs.	= 817 lbs./lin. ft.
Weight of beam .		= 510 lbs./lin. ft.
	Total weight	= 2,000 lbs./lin. ft.

By the theorem of three moments the following bending moment coefficients are obtained :—

$$-\frac{wl_1^2}{2} \qquad +\frac{wl^2}{16 \cdot 4} \qquad -\frac{wl^2}{10 \cdot 82} \qquad +\frac{wl^2}{30 \cdot 6}$$

A B C D

l_1 l l l l_1

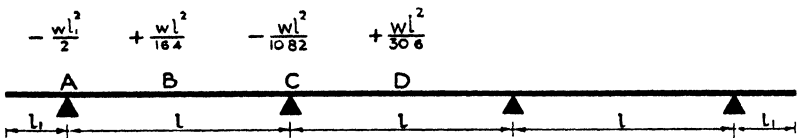



Fig. 109.

The bending moments due to the dead load are therefore :—

$$\text{At Section A} \quad -\frac{2{,}000 \times 10^2 \times 12}{2} = -1{,}200{,}000 \text{ inch-lbs.};$$

$$\text{At Section B} \quad \frac{2{,}000 \times 36^2 \times 12}{16 \cdot 4} = +1{,}900{,}000 \quad „$$

$$\text{At Section C} - \frac{2{,}000 \times 36^2 \times 12}{10{\cdot}82} = -\ 2{,}870{,}000 \text{ inch-lbs.}$$

$$\text{At Section D} \quad \frac{2{,}000 \times 36^2 \times 12}{30{\cdot}6} = +\ 1{,}020{,}000 \quad \text{,,}$$

The *live load*, as given in the general conditions, consists of a number of trains of the Ministry of Transport's standard loading. The proportion of the load carried on one beam is obtained from a study of the cross section. Two adjacent parallel trains can be assumed to be travelling in the same direction, and the distance between the outer edges of the widest wheels not less than 12 inches.

The maximum actual concentrated load that may possibly come

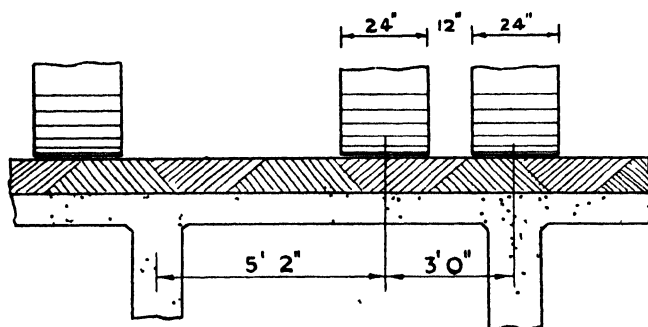



FIG 110.

upon any roadway beams, expressed as an increase of one train of wheels, is $1 + \frac{5{\cdot}17}{8{\cdot}17} = 1{\cdot}632$ (see Fig. 110).

The resultant train of maximum loads on the beam is given in Fig. 111.

Note.—The alternative distribution of loading given in Art. 10 can be adopted and the following method of calculation also applied. In this case the increase on one train of wheels would be, with the

14700 lbs 40200 lbs 18300 lbs 18300 lbs 18300 lbs 18300 lbs 18300 lbs 18300 lbs

10' 12 8 10 8' 10' 8'

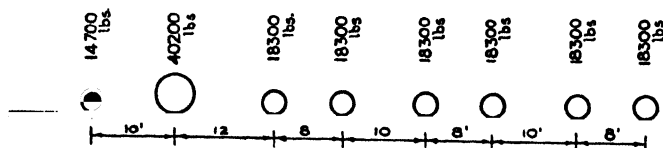



FIG. 111.

spacing adopted in this example, approximately as above, viz., 1 634.

The maximum bending moments, shearing forces and reactions at various sections in the beams are found by applying the above series of loads to the influence line diagrams for the appropriate sections.

The position of the train of loads in Fig. 111 producing the maximum effects at the section under investigation is given in the diagram with each calculation, and the values of the influence line ordinates given in the calculations may with advantage be checked by the reader.

Calculation of Maximum Bending Moments at Selected Sections.

At Section A (at end support) (Fig. 112):—

Fig. 112.

Live load bending moment = 40,200 × 10′ × 12 = − 4,824 000 inch-lbs.

At Section B (distance $0{\cdot}45l$ from end support) (Fig. 113):—

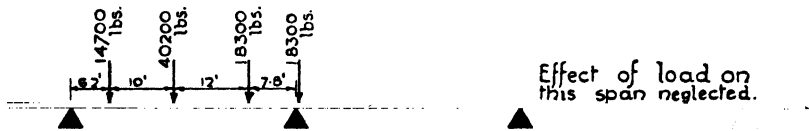



Fig. 113.

Live load bending moment =

14,700 × 0·074 × 36′ × 12 = + 470,000 inch-lbs.
40,200 × 0·2043 × 36′ × 12 = + 3,550,000 ,,
18,300 × 0·061 × 36′ × 12 = + 482,000 ,,

Total = + 4,502,000 inch-lbs.

At Section C (at intermediate support) (Fig. 114):—

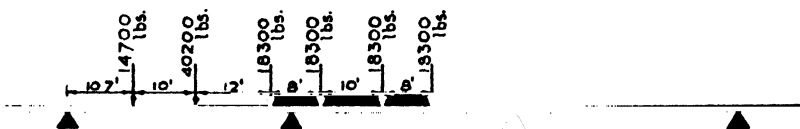



Fig. 114.

Live load bending moment =

14,700 × 0·0725	× 36′ × 12	= −	460,000	inch-lbs.
40,200 × 0·1025	× 36′ × 12	= −	1,780,000	,,
18,300 × 0·0455	× 36′ × 12	= −	359,000	,,
18,300 × 0·0463	× 36′ × 12	= −	366,000	,,
18,300 × 0·08	× 36′ × 12	= −	632,000	,,
18,300 × 0·06	× 36′ × 12	= −	474,000	,,
	Total	= −	4,071,000	inch-lbs.

At Section D (distance $0{\cdot}5l$ from intermediate support centre span). (Fig. 115).—

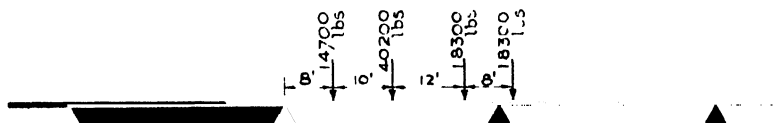



FIG. 115.

Live load bending moment

14,700 × 0·0607	× 36′ × 12		385,000	inch lbs.
40,200 × 0·175	× 36′ × 12		3,040,000	,,
18,300 × 0·0425	× 36′ × 12,	−	336,000	,,
18,300 × 0·009	× 36′ × 12	−	71,000	,,
	Total		3,690,000	inch-lbs.

Calculation of Reinforcement required in the Selected Sections. The reader is presumed to be acquainted with the customary method of determining the stresses in reinforced concrete members subjected to bending. No explanation of this will therefore be given.

Section A.

Total negative bending moment—

(*a*) Produced by dead load .	1,200,000 inch-lbs.
(*b*) Produced by live load .	4,824,000 ,,
	6,024,000 inch-lbs.
Total depth of section . . .	= 62 inches. (See Section 1—1, Fig. 123.)
Effective depth, 62″ − 2¼″ . .	= 59·75 inches.
Lever arm	= 54 ,,

Area of tensile steel . $\dfrac{6{,}024{,}000}{16{,}000 \times 54} = 7$ square inches.

Tension bars . . .	*4 bars* $1\frac{1}{2}$ *inches dia. in top.*
Compression bars . .	*3 bars* $1\frac{1}{4}$ *inches dia. in bottom.*

Section B.

Total positive bending moment—

(*a*) Produced by dead load .	1,900,000 inch-lbs.
(*b*) Produced by live load .	4,502,000 ,,
	6,402,000 inch-lbs.

Total depth of section . . .	= 38 inches. (See Section 2—2, Fig. 123.)
Effective depth, 38″ — $3\frac{1}{2}$″ . .	= $34\frac{1}{2}$ inches.
Lever arm	= 31·2 inches.
Area of tensile steel $\frac{6,402,000}{16,000 \times 31 \cdot 2}$	= 12·8 square inches.

4 *bars* $1\frac{1}{2}$ *inches dia.* plus 4 *bars* $1\frac{3}{8}$ *inches dia. in bottom.*

The top flange of the T beam, having an effective breadth equal to 0·75 × 8 feet 2 inches = 6 feet $1\frac{1}{2}$ inches, has a maximum stress of 533 lbs. per square inch.

Section C.

Total negative bending moment—

(*a*) Produced by dead load .	2,870,000 inch-lbs.
(*b*) Produced by live load .	4,071,000 ,,
	6,941,000 inch-lbs.

Total depth of section . . .	62 inches. (See Section 3—3, Fig. 123.)
Effective depth, 62″ — 3″ . .	59 inches.
Lever arm	54 ,,
Area of tensile steel $\frac{6,941,000}{16,000 \times 54}$	8·04 square inches.

Tension bars . 4 *bars* $1\frac{3}{8}$ *inches dia.* plus 2 *bars* $1\frac{1}{4}$ *inches dia. in top.*

Compression bars 4 *bars* $1\frac{3}{8}$ *inches dia. in bottom.*

Section D.

Total positive bending moment—

(*a*) Produced by dead load .	1,020,000 inch-lbs.
(*b*) Produced by live load .	3,690,000 ,,
	4,710,000 inch-lbs.

Total depth of section . . . . = 38 inches. (See Section 4—4, Fig. 123.)
Effective depth, 38″ − 3½″ . . = 34½ inches.
Lever arm . . . . . . = 31·4 inches.

$$\text{Area of tensile steel} = \frac{4{,}710{,}000}{16{,}000 \times 31{\cdot}4} = 9{\cdot}4 \text{ square inches.}$$

4 *bars* 1½ *inches dia.* plus 2 *bars* 1¼ *inches dia. in bottom.*

The maximum stress in the top flange of the T beam, having a breadth of 6 feet 1½ inches, is 422 lbs. per square inch.

Calculation of Negative Bending Moments in the Centre of the Spans.—The maximum negative moment in the centre of the *end*

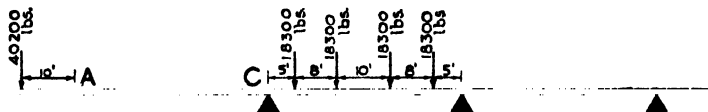



FIG. 116

spans is produced when the live loads are in the positions given below (Fig. 116).

The negative moment at section $0{\cdot}45l$ from the end support, produced by the loads on the centre span, can be read off the influence lines for the bending moment at this section. The load on the end of the cantilever produces a negative bending moment at support A of magnitude Ma and a positive moment at support C equal to $\frac{4}{15}$ Ma, as shown by the bending moment diagram below (Fig. 117).

Note.—This result is given by the theorem of three moments or by the graphical method given in Chapter IV.

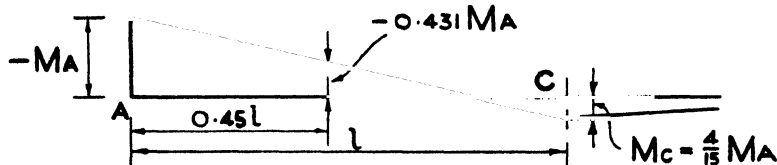



FIG. 117.

The negative moment at section $0{\cdot}45l$ is therefore equal to—

$$-\frac{19}{15}\,\text{M}a\left(\frac{l - 0{\cdot}45l}{l}\right) + \frac{4}{15}\,\text{M}a = \quad 0{\cdot}431\ \text{M}a \text{ (by proportion).}$$

The total negative bending moment at the chosen section is :—

0·431 × 40,200 × 10′ × 12	= − 2,080,000	inch-lbs.
18,300 × 0·022 × 36′ × 12	= − 174,000	,,
18,300 × 0·036 × 36′ × 12	= − 284,000	,,
18,300 × 0·027 × 36′ × 12	= − 213,000	,,
18,300 × 0·009 × 36′ × 12	= − 71,000	,,
	− 2,822,000	,,
Deduct dead load bending moment . . .	+ 1,900,000	,,
Resultant negative moment	= − 922,000	inch-lbs.

Total depth of section . . = 38 inches.
Effective depth . . . = $34\frac{1}{2}$ inches.
Lever arm . . . . = 31·4 inches (approx.).

$$\text{Area of tensile steel} = \frac{922,000}{16,000 \times 31 \cdot 4} = 1 \cdot 83 \text{ square inches.}$$

Ample reinforcement is provided by carrying through 2 *bars* $1\frac{1}{2}$ *inches dia. from the top steel* at the end support.

Maximum Negative Moment in the Centre of the Middle Span.— This moment is produced when the loads are on the end spans in the position given in Fig. 118.

FIG. 118.

From the appropriate influence line the total negative bending moment is :—

14,700 × 0·028 × 36′ × 12	= − 178,000	inch-lbs.
40,200 × 0·038 × 36′ × 12	= − 658,000	,,
18,300 × 0·012 × 36′ × 12	= − 95,000	,,
18,300 × 0·036 × 36′ × 12	= − 284,000	,,
18,300 × 0·035 × 36′ × 12	= − 276,000	,,
18,300 × 0·02 × 36′ × 12	= − 158,000	,,
	= − 1,649,000	,,
Deduct dead load bending moment . . .	+ 1,020,000	,,
Maximum negative moment in middle span . .	= − 629,000	inch-lbs.

$$\text{Area of tensile steel} = \frac{629{,}000}{16{,}000 \times 31{\cdot}4} = 1{\cdot}25 \text{ square inches.}$$

2 bars 1⅜ inches dia. in the top provide ample reinforcement.

Note.—In cases where the spans are considerable or numerous, it is sometimes economically desirable to investigate sections at frequent intervals (say, at ten equally spaced sections), and to prepare an envelope of maximum bending moments. The required distribution of longitudinal reinforcement can then be accurately determined in the customary manner. For an ordinary case, such as that of the above bridge, it is sufficient to investigate the principal sections and to provide an excess of reinforcement in the remaining sections, as shown in Fig. 123.

Shearing Forces.—Two sections will be investigated for the shearing forces :—

(1) At a section immediately to the right of the left-hand end support ;

(2) At a section immediately to the left of the left-hand intermediate support.

When the area and the spacing of the stirrups have been ascertained for these sections, the spacing between them can be determined by means of shearing force influence lines for intermediate sections. With experience, however, this can usually be arranged with sufficient accuracy by inspection.

(1) *Section to the Right of the Left-hand End Support.*—The shearing force produced by the dead load is obtained by taking moments to the left about support C :—

$$\text{Dead load shearing force} = \frac{1}{36}\left(\frac{wl^2}{2} + Ma - Mc\right)$$

$$= \frac{1}{36}\left(\frac{2{,}000 \times 36^2}{2} + \frac{1{,}200{,}000}{12} - \frac{2{,}870{,}000}{12}\right)$$

$$= \frac{1}{36}\left(1{,}296{,}000 + 100{,}000 - 240{,}000\right)$$

$$= \frac{1{,}156{,}000}{36} = 32{,}100 \text{ lbs.}$$

By placing the rolling loads on the influence line diagram for the shearing force at the above section in the positions shown below

(Fig. 119) the maximum shearing force due to the live load is determined :—

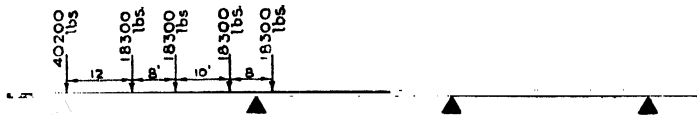



FIG. 119.

40,200 × 1	= 40,200 lbs.
18,300 (0·58 + 0·35 + 0·1 − 0·025) . .	= 18,400 ,,
Maximum live load shearing force on section	= 58,600 lbs.
Total shearing force at the section 32,100 + 58,600	= 90,700 lbs.

Ignoring the resistance of the concrete in shear and providing ½-*inch dia. stirrups having four branches* with a total area of 0·7854 square inches, their pitch, or spacing, will be given by the formula

$$\frac{As.fs.a}{Fm}$$

where As = total area of stirrups ;
fs = allowable shearing stress on ditto ;
a = lever arm ;
Fm = total shearing force.

$$\therefore \text{Pitch} = \frac{0{\cdot}7854 \times 12{,}000 \times 54}{90{,}700} = 5{\cdot}6 \text{ inches, } say\ 5\tfrac{1}{2}\ inches.$$

(2) *Section immediately to the Left of the Left-hand Intermediate Support.*—The dead load shearing force will be the difference between the total load on the end span and the shearing force at A, *i.e.*

= 2,000 lbs. × 36 − 32,100 lbs. ;
= 72,000 − 32,100 = 39,900 lbs.

The maximum live load shearing force obtained from the influence line diagram is obtained when the loads are in the positions shown in Fig. 120, and is as follows :—

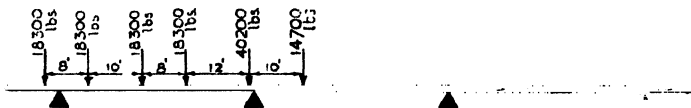



FIG. 120.

14,700 × 0·07	= 1,000 lbs.
40,200 × 1	= 40,200 ,,
18,300 × 0·75	= 13,700 ,,
18,300 × 0·53	= 9,700 ,,
18,300 × 0·19	= 3,500 ,,
Maximum live load shearing force on section	= 68,100 lbs.
Total shearing force = 39,900 + 68,100	= 108,000 lbs.

Providing ½-inch dia. stirrups having four branches (as before),

$$\text{Pitch} = \frac{0{\cdot}7854 \times 12{,}000 \times 54}{108{,}000} = 4{\cdot}7 \text{ inches, say } 4\tfrac{1}{2} \text{ inches.}$$

Girder Bridge Supports.—There are in general three types of supports applicable to reinforced concrete girder bridges :—

(1) Mass concrete or masonry piers ;
(2) Reinforced concrete walls ;
(3) Reinforced concrete trestles composed of columns or piles with suitable transverse stiffeners.

In this example the supports will consist of columns resting on reinforced concrete bases which will spread the load from the bridge so that the presumed permissible ground pressure is not exceeded.

The columns are spaced either under each longitudinal beam or at greater intervals. When the spans of the bridge are great, and consequently the load from each beam is considerable, it is usually economical to place a column under each beam. On the other hand, if the height of the bridge above foundation level is such that the columns have to be massive, then it is advisable to space the columns so that the full area of the column is working at approximately its permissible working stress. The longitudinal deck beams are then supported by a stiff transverse beam connecting the heads of the columns.

The former column spacing will be adopted in this example.

With a bridge of the type given in this example, the end columns have to be capable of a small but definite lateral movement at the top to permit the deck platform to expand and contract under changes of temperature. It is therefore desirable to employ relatively thin members, and, in order to reduce their areas to a minimum, use is made of the greater compressive resistance and ductility given by spiral reinforcement.

It is not usual to introduce into the calculations for the vertical columns the question of lateral displacement at the upper ends,

The permissible movements of the columns of varying sizes and lengths have been deduced, and it is sufficient to ascertain that the maximum possible movements due to temperature changes are within these allowable limits.

In the above example the probable maximum movement of the end columns is 0·128 inches, which is well within that permissible. For trestle bridges having lengths causing the above limits to be exceeded, roller bearings or sliding joints may be introduced under the longitudinal beams where these rest upon the column or other supports in question.

Load on the Columns due to the Dead Load.—The dead load on the beam per lineal foot is 2,000 lbs. The load on the end columns is found by taking moments about the adjacent interior columns,

i.e.
$$Ra \times 36 = 2{,}000 \times 46 \times 23 - Mc$$
$$= 2{,}116{,}000 - \frac{287{,}000}{12}$$
$$Ra = \frac{1{,}876{,}000}{36} = 52{,}100 \text{ lbs.}$$

and the load on the interior column is equal to half the load on the bridge, minus the load on the side columns

$$Rc = \frac{2{,}000 \times 128}{2} - 52{,}100 = 75{,}900 \text{ lbs.}$$

Load on the Columns due to the Live Load.—The simplest means of ascertaining the maximum reaction produced by a train of point loads is to apply the series of loads to the appropriate influence line.

The following load positions (Figs. 121 and 122) produce the maximum reactions on the columns, and the ordinates of the influence line under the loads enable the magnitude of this reaction to be found.

End Support.—

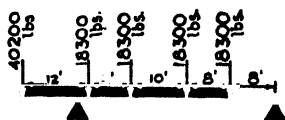



FIG. 121.

40,200 × 1	= 40,200 lbs.
18,300 (0·925 + 0·645 + 0·345 + 0·135) .	= 37,500 „
	77,700 lbs.

Intermediate Support.—

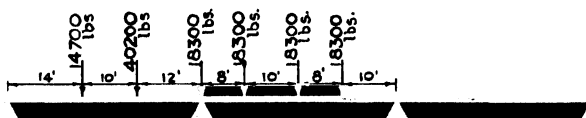



FIG. 122.

14,700 × 0·59	=	8,700 lbs.
40,200 × 0·88	=	35,400 ,,
18,300 (1·0 + 0·885 + 0·575 + 0·3) . .	=	50,500 ,,
		94,600 lbs.

Design of End Columns.—The columns are approximately 15 feet high from top of base to underside of beam.

Dead load	=	52,100 lbs.
Weight of top tie beam	=	1,900 ,,
Weight of columns.	=	2,600 ,,
Live load	=	77,700 ,,
Maximum load on end column . .	=	134,300 lbs.

The compressive resistance of a 14-inch octagonal column (Fig. 124) reinforced with 8 *longitudinal bars, $\frac{3}{4}$-inch dia.*, and *$\frac{5}{16}$-inch dia. spiral at 2-inch pitch, is* 138,700 *lbs.*, calculated as follows (see Second Report of R.I.B.A. Joint Committee on Reinforced Concrete) :—

Increased stress on core due to spiral $= 600\left(1 + \dfrac{32 \times 17 \cdot 8}{1360}\right)$

852 lbs. square inch.

Resistance of core area = 852 × 0·7854 × 12″ × 12″ = 96,500 lbs

Resistance of vertical steel = 852 × (15 − 1) × 3·534 = 42,200 ,,

138,700 lbs.

The total working resistance of the columns is therefore greater than the total load to be carried.

It may be noted that the calculated strength of the above column provides an absolute factor of safety in the neighbourhood of 4·72. The load calculated to come upon the column may be regarded as an absolute maximum, and the increase of compression on one face, due to possible distortion resulting from atmospheric changes, can therefore be accommodated without overstress.

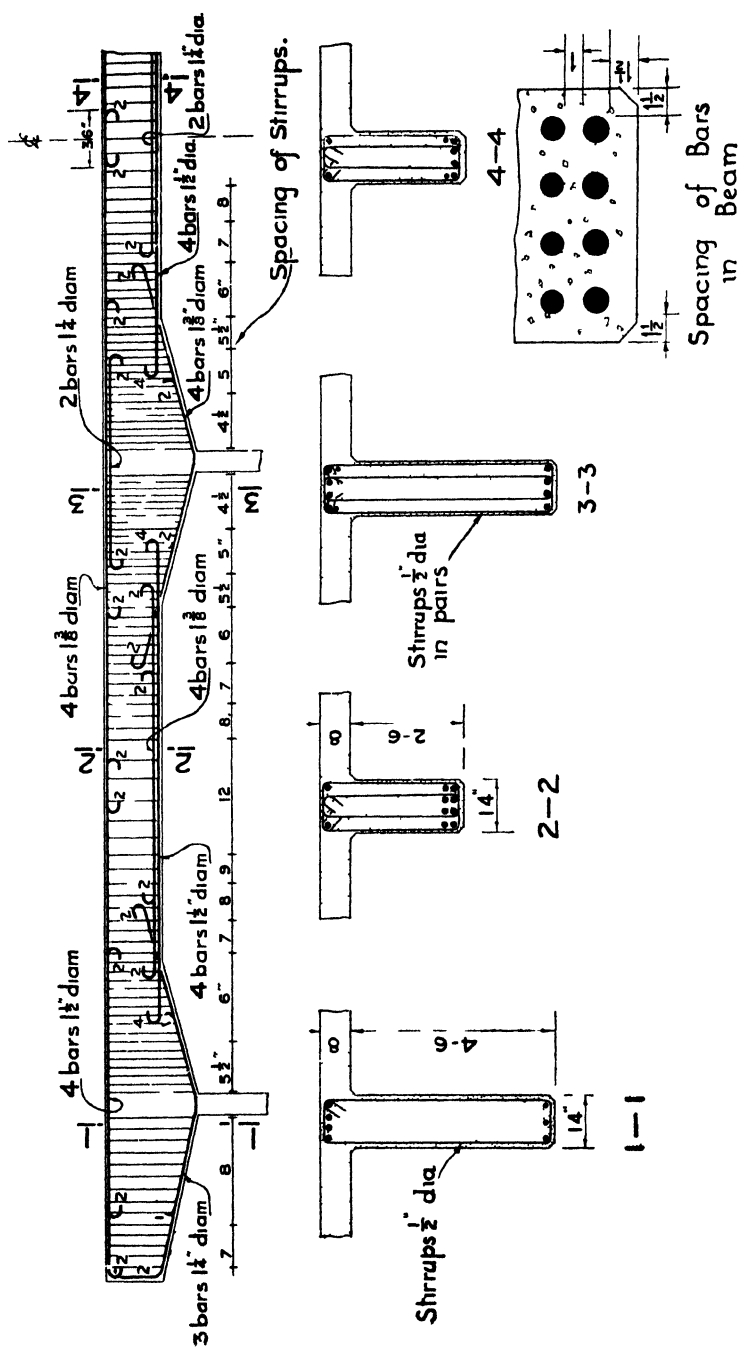



FIG 123 —Arrangement of Reinforcement in Main Longitudinal Beams
(See Art 107)

Design of Intermediate Columns.—

Dead load	=	75,900 lbs.
Weight of top tie beam	=	1,900 ,,
Weight of column	=	2,600 ,,
Live load	=	94,600 ,,
Maximum load on intermediate column .	=	175,000 lbs.

The compressive resistance of a 14-inch octagonal column (Fig. 124) reinforced with 8 *longitudinal bars* $\frac{7}{8}$*-inch dia. and* $\frac{3}{8}$*-inch dia. spiral at 2-inch pitch* is 174,000 lbs., calculated as follows :—

Increased stress on core due to spirals $= 600\left(1 + \frac{32 \times 25{\cdot}6}{1{,}360}\right) =$ 964 lbs. per square inch.

Resistance of core area $= 964 \times 0{\cdot}7854 \times 12'' \times 12'' =$ 109,000 lbs.
Resistance of vertical steel $= 964 \times (15 - 1) \times 4{\cdot}81 =$ 65,000 ,,
174,000 lbs.

This total resistance is approximately the same as the total load on the columns.

Foundations.—The foundations to the bridge supports vary with every site, depending on the bearing capacity of the ground, whether the work is under water, tidal or otherwise, and numerous other practical conditions which have to be considered individually.

FIG. 124.—Arrangement of Column and Reinforcement.

For an average site, as, for instance, when the bridge is over a railway, the best form of base would probably be a narrow continuous foundation beam having a bottom slab of sufficient width in order that the allowable pressure upon the ground is not exceeded.

No special calculations will be made for this portion of the work, as they would only apply to a specific case and would not differ from those of other engineering structures.

The complete bridge is shown in longitudinal and cross sections in Fig. 108.

The detail showing the complete reinforcement of a roadway beam is given in Fig. 123, and a detail of one of the column supports is given in Fig. 124.

CHAPTER VIII

BOWSTRING GIRDER BRIDGES

108. General Description.—Any girder of the open type having a curved upper boom and horizontal lower member is usually referred to as a bowstring girder. When constructed in reinforced concrete, these girders generally comprise a curved top rib with their ends connected together at the level of the springings by a tie member, or "string," supported at various points along its length by vertical posts or suspenders connecting the tie beam to the arched rib.

Diagonal shear members as employed for steel girders have been found from experience to be unsatisfactory for reinforced concrete girders. The calculations for such systems are based on the assumption that the joints between the members are perfectly free to rotate. This assumption is not realised in reinforced concrete work, and any attempt to triangulate reinforced concrete structures, such as bowstring girders, results in serious secondary stresses at the points of intersection of the component members. These latter stresses prove a source of weakness unless the section and steel reinforcement at the connections are increased to such an extent as to be uneconomical.

A bowstring girder bridge made by Considère in 1907,* and tested to destruction, whilst proving the almost miraculous resistance of such a structure, showed that the presence of the diagonal compression members when under heavy loading had a bursting effect upon the longitudinal members and was directly resultant in the failure of the entire structure.

Admittedly this failure was produced by a load more than four times that for which the bridge was designed, but the presence of these members was nevertheless held to be a source of weakness, and they are not now employed.

Two remaining types have vertical members only connecting the curved upper boom to the lower horizontal boom.

The essential difference of these two latter types lies in the relative stiffness of the "vertical suspenders" in the plane of the girder. Where these vertical members are made stiff in this direction by providing them with suitable cross sections, and particularly by stiffening them at their junctions with the main longitudinal members

* "Reinforced Concrete," by A. Considère, p. 203.

by means of gussets, in such a way that they resist distortion, then the girder so formed must be regarded as an arched beam and designed as such. Moreover, to a great extent the objections existing in the case of the girder diagonally braced are present in these latter beams, in which the shearing forces are taken by the rigidity of the verticals instead of by the diagonal bracing under direct stress.

FIG 125 —Elevation of Bowstring Girder Bridge.

The remaining type of bowstring girder is that found to be most satisfactory, and is now always employed. It consists of a curved "bow" or arc connected with a horizontal tension member in the form of a "string" or tie, which is suspended from the bow by means of vertical hangers or posts This type of girder is in reality an arch in which the horizontal thrust is taken by the tie, the vertical members being of secondary importance, *i e.*, to give the necessary support to the horizontal tie, which, owing to its relative flexibility, would otherwise "sag."

109. Advantages of Bowstring Girder Bridges. — The principal advantage gained by this type of structure is the greater headroom available under the bridge. In addition, it is possible to span great lengths without the necessity of any inter-

mediate supporting piers, and without the provision of the expensive abutment supports so frequently necessary in the case of a big span arch. This latter point often results in an economy for the bowstring girder bridge over other types, having due regard, of course, to the advantages of span and headroom. The abutment supports for a bowstring girder can be constructed with an absolute minimum of excavation and expensive foundation work. The bearings at either end of the bridge can be inclined so as to balance the component of the earth thrust, thereby eliminating the necessity of balancing this thrust by providing a large area to the bottom slab of the abutment walls with expensive counterforts.

110. Arrangement of Principal Members.—Fig. 125 shows an elevation of a bowstring girder bridge for a span of 145 feet. It will be apparent to the reader that in bridges of this type it is necessary to arrange the beams supporting the deck slab transversely, that is at right angles to the direction of the roadway, as shown in Fig. 126. The spacing of these beams is governed in the case of heavy rolling loads by the required thickness of the deck slab. To carry a heavy concentrated load in a satisfactory and economical manner, the deck slab requires to have a certain minimum thickness in order that the shearing stresses shall not become serious, and a slab of this minimum thickness is capable of spanning a certain distance economically. This distance is determined and adopted for the spacing of the transverse beams supporting the deck slab.

FIG. 126.—Cross Section of Bridge shown in Fig. 125.

Occasionally the width of the bridge is such as to make it desirable to place the above transverse supporting beams closer together than is necessitated by the above requirements of the deck slab.

It will be noted from Fig. 126, which gives a cross section of the bridge illustrated in Fig. 125, that the deck platform is placed at the level of the tie beam. The desirability in doing this will, of course, be obvious to the reader, as will also the practice of providing vertical suspenders directly opposite and supporting each end of the transverse deck beams. By doing this an inherent transverse stiffness at

frequent intervals is provided, and the necessity for introducing overhead wind bracing between the curved ribs frequently obviated.

111. Stability against Wind Pressure.—The lateral resistance of bowstring girders against wind pressure requires to be carefully investigated, as, owing to their form, they offer considerable resistance to the wind and consequently require proper provision to prevent them being overstressed from this cause.

In Fig. 126 it will be seen that the vertical suspenders are increased in width towards their lower ends, and these members are designed to take the overturning moment produced by the wind pressure acting upon the exposed windward face. Each girder is treated as a separate unit and designed to be self-supporting. This is found to be more economical than connecting the two ribs, which only has the merit of equalising the pressure; unless the bridge is narrow any arrangement of transverse overhead stiffening would require to be made very rigid.

In calculating the wind pressure on the vertical suspenders, it is necessary to consider the area of the windward face of the suspender itself, in addition to the portion of curved rib laterally supported by this member, the resultant horizontal forces being applied at the centres of the respective areas. The bending moment produced is considered in conjunction with the uniform tension in the suspender resultant from the vertical loading carried by it. This is done in the orthodox manner, it being noted that the predominant force is tension instead of compression, as is the case for arches and columns eccentrically loaded.

It should be further noted that the increased tension produced by the bending action may act on either face. It is usual, in calculating the stress induced by wind pressure on the individual suspenders, to ignore the protection that may be given by the windward girder to the windward face of the leeward girder.

In addition to designing the suspenders to resist lateral bending moments, the horizontal pressure transmitted to the deck platform by them has to be investigated. The total wind pressure produced by the wind action simultaneously upon both girders is calculated as follows :—

Total horizontal wind pressure on deck platform—

$$P_w = p.b\left(2 - \frac{b}{a}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where p = assumed unit wind pressure, b = nett surface area of one girder, and a = total area enclosed by the underside of the deck construction and the top surface of the parabolic rib.

The above formula assumes a reduction of pressure upon the leeward girders, justified in the case of most bowstring

bridges met with in practice. To calculate the maximum lateral bending moment produced by the above wind force, the latter may be assumed with sufficient accuracy as a uniformly distributed load and the span as freely supported. The maximum bending ascertained by this means, of course, occurs in the centre of the bridge, and, as in the case of the vertical suspender, is reversible. Additional steel is usually added throughout the length of the longitudinal ties to provide for the increased tension so produced.

The increased pressure upon the leeward abutment supports, owing to the eccentricity of the lateral wind pressure in the vertical plane, is usually negligible, but in narrow bridges in an exposed position this factor should be considered.

112. Design of Bowstring Girders.—In most cases the design of a bowstring girder resolves itself into the separate investigation of the three types of members of which it is composed. In special cases, however, this is not possible. Where the ties are incorporated with the parapets, thereby forming members of some appreciable stiffness relative to their unsupported length, they will inevitably contribute to the moment of resistance of the composite whole, this latter contribution being quite distinct from their essential function as ties (see Fig. 176).

In such cases the portion of the bending moment taken by the curved rib and tie beams respectively depends upon their relative moments of inertia.

It is customary to assume the length of the suspenders as unalterable in making the above calculation.

There is an important difference in the design of a bowstring girder from that of a two-pinned arch. In the latter, the pins or hinges at the springings are assumed to permit rotation, but to be incapable of relative displacement. This has the effect of introducing a bending moment into the arch when it expands and contracts due to atmospheric changes. In the case of the bowstring girder, however, any change in the length of the curved rib or bow, causing the span to vary, is permitted by the rocker bearings. The bearings are caused to rotate to the extent required to accommodate the rib movement by the simultaneous variation in length of the horizontal ties and bridge deck due to change of temperature.

Consequently no appreciable stress is produced in any part of this type of girder from temperature effects, and the linear variations from this cause are not taken into account in design.

In addition to the bending produced by the superimposed rolling loads in the curved rib of a bowstring girder, bending moments are induced by the elastic extension of the horizontal tie resultant from the tension in this latter member.

In bridges having spans from 100 feet upwards, this bending moment becomes important, and should be taken into account. Where this is done, it is customary to consider the whole of the tension to be taken by the longitudinal steel reinforcement in the ties and to calculate the total elastic extension in these members upon this assumption. The equivalent horizontal force acting upon the curved " bows " or ribs from this movement is then made in the same manner as in the case of linear variation for the two-hinged arch, as explained in the chapter on arches (Art. 80). In this connection the reader should note that the effect of an increase in distance between the bearings of a bowstring girder, due to the elasticity of the horizontal ties, is similar in effect to a settlement of the abutments in a two-hinged arch. That is to say, a horizontal force is induced at the springings tending to flatten the curved rib.

For bowstring girders of moderate span, in view of the combined resistance of the ties and the entire deck platform, no appreciable elastic extension will take place, and its effect upon the deformation of the curved ribs is therefore neglected.

113. Design of Curved Rib.—For purposes of the design, the curved rib members may be considered as two-hinged arches, and are preferably made of parabolic form. The span is measured between the points of supports in place of the distance between the articulations, as in the case of the two-hinged arch. In order to arrive at an economical arrangement, the rise of the curved rib should be about one-sixth of the above span. It might also be mentioned that any greater rise than this, under normal conditions, is unnecessary and uneconomical if the ribs are made of suitable section and provided with reinforcement properly arranged.

With regard to the latter points, octagonal sections reinforced with longitudinal rods enclosed with a closely pitched spiral throughout their length have been found to give the most satisfactory result both from the point of view of economy and ductility, the latter being a factor of the greatest importance in members of this type.

In bowstrings of large span, it is sometimes found desirable to form the curved ribs by means of two octagonal booms, reinforced as above, and connected by means of a relatively thin web of concrete encasing reinforcement in the form of stirrups (see Fig. 172).

In addition to calculating the bending moments produced by the vertical superimposed loading, the additional moments induced by the elastic extension of the horizontal ties should be computed in the case of large span bridges (see Art. 112).

Further, when the inertia of the tie is such as to appreciably

resist, as already mentioned, the bending moments caused by the vertical loading, this should be taken into account as follows :—

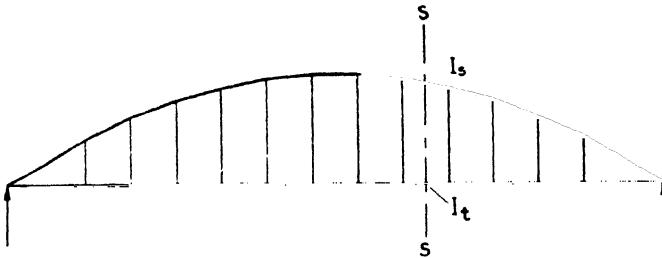



Fig. 127.

Proportion of total bending moment taken by curved rib at section

$$SS \qquad \frac{Ms}{Is + It} \cdot Is \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

where Is = moment of inertia of rib at section considered ;
It = moment of inertia of tie (steel only) ;
Ms = bending moment at section SS.

From (2) the compressive and tensile stresses at rib section SS can usually be calculated as follows :—

$$S \text{ max.} = \frac{H}{\cos \alpha . Ae} \pm \frac{Ms . y}{Is + It} \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

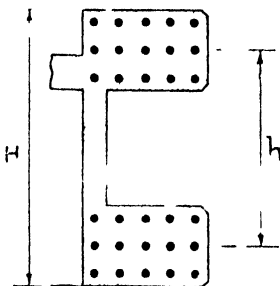
where S = stress at distance y from neutral axis of curved rib ;
H = horizontal thrust ;
Ae = total equivalent cross sectional area ;
α = angle of tangent to rib axis with horizontal at section SS.

Note.—In finding moment of inertia of the tie the concrete is neglected and the moment of inertia of the longitudinal rods only considered, the area of these being increased by the assumed modular ratio $\frac{Es}{Ec}$ for this purpose.

Fig. 128.

114. Design of Horizontal Ties.—The horizontal ties have to be designed to resist the action of three efforts, as follows :—

(1) The uniform tension due to horizontal thrust from the curved ribs = H ;

(2) Varying tension due to the bending moment (Mw) induced in the deck platform by the presence of the wind = $\frac{Mw}{B}$

B = width of bridge (between centres of ties);

(3) Bending moment resultant from deformation due to vertical loading $= \frac{Ms}{Is + It} \cdot It.$

The total maximum tension occurring in either the upper or lower boom of the tie can be found from the three above efforts, and may be expressed approximately as follows :—

$$T = \frac{H}{2} + \frac{Pw.l}{16B} + \left(\frac{Ms}{Is + It} \cdot It\right)\frac{1}{h} \quad . \quad . \quad . \quad (4)$$

where l = span or length of bridge;
B = lateral distance between centres of ties;
h = distance between centres of area of top and bottom groups of rods (Fig. 128).

Note.—In the event of the supporting rockers being at an angle with the vertical, the horizontal component of the inclined reactions tends to reduce the tension in the tie and may be subtracted from it.

115. Design of Vertical Suspenders.—Regarding the design of the suspenders, it will, of course, be noted that the whole of the tension coming upon them is assumed to be carried by the steel reinforcement for which a low working stress is usually adopted ; 14,500 lbs. per square inch is a suitable figure for this latter working stress. When calculating the moment of inertia the vertical rods only are considered, as for the case of the horizontal ties.

The bending moment produced by wind acting upon the ribs and suspenders themselves requires to be added to the uniformly distributed tension resultant from the vertical loading.

It is customary to provide the same area of reinforcement on both faces of the suspenders, although a lesser windage, and therefore tensile stress, is likely on the inner face owing to the partial protection given by the opposite girder.

It is particularly important in designing the vertical suspenders to make them flexible in the plane of the girders, so that they may accommodate themselves without undue strain or resistance to the slight but inevitable relative movement between the rib and ties.

Care must also be taken, in arranging the reinforcement at the junction of the vertical suspenders and cross-deck beams, to provide for the bending moments at these points.

116. Method of Supporting Bowstring Girder Bridges.—In order that a bowstring girder may act in accordance with the assumptions made in design, it is essential that its supports shall permit the required rotation and that one support shall allow

lateral displacement. For this reason, roller or rocker bearings are provided at one end of the bridge. Fig. 26 gives a type of reinforced concrete rocker bearing that has been successfully used in practice. Where the supporting members are composed of roller bearings, these may be of cast steel, of the type illustrated in Fig. 27.

117. Connection of Curved Ribs with Horizontal Ties.—A point of vital importance, and that requiring most careful attention in detailing the reinforcement and during construction, is the connections between the curved rib and tie.

The distribution of stress at these points is by no means uniform, and great care is required to design the junction of these members in such a way that the tension in the horizontal tie is gradually absorbed from the longitudinal reinforcement and transmitted to the curved rib, where it neutralises the horizontal component of the normal compression force in this member. Shearing reinforcement is put in for the purpose, which assists the concrete at the points of greatest stress concentrations.

CHAPTER IX

TEMPORARY AND PERMANENT HINGES

118. Advantages of Temporary Hinges.—For a number of practical reasons it is found that arches of the fixed or hingeless type are most suitable for the majority of cases met with in practice. Certainly the hingeless arch lends itself more freely to architectural treatment, inasmuch as it is unnecessary to provide an expansion joint in the bridge superstructure at the crown as well as over the springings of the arch. This may appear a point of little importance, but the difference in appearance, and in some cases the difference in cost, is marked. At the same time there are the theoretical considerations with the hingeless type that present difficulties.

It has already been explained (Art. 28) that the following effects produce shortening, or similar effect, in an arch member :—

(1) Shrinkage of concrete during setting and hardening ;

(2) Elastic compression in arch due to superimposed loading ;

(3) Settlement of abutments under arch thrust.

In a hingeless arch this shortening produces bending moments which permanently stress the arch and which become serious if the arch is of big span or of low rise-span ratio. If hinges are introduced at the springings and crown, the arch can freely accommodate, without appreciable strain, any slight alteration in length by a rise or fall of the crown.

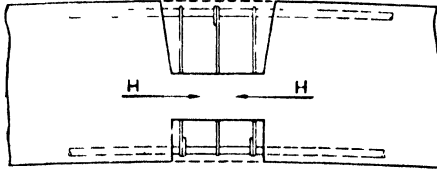



Fig. 129.—Temporary Hinge at Crown of Arch Rib.

Furthermore, the reader will note that this shortening is produced for the most part during construction, since, firstly, most of the shrinkage takes place during this period, and, secondly, in most arch bridges the weight of the structure is much greater than the total superimposed moving loads coming upon the bridge.

It is principally with the object of eliminating the stresses resultant from this arch shortening that temporary hinges are introduced.

An additional advantage obtained by their use is that the bending moment caused by the smaller proportion of arch shortening taking place after the temporary hinge sections are filled can be

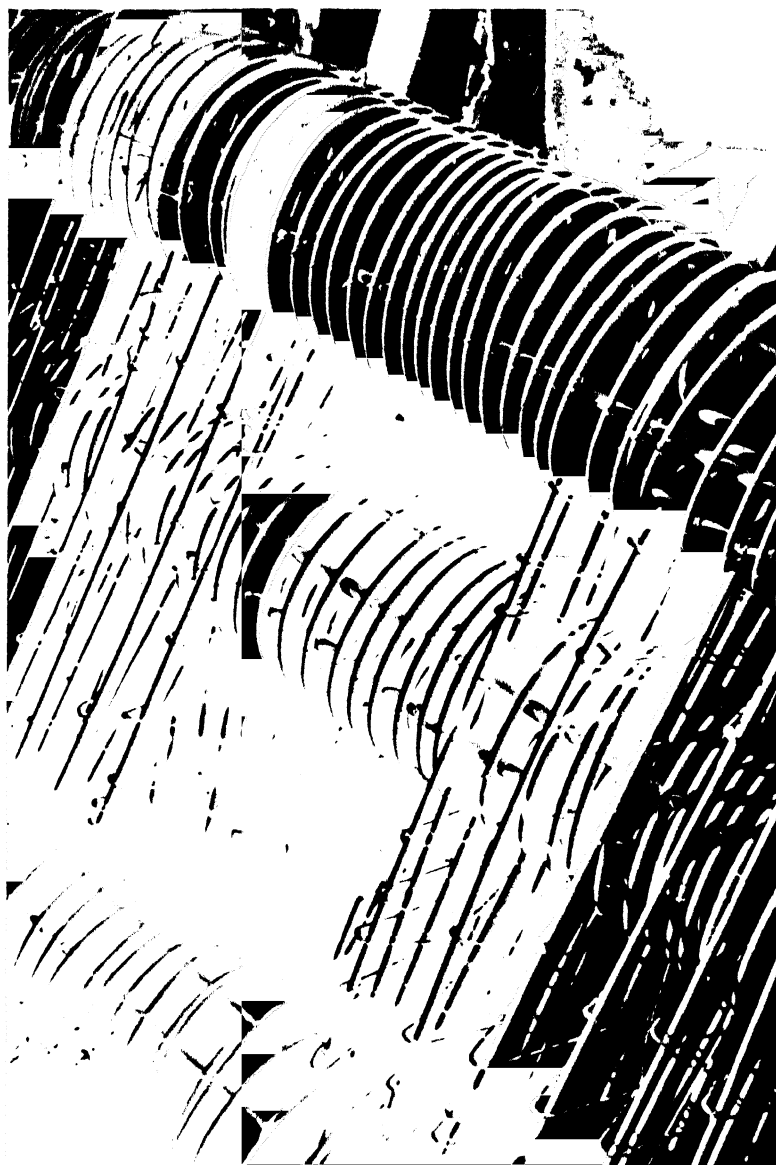



Fig. 130. Temporary Hinge at Springings of Vesubie Bridge. (Before Concreting.)

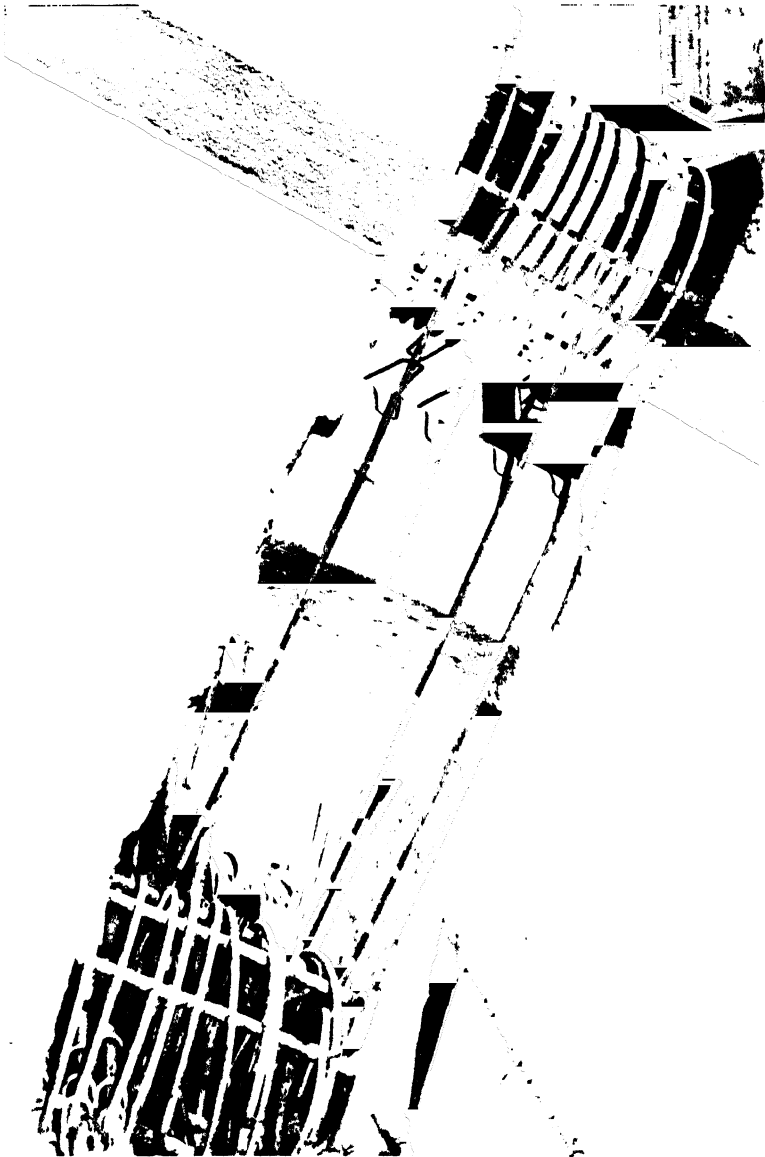



PLATE V.

Fig. 131. Temporary Hinge at Springings of Vesubie Bridge (Under Load.)

FIG. 132. VIEW OF TEMPORARY HINGE SUITABLE FOR SPANS UP TO 80 OR 100 FT.

PLATE VII.

neutralised by suitably arranging the hinges in regard to their cross-sectional positions. For an explanation of this, see "Eccentricity of Hinges" (Art. 123).

The employment of temporary hinges also has the merit of considerably simplifying the design of an arch bridge as well as enabling the stresses produced by the dead weight of the structure to be estimated to a greater degree of known accuracy than in the case of a hingeless arch not provided with flexural hinges.

119. Description of Temporary Hinges.—By temporary hinges is meant the provision of flexural hinges which act as such only during the construction of an arch bridge.

First employed by Considère, hinges of this type consist essentially of a local reduction in the concrete cross section, so loaded and reinforced that the reduced area is ductile, and resistance to slight rotation is practically eliminated, and the reaction from the loads consequently caused to pass through them (Fig. 129). It should be noted that the temporary staging or centring is slackened as soon as the concrete in the arched ribs is sufficiently hardened to permit this being done.

After completion—that is, when the prearranged amount of the dead or permanent weight has been added to the structure—the arch sections at each of the hinges are made good with concrete, and thereafter it acts as a fixed arch.

One of the largest span bridges in which temporary hinges have been employed is that over the River Vesubie, described in Chapter XII. This structure has a clear span of 315 feet, and the thrust taken by each of the hinges was 400 tons.

A view of one of the springings' hinges is given in Fig. 130, showing the reinforcement in position. The view in Fig. 131 illustrates the completed hinge actually under load. The average compressive stress upon this hinge, calculated upon the gross cross sectional area, was 2,850 lbs. per square inch.

Fig. 132 gives a view of a temporary hinge suitable for small bridges having spans up to 80 or 100 feet with moderate rise-span ratios.

120. Efficiency of Temporary Hinges.—It may be mentioned here that criticism has been advanced against temporary flexural hinges to the effect that such hinges offer considerable resistance to even the very slight rotation they are called upon to permit. This resistance is held to be provided by the concrete hinge section acting in compression and the longitudinal reinforcement on one face in tension, it being apparently admitted that the compression reinforcement will tend to buckle and thereby not oppose rotation.

A brief study of the question will make it clear that this criticism is not in accordance with practical fact, and could only have originated through incomplete knowledge of this type of hinge and its function. The longitudinal section of the hinge is so arranged that the upper and lower curved rods are exposed for a length sufficient to permit them to accommodate themselves without undue strain to the very slight movement produced by the hinge section.

In a properly designed spiral temporary hinge the shortening of

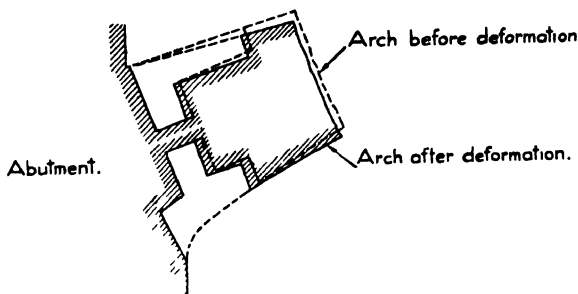



FIG. 133.—Diagrammatic View of Hinge Action.

the hinge itself is such that little, if any, tension is imposed upon the longitudinal reinforcement. This is illustrated in Fig. 133, the movement being greatly exaggerated for clarity. The above statement applies only to hinges which are spiralled and not to other types in which the reduction of cross sectional area is more gradual and consequently less ductile.

121. Arrangement of Reinforcement in Temporary

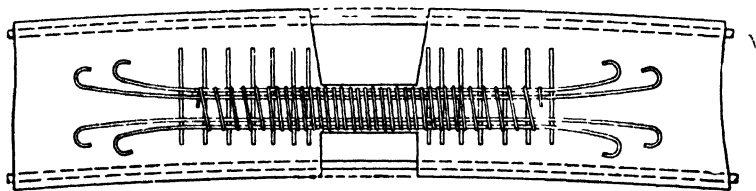



FIG. 134.—Longitudinal Section through Temporary Hinge, showing Arrangement of Reinforcement.

Hinges.—Fig. 134 indicates the longitudinal section of a temporary hinge of the type employed by Considère for arch rib members having a rectangular cross section.

This particular hinge is that situated at the crown. The arrangement of reinforcement is clearly shown, and may be taken as typical, the size and spacing of the reinforcement only being varied to accommodate the thrust to be taken. The cross sectional area of

concrete required at the hinges is, of course, directly dependent upon the thrust given by the dead weight of the bridge, and in order to keep this area as small and ductile as possible it is enclosed by one or more closely pitched spirals, the precise number depending upon the cross sectional form of the arched rib member.

Fig. 135 shows the cross section of the crown hinge employed for

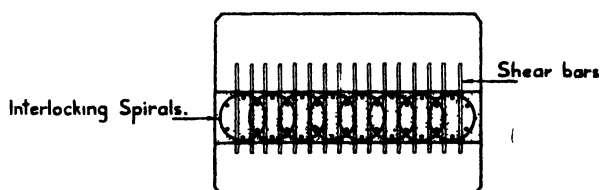



FIG. 135.—Cross Section of Crown Hinge, Warrington Bridge.

the Warrington Bridge (see Art. 152), and Fig. 136 gives a view of one of the ribs of this bridge during construction.

122. Design of Temporary Hinges.—With regard to the design of temporary hinges, having ascertained the thrust coming upon a particular hinge and knowing the shape of the cross section of the arch member at this point, the minimum area and depth of the hinge section can be easily calculated, as shown in Art. 124. The maximum compressive stress calculated upon the cross sectional area of the hinge within the spiral is usually about 2,000 to 2,500 lbs. per square inch, depending upon the mixture of concrete employed. This high compressive stress is necessary in order that the concrete shall be suitably ductile, this latter quality being essential for its efficient action. In order that the concrete shall be capable of withstanding this very high compressive stress, the required percentage of lateral reinforcement is considerably greater than is ordinarily employed.

For large bridges upon the Continent stresses exceeding 3,000 lbs. per square inch, calculated upon the cross sectional area of the concrete alone, have been employed with complete success, this figure being greater than the specified crushing strength of the concrete employed. As mentioned above, it is essential for such limiting compressive stresses that the section be suitably spiralled, and the greatest care taken with regard to the qualities of the material employed and its construction.

The hinge spiralling should be arranged on the lines indicated in Figs. 134 and 135. The length of the reduced hinge section should not exceed twice the least dimension of its cross section, and care should be taken to ensure the compressive stress in the normal rib section being concentrated to the reduced section without distressing

the concrete, which may be unspiralled on either side of it. A suitable method of providing for this concentration is shown in cross section and in elevation in the above figures.

123. Eccentricity of Temporary Hinges.—It is obviously desirable in an arch member to ensure, as far as possible, that the maximum eccentricity of thrust acting upon any cross section shall be equal in amount on either side of the neutral axis of the section.

If the effects of elastic arch shortening, due to compression, shrinkage, and settlement of abutments, be ignored, and an envelope of maximum bending moments resultant from superimposed loading be plotted, it will be seen that this condition obtains. It has already been explained that the above effects produce no stress whilst the temporary hinges are present, and it remains, therefore, to provide against any eccentricity of thrust induced by these effects *after* the hinge sections have been concreted in.

Practical considerations usually make it desirable to complete the concreting at the temporary hinges before the construction of the bridge is finished, and a certain proportion of the permanent, as well as the whole of the superimposed, loading may therefore come upon the structure when it is in its fixed or hingeless condition. A small percentage of the shrinkage of the arch member may also take place when the arch is in this state.

The deformation produced by this latter loading and shrinkage causes a permanent eccentricity to the line of pressure of the arch, which changes sign at the level of the "elastic centre" of the arch axis, producing negative moments below this level and positive moments above it.

These bending moments at each of the hinge sections can be determined when the amount of arch shortening taking place after the hinge sections have been concreted is known.

This figure depends upon the type and amount of permanent loading placed before the arch is fixed, and may vary very considerably.

The equivalent horizontal force acting through the "elastic centre" of a parabolic arch axis required to produce the above bending moments is approximately as follows :—

$$Hs = \frac{45n . E . Ic}{4f^2}$$

where $n = \dfrac{\text{total contraction}}{\text{length of arch axis}}$ Having ascertained the value H*s*, the resultant bending moments at the springings and crown hinges can be readily calculated.

In order to neutralise these bending moments, the temporary

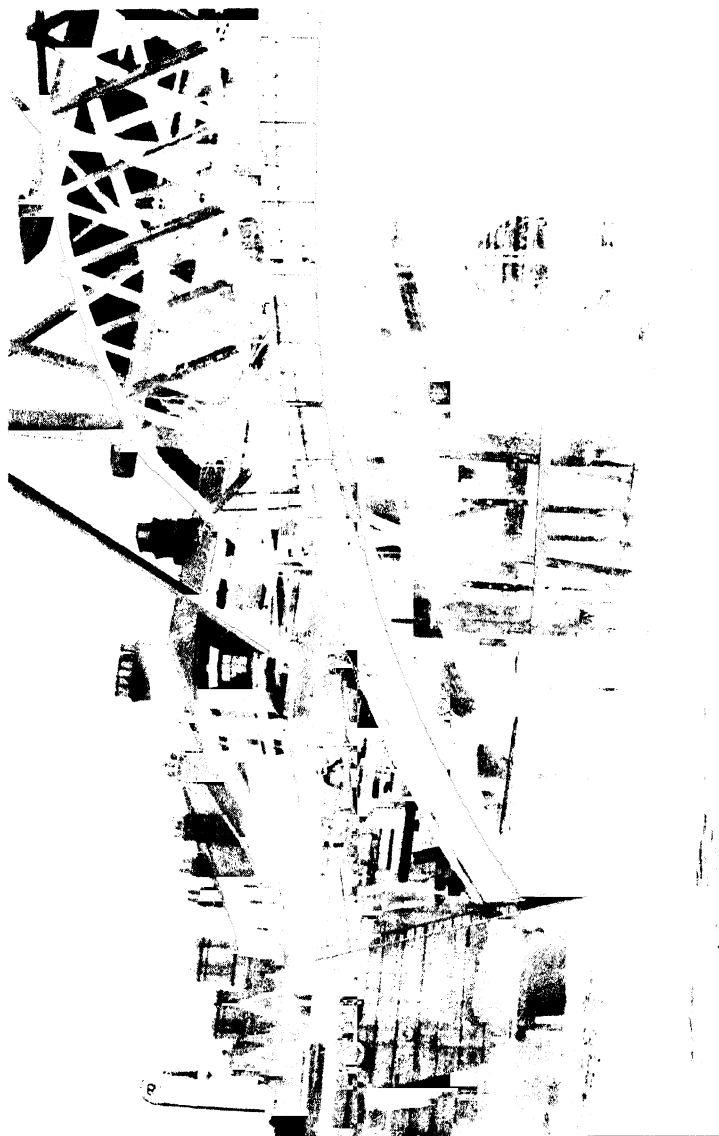



FIG. 136.—VIEW OF ARCH RIB, SHOWING TEMPORARY HINGES.

[To face page 188.]

hinges are placed at a distance from the axis of the arch section so that they induce moments equal in amount, but opposite in sense, throughout the arch.

For example, in the case of a hinge at the springing it would be placed at a vertical distance $\frac{M}{H(d+s)}$ *above* the arch axis, where M = moment at hinges produced by arch shortening; $H(d + s)$ = horizontal thrust from dead load and arch shortening.

No account is taken of temperature stresses in making the above calculations, as the hinges are filled in at an assumed mean temperature, in which case these stresses have equal maximum positive and negative values.

It should be noted that when temporary hinges are employed, and the above allowance for arch shortening made, the quantity $\frac{45\ Ic}{4\ Av.f^2}$ contained in the formula for horizontal thrust is neglected (see Art. 66). This greatly simplifies the investigation of a parabolic hingeless arch, and enables standard influence line to be employed.

124. Calculation for Temporary Hinge Sections.—The method of finding suitable dimensions and reinforcement for a temporary hinge is as follows:

Maximum normal thrust upon hinge under dead weight = N tons. Employing say 8 per cent. of lateral reinforcement in order to keep the hinge area as small as practicable, and also to give the concrete core the necessary ductility, the permissible stress upon the spiralled core

$$Sp = Sc\left(1 + \frac{8 \times 32}{100}\right) = 3{\cdot}56.Sc$$

Note.—Sc = permissible stress in pounds per square inch upon unspiralled concrete.

Required area of spiralled concrete core $As = \frac{2240.N}{Sp}$

Assuming As to be in square inches, the required volume of spiral per inch length of hinge = 0·08 As cubic inches, from which a suitable diameter and pitch can be adopted.

In the event of several spirals being employed, as is desirable with a wide rectangular section of arch member, the volume of total spiralling necessary must, of course, be divided by the number of spirals found suitable before the diameter and pitch of the lateral reinforcement can be found.

Suitable longitudinal reinforcement, not less than 0·8 per cent. in volume of the spiralled core, should be introduced.

125. Permanent Hinges.—In the case of arches where the loading may be unusually heavy, or when the rise span-ratio may be lower than is usual, it is sometimes desirable to provide permanent articulations or hinges. Also in cases where the range of temperature is likely to be great it is sometimes found economical to provide permanent hinges.

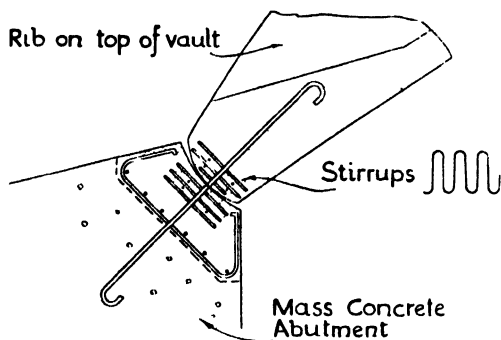



Fig. 137.—Reinforced Concrete Arch Hinge, Contact Type.

These hinges may be introduced into arches having rib members, or into those consisting of plain slabs or vaults, with continuous bearings along the abutment supports. Either of these types of bridges may require to be constructed on the skew, that is, so that the direction of the bridge makes an angle other than 90° with the direction of its abutments. In these latter cases, the sliding tendency caused by the arch thrust making an angle with the normal reaction from the abutment must be taken into account.

Hinges employed for reinforced concrete arch bridges may be of metal or concrete. When constructed of the latter material they may be either of the curved contact type (Fig. 137) or alternatively of the flexural type (Fig. 138).

Metal hinges are usually of cast steel. Two principal forms are employed, there being a variety of each so far as the detailed finish is concerned.

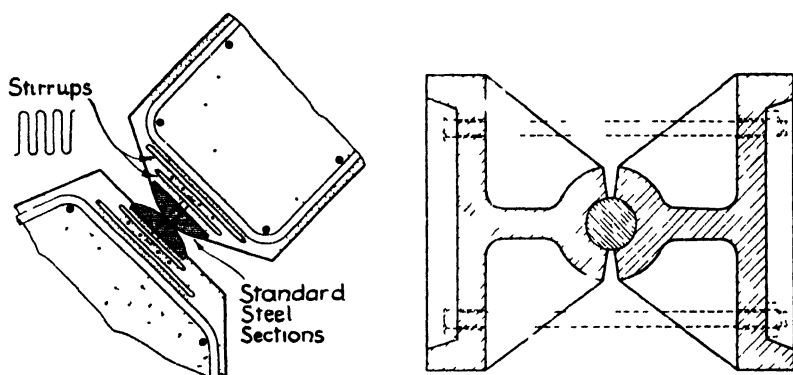
Fig. 138.—Reinforced Concrete Arch Hinge, Flexural Type.

Figs. 139 and 140 indicate typical metal hinges.

The hinge shown in Fig. 139 may be composed of commercial sections suitably bored and with mild steel pin cut from a rod of the appropriate diameter ; the hinge indicated in Fig. 140, on the other hand, requires to be especially cast, and is, therefore, more

expensive to employ, and is only adopted in the cases where the former type is unsuitable.

For arches having continuous bearings along their abutment supports the types in Figs. 137, 138 and 139 may be employed. Where the thrust is taken by intermittent ribs it is usual to employ permanent hinges of cast steel, as shown in Fig. 140, or an adaptation of that illustrated in Fig. 139.

In the case of skew arched bridges, where ribs are adopted, the hinges should be designed and set normal to the axis of the rib and the abutment supports of these members designed accordingly. It should be mentioned, however, that in most cases met with in practice where the arch is of the open spandril rib type temporary hinges are used, the finished rib being monolithic with the abutments.

Figs. 139 and 140—Types of Steel Hinges for Arch Bridges.

Only in special cases, where the loading is severe and the arch very flat, are permanent hinges a necessary provision.

For skew arches having continuous bearings along their abutments in order to provide against the sliding tendency mentioned above, it is necessary either to " step " the hinges, keeping the planes of contact normal to the longitudinal bridge axis, or alternatively to provide sufficient steel to take the unbalanced tangential force by shear, or, if the reinforcement be inclined on plan, wholly or partially by tension.

For the former arrangement with the " stepped " hinge types in Figs. 137 and 139 are suitable, or for straight continuous hinges that shown in Fig. 137, with the connecting rods designed to take the whole of the resultant shear. If these latter connecting rods are placed inclined on plan, so as to be in the direction of the unbalanced component of the arch thrust, they should be calculated to take the whole of the resultant tension.

A variety of permanent hinges have been employed upon the Continent, but the selection given above may be taken as representing suitable types for almost any condition of arch, and also as representing those most economically suited to use in this country.

126. Curvature of Contact Surfaces.—There are several methods in use for ascertaining the radius of curvature of contact surfaces under compression. The best known is that of Hertz, but this method has been found from repeated experiments to give dimensions which are in many cases unnecessarily large.

The compressive stresses permissible upon hinges when calculated upon the actual areas in contact are much higher than those ordinarily employed, since the areas so stressed are embraced by material stressed to a lesser extent, and which, therefore, relieves the higher stresses upon the small areas actually receiving the load.

The following method gives suitable radii for expansion rollers, arch hinges, etc.

Referring to Fig. 141, the general formula giving the required radii R and r in inches for the contact faces applicable to either concrete or steel hinges is

$$\frac{1}{P} = \frac{1}{K}\left(\frac{1}{r} + \frac{1}{R}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

K being a constant depending upon the material employed and P the pressure per inch of length in pounds; r and R are considered to be of the same sign if the curves are opposed, or contrary if the curved faces are of similar sign.

Case (a), where a concave face supports a convex face, the former having a radius of twice the latter:—

$$\frac{1}{P} = \frac{1}{K}\left[\frac{1}{r} + \left(-\frac{1}{2r}\right)\right] = \frac{1}{2K.r}$$

from which
$$r = \frac{P}{2K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1\text{A})$$

Case (*a*). Case (*b*). Case (*c*).

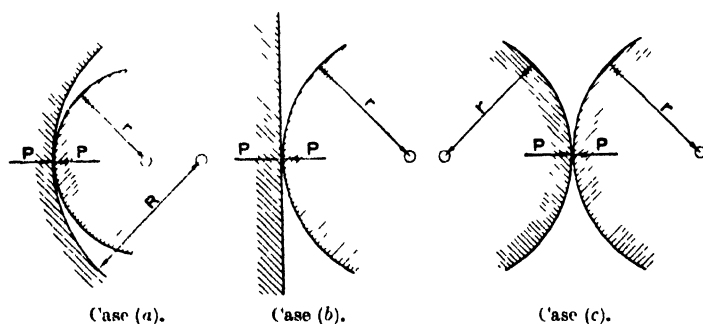



Fig. 141.

Case (b), where one face is flat —

$$\frac{I}{P} = \frac{I}{K}\left(\frac{I}{r} + \frac{I}{\infty}\right) = \frac{I}{K.r}$$

$$\therefore \quad r = \frac{P}{K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1\text{B})$$

Case (c), where curves are equal with both faces convex :—

$$\frac{I}{P} = \frac{I}{K}\left(\frac{I}{r} + \frac{I}{r}\right) = \frac{2}{K.r}$$

$$r = \frac{2P}{K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1\text{C})$$

For concrete hinges, expansion rollers and rocker bearings it is essential to employ concrete of good quality, and the mixture should not be poorer than 1 : 1½ : 3, in which case the constant K in the above formulæ may be given the value 342.

For cast steel the value of the above constant K is 2,840.

127. Construction of Concrete Hinges.—For hinges shown in Fig. 137 care must be taken in construction to ensure the curved bearing surfaces being accurately formed. This may be done by screeding the concave surfaces with a wooden screed and by introducing filling pieces of soft wood so as to form the convex faces. Alternatively the latter curve may be achieved by means of plaster of Paris, screeded to receive the convex faces in a similar manner to that employed for the concave faces. The centre hinges are similar in form, and may be constructed in the same manner as those at the abutments.

As in the case of rocker bearings, it is necessary to provide appropriate lateral reinforcement directly above and below the hinges where these are of concrete, and also for steel hinges where the intensity of stress immediately above and below the bedplates may be higher than that permissible upon plain concrete. This is best done by means of placing several layers of stirrups of the "gridiron" type alternately at right angles to one another, as shown in Figs. 137 and 138.

128. Use of Hydraulic Jacks.—A device more recent, but having the same object as temporary hinges, is the "opening" of an arch at the crown by introducing hydraulic jacks. This has been successfully accomplished for several bridges in France, and was adopted for the three span arch bridge over the River Thames at Chiswick. It is found to be particularly suitable for large arches where temporary hinges present practical difficulty on account of the forces to be dealt with, and also for bridges faced with stone-

work, where the subsequent filling-in of the temporary hinge sections at the springings would lead to constructional complications.

An example of the former is the Plougastel Bridge over the Elorn River, Art. 162 (see page 240), and of the latter, the Chiswick Bridge mentioned above, which is faced with Portland stone.

A further case where the use of hydraulic jacks is preferable to temporary hinges is that of arches composed of a number of relatively narrow ribs connected by slabs at the intrados or extrados. With this type, the cross-section of each individual rib is of " T " section, which makes the introduction of temporary hinges difficult.

Although hydraulic jacks are introduced with the same object as temporary hinges, the manner of attaining the result, *i.e.*, the elimination of the cumulative secondary effects, is quite different. With temporary hinges the arch is enabled to settle during construction without appreciable strain, whilst by the action of hydraulic jacks, the deformed arch is lengthened and the secondary strains thereby eliminated.

The precise use of hydraulic jacks may be explained as follows :—

Assume that the arch has been concreted upon a temporary stage which was erected originally to the correct theoretical profile of the arch intrados.

Also, that the stage is lowered without any compensation being made to the arch.

It will be found that the arch has settled, partly due to the deformation of the temporary staging, and in part to initial shrinkage and elastic compression of the concrete. The effect of such a settlement of the arch is to induce bending moments throughout the arch causing unequal stresses at all sections.

To replace the arch into its correct position, it is necessary to lengthen it, which may be done by the insertion of a wedge-shaped voussoir of the required proportions at the crown.

Subsequent to its completion the concrete arch will suffer further shrinkage and further elastic shortening due to the loading then added.

By increasing the width of the above voussoir, the arch can be lengthened still further, and the secondary stresses that would subsequently be induced can thereby be prevented.

By the introduction of hydraulic jacks the arch is forced open at the crown so as to provide a space for these voussoirs. This space is then filled with cement mortar, as described in the next article, and the jacks removed. The intermittent openings provided to accommodate the jacks are also filled solid with concrete.

In practice the temporary staging is not struck until the jacks have been used, the arch being lifted off the centering by their action.

It will, of course, be appreciated that the arch is entirely severed transversely at the crown, and that no tension can take place at this section, either before or after completion. Unless there are exceptional circumstances, this condition is realised in the design.

The effect of opening the arch with the jacks for the insertion of the crown voussoir is to produce a compensating moment at the springings, and by placing the jack eccentrically at the crown, a suitable compensating moment is also produced at this section. Bending moments tending to equalise the stresses are also induced throughout the arch by the above operations.

The amount of compensation to be realised by the jacks is ascertained partly by observation and partly by calculation. There are, however, three methods by which the actual operation can be controlled :—

(1) By the force applied, or the pressure of the jacks.

(2) By the amount of opening of the jacks, or the travel of the rams.

(3) By the change in level of the arch crown.

The first method of control is unsatisfactory in practice, owing to the uncertainty of obtaining absolutely accurate readings from the pressure gauges.

The second method is also unsatisfactory, since it would involve obtaining by calculation the movement of the jacks required to compensate the initial vertical displacement which is recorded by observation.

The last method—that of measuring the rise in level of the arch —is therefore usually adopted. The arch crown is raised the requisite amount so as place it at its theoretical level, this amount being obtained by direct observation. A supplementary lift is then given to compensate for future shrinkage, etc., this latter lift being estimated by calculation.

Since the principal compensation to be made is that recorded by direct observation, any slight error involved in the calculation of the supplementary raising is unimportant.

In order to make clear the method of ascertaining the supplementary raising of the arch and eccentricity of the jacks required to induce the compensating bending moments, it is useful to consider the maximum stresses that would be developed at the principal sections were the secondary effects in question not compensated. These maximum stresses would be found to differ widely at the extrados and intrados, thus at the crown the maximum stress realisable at the extrados would be considerably more than that at the intrados and conversely for the springings (see Fig. 142).

These variations are due to the shortening of the arch, as already

explained, as well as in a lesser degree to moving concentrated loads, such as would be imposed by the engine to the Ministry of Transport's load train.

To equalise the maximum intrados and extrados stresses at the crown, the jacks are placed eccentrically in much the same manner as for temporary hinges.

Eccentricity of Jacks.—The required eccentricity is ascertained as follows :—

Let the total maximum compressive stresses on the crown section be S_1 (extrados) and S_2 (intrados) and those at the springing sections be S_3 (intrados) and S_4 (extrados).

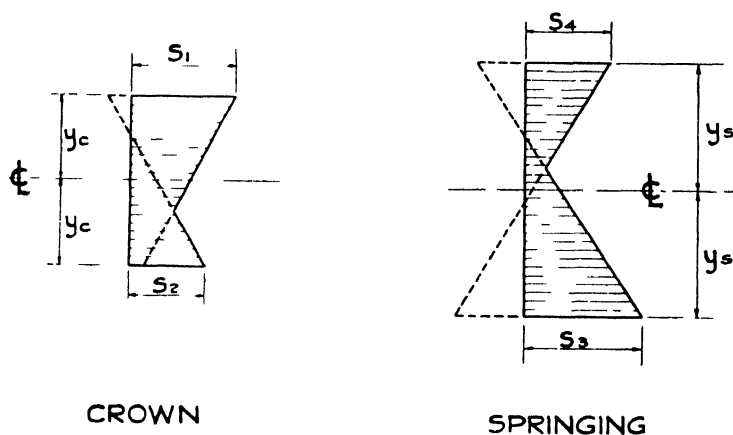



FIG. 142.

To equalise these stresses the following bending moments must be introduced :—

$$M \text{ (at the crown)} = \frac{(S_1 - S_2)}{2} \cdot \frac{Ic}{y_c} \quad \text{(Negative)}$$

$$M \text{ (at the springings)} = \frac{(S_3 - S_4)}{2} \cdot \frac{Is}{y_s} \quad \text{(Positive)}$$

Where y_c and y_s are the distances of the extreme fibres from the mean fibre or neutral axis of the sections (see Fig. 142).

The accompanying horizontal thrust (H_2) which would develop these moments acts at a distance u_2 below the crown, so that

$$H_2 . u_2 = M \text{ (at the crown)}$$
$$\text{and } H_2 (f - u_2) = M \text{ (at the springings)},$$

from which H_2 and u_2 can be found.

Assume that the jacks are used when only the arch is constructed, and that H_1 and M_1 are the horizontal thrust and bending moment respectively at the crown, due to the weight of the arch alone.

The crown section may then be considered as subjected to two thrusts, H_1 (from the arch alone), and H_2 (the supplementary thrust).

The eccentricity of thrust $(H_1) = u_1 = \frac{M_1}{H_1}$

The eccentricity of thrust $(H_2) = u_2$

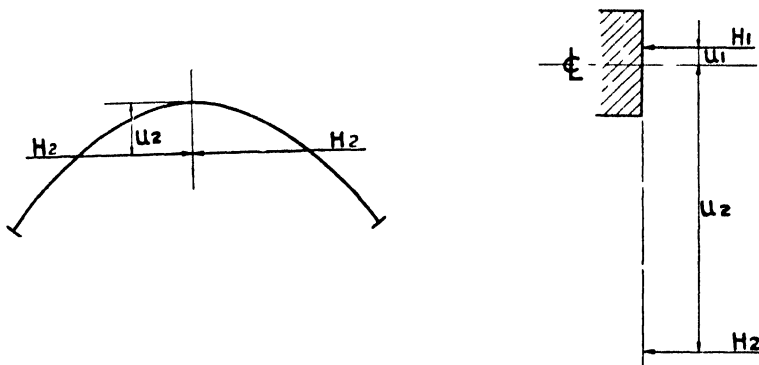



FIG. 143.

These two thrusts can be combined, and by taking moments about the mean fibre, the required eccentricity of the jack will be :—

$$u = \frac{H_1 . u_1 - H_2 . u_2}{H_1 + H_2}.$$

$$= \frac{M_1 - M_2}{H_1 + H_2}.$$

Vertical Raising of the Crown.—As stated, the amount of lift required to raise the arch to its theoretical level is usually found by observation.

The supplementary raising to compensate for subsequent deformation is found by the calculation of the vertical deflection of a curved member. This is obtainable from formula (11), page 74,

$$i.e., \int \frac{M . x . ds}{E . I}.$$

Thus the vertical displacement of a curved encastré rib depends upon the moments acting on all sections throughout the length of the rib. It has been assumed when calculating the eccentricity of the jack that only the arch has been constructed. The moments (M), therefore, are the bending moments due to the weight of the arch plus the moments of the secondary thrust, H_2 acting at a distance u_2 below the crown. These moments can be expressed as

$$M = M_f - H . y_1 - U - H_2(y_1 + u_2 - f . g).$$

For convenience, the integration is replaced by a summation. Values of M are calculated for a number of equidistant sections along the half span, each being multiplied by the appropriate value of the factor $\frac{x}{EI}$, the distances x being measured from the vertical centre line of the arch. The summation of the products is made by Simpson's rule, and the resultant total gives the vertical displacement of the arch at the crown. If all linear dimensions in the values M, x, E and I are in inches, the required supplementary movement will also be in inches.

129. Practical Application of Hydraulic Jacks.—The number of jacks required, the mode of operation, and, consequently, the cost involved by their use, is to a large extent dependent on the size and type of bridge to be opened.

Where there are several spans, it is usually possible for the outfit to be re-used. Such re-use is sometimes possible even with a single span. For example, should the arch comprise ribs connected together by a vault slab, it may be constructed as a number of separate longitudinal units of suitable width by the temporary omission of a strip (or strips) of the vault slab between two adjacent ribs. Each unit may then be opened separately, using the same jacks for each operation, after which the units can be joined and the arch made good by concreting the portions of vault slab previously omitted.

Where possible, the jacks are placed in a single horizontal row at the calculated distance below the crown axis. Due either to the span being great, or to a small rise-span ratio, it may be found that the total thrust to be realised requires a greater number of jacks than can be arranged in a single row. In such cases, they may be placed in pairs, or otherwise grouped, but always so that the combined force acts equivalently to a single force at the required eccentricity (calculated according to the method given in Art. 128).

Temporary pockets are provided to receive the jacks, and these openings must be sufficiently large to enable them to be installed and dismantled conveniently.

The intensity of compressive stresses on the relatively small areas in contact with the jacks is very great, in some cases reaching 3,000 lbs. per square inch. It is consequently desirable that the pockets provided for the jacks be arranged opposite the main ribs, so as to prevent lateral eccentricity of the compressive forces during the operation of opening.

To enable such high local stresses safely to be imposed, special reinforcement is introduced near the surface of the concrete in contact

Fig. 144.—Construction at Crown, showing Openings for Jacks.

FIG. 145. VIEW SHOWING INSTALLATION OF PUMPS, GAUGES, JACKS, ETC.

PLATE X

with the jacks. This reinforcement is of the form employed for reinforced concrete rocker bearings as described in Chapter III.

The portions between the pockets are made solid to take the arch thrust before and after opening. Mild steel plates about $\frac{3}{16}$-inch thick are introduced, severing these solid portions and thereby ensuring a complete transversal plane of separation across the crown section (see Fig. 144).

To permit the concrete in the vicinity of the jacks to harden properly before the high local stresses referred to above are imposed, the concreting at the crown should be carried out as long as possible before the opening.

For single spans it may be desirable to deposit a transverse strip of the concrete at the crown in advance of the remainder. Where there are several spans, the above remarks would, of course, apply to the arch lastly to be concreted.

The whole of the apparatus employed, including jacks, pumps, tubing, pressure gauges, etc., may be obtained from any appropriate firm, usually without involving special manufacture.

An important point to be noted is that the jacks must be capable of permitting a slight rotary movement, so that the bearing faces maintain uniform surface pressure upon the concrete during the opening process. Jacks having a capacity of from 250 to 350 tons are found suitable for the purpose.

The illustration (Fig. 145) shows more clearly than a written description the apparatus in question. The four hand pumps in the foreground worked eight jacks, one of which can be seen in the process of being lowered into position by tackle.

When the jacks are expanded and the arch opens, small gaps or slits are formed at the steel plates. The latter are then removed and the spaces filled with cement mortar as described below.

At this time the arch will have risen off the centering slightly, and it is necessary to deposit at the bottom of the gaps a layer of sand or other suitable material to prevent the escape of the liquid cement mortar. For the same reason the vertical edges of the gaps should be caulked.

It is found advantageous in practice to increase the pressure in the jacks very slightly beyond the amount calculated, and to release this shortly after the grouting operation is complete. This squeezes the mortar and consolidates it.

Following this, the pressure is further reduced, reaching zero in about two hours.

The composition of the mortar used in filling the gaps is important. A suitable mixture is 2¼ parts of sand to one part of cement. A dry mix is essential and about 8 per cent. of water is recommended.

The sand used should be screened to obtain uniform sized grains or particles having a diameter of about one-fifth of the gap dimension. This gives to the mortar a frictional resistance against compression before the setting action is complete.

The mortar should be deposited and rammed in layers of a few inches in height, each layer extending across all the gaps for the full width of the arch or section opened.

The total period required for the complete opening should not exceed eight hours in ordinary cases, and care must be taken to ensure that the arch during this time is not loaded appreciably above that presumed in the calculations, and any superimposed loads from plant or the like, which might cause additional thrust, should be removed until the gaps are filled and the mortar matured.

It has been stated that the opening would cause the arch to rise off the staging. While this is the case for the central portion of the arch, the springings usually remain in contact, owing to the elasticity of the staging exceeding the very small lifting movement, and some arrangement of wedges or sand boxes is advisable to facilitate the lowering of the staging at these points.

After the gaps have been made good with cement mortar, as described above, the jacks are removed from the temporary openings or pockets, and these are then filled solid with concrete.

CHAPTER X

FOUNDATIONS AND ABUTMENTS

130. General.—The most suitable foundations for any particular bridge are dependent on so many varying factors that it is quite impossible to suggest any form as being suitable to specific bridges. In the case of some large span reinforced concrete bridges, the question of simplicity and economy of foundation work is one of the first considerations, and frequently governs the design of the bridge superstructure.

Bridge foundations may be grouped into two principal classes :—

(1) Those carrying purely vertical loading from the bridge superstructure and
(2) Those in which the reactions are sufficiently inclined to require provision being made for it in the design. The latter case is always necessary in the design of arch bridges.

131. Girder Bridge Foundations.—For ordinary cases unaffected by water, and where a stratum capable of safely taking a working pressure of 3 tons per square foot and upwards is found at a depth of a few feet below surface, the load may be distributed over the required area by means of reinforced concrete bases. Where the depth of suitable strata upon which to found is known only approximately, it is sometimes convenient to fix an appropriate level for the reinforced concrete bases and to provide mass concrete under them, which can be carried down to any lower level that may subsequently be found to be necessary. It is usually more economical to construct piers of mass concrete than reinforced concrete columns and bases when these are required to be more than 3 or 4 feet below ground level.

132. Foundations for Intermediate Bridge Supports.—For the abutments and foundations to the intermediate supports of a bridge, where these may be wholly or partially submerged in water, mass concrete construction is frequently resorted to, owing to the simplicity and rapidity with which such foundations can be constructed. If the minimum depth of water to be dealt with is inconsiderable, say up to 10 feet, such foundations can be placed within sheet piling, which may be left permanently in position, or

alternatively the piles can be drawn, after slacking the temporary ties and stiffening members at their heads, and re-used.

If either of these methods is employed, steel sheeting of the interlocking type is driven to the shape of the base and the head of the piling suitably stiffened and braced. The interior spoil is then excavated either by hand or by " grab " to the depth required and the mass concrete deposited to the requisite height. If the top of the sheet piling is kept 2 feet or so above the maximum recorded water level, the superimposed reinforced concrete pier can also be constructed to some level above water level before the piled enclosure is removed. In order to do this, the sheeting must be reasonably watertight and pumping resorted to. Additional courses of timbering may be required to the sheeting to resist the external water pressure, depending upon the depth to which the water is lowered.

If this method is not practicable, it is necessary to construct a coffer-dam of larger size than the base in question and to construct the whole of the bridge support " in the dry."

It should be noted that the above mass concrete foundations and piers can, if required, be carried upon reinforced concrete piling driven after the necessary sheeting is in position and the ground excavated to the required level.

It is sometimes economical to provide circular piers carried upon groups of piles and to form these piers by precasting a thin outer shell or cylinder in convenient lengths of reinforced concrete, and having prepared a suitable bed and lowered the cylinders into position, to fill the interior space with mass concrete. Where such concrete filling is required under water, it should be deposited through a tube, usually about 8 inches diameter, and the lower end kept at, or slightly below, the level of the concrete being deposited. At the upper end a hopper is fitted, enabling a full charge of concrete to be maintained in the tube.

Where the foundations for the intermediate, and occasionally the end, supports of a bridge spanning a river carrying heavy loading, as in the case of several long spans, require to be carried down to a depth exceeding 15 or 20 feet below the river bed, reinforced concrete caissons are sometimes employed. This is particularly the case where piles may, for a variety of reasons, be impracticable or prohibited, say by the proximity of the new foundations to one or more existing structures.

Such caissons may be rectilinear or curvilinear on plan, and may be constructed upon a stage erected immediately over their final positions and lowered as the work proceeds by means of chains and " jacks." Alternatively they may be constructed in the above manner, but upon a single stage placed in a convenient position. In

this case they require to be temporarily sealed at their lower ends, in order that they may be floated to their respective positions. They are then lowered on to a prepared bed by admitting water into their interiors, after which the temporary bottom can be removed and sinking operations commenced.

It is advisable, if possible, to construct the caissons to their full height before sinking operations are commenced, so that the latter may proceed continuously.

Reinforced concrete caissons may be sunk either by means of compressed air or in the ordinary manner by grabbing. The method

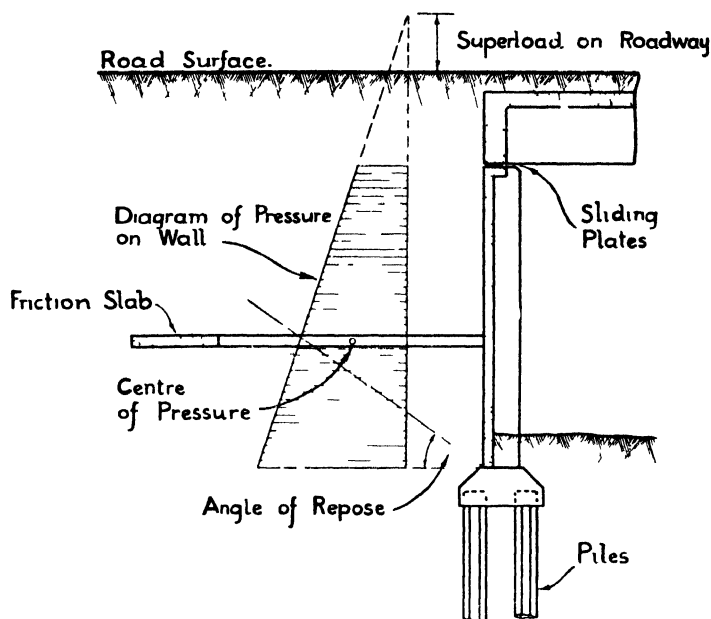



FIG 146 —Abutment Retaining Wall of Girder Bridge carried upon Piles.

of sinking caissons constructed of this material is similar in principle to that employed for steel caissons. Some useful data and information upon this branch of foundation work are given in "Bridge Engineering," Vol. I., p. 981, by J. A. L. Waddell, C E., B.A.Sc., etc.

133. End Supports to Girder Bridges.—It is usual to design the end or abutment supports and wing walls of an ordinary girder bridge, or the latter for an arch bridge, with a vertical front screen wall, supported and stiffened by a bottom slab and counterforts, spaced from 8 to 15 feet apart, depending upon the height and nature of the retained material.

Abutment retaining walls of this type carrying vertical loading will, however, only be found suitable where the ground upon which

they are founded is capable of carrying pressures of from 1 ton per square foot upwards. Where the vertical loading is heavy, and the permissible ground pressure low, piling should be resorted to and a type of abutment wall upon the lines of that illustrated in Fig. 146 employed.

For the abutment walls supporting beam bridges it will usually be found economical to arrange a counterfort immediately under each of the longitudinal beams and, if sliding joints are used, to set these back as far as practicable from the front face of the wall, so as to assist its general stability.

In many small bridges of the beam type, it is found that the superimposed rolling loads when immediately over the end supports produce reactions which are relatively serious. So far as the foundations are concerned, however, this concentration of loading may be taken as spreading itself through the retaining wall and interesting a length of foundation approximately equal to the height of the wall itself. When making this calculation it is, of course, necessary to take into account the maximum rolling loads possible on the adjacent beams.

Where the abutment walls are carried upon piles, as in Fig. 146, the distribution of the maximum reactions from the beams through the vertical screen wall is a great advantage, as without this distribution the number of piles necessary under each of the counterforts would have to be such as to carry the total maximum reaction (including the superimposed loading) in addition to the weight of the wall, counterforts, etc., coming upon them.

When designing an abutment retaining wall, allowance must be made for the possible pull resultant from the friction between the beam and counterfort where sliding joints are employed. When copper or zinc contact plates are used for this purpose, the friction varies from one-sixth to one-fifth of the maximum vertical load from the beam.

The above horizontal force would act in the same direction as the thrust from the earth filling retained when a fall in temperature occurred causing the deck platform to contract.

The contraction necessary to produce movement causing a pull on deck beams relative to the abutment walls would be spread over an appreciable period (Chapter III.). Although, in making the calculation, it is usual to assume the maximum possible load on the beam, it is very unlikely, in fact, that the superimposed loading necessary to give the maximum beam reaction would be present during this period.

In cases where the load from the deck platform is carried independently of the abutment retaining wall, as, for instance, with the

introduction of flexible column construction in front of it, the pull mentioned above does not, of course, exist, but the distributing effect of the wall with the latter arrangement is absent, and the full load from the columns has to be carried usually by means of a deep toe formed in front of the abutment wall, of sufficient stiffness to distribute the loading and so enable the piling to be arranged continuously in the direction of the toe.

When the vertical loading upon the abutment wall is carried upon piling and the horizontal thrust from the retained material taken by ties and anchor slabs, as illustrated in Fig. 146, it is usual to place the ties at the level of the resultant of this latter thrust.

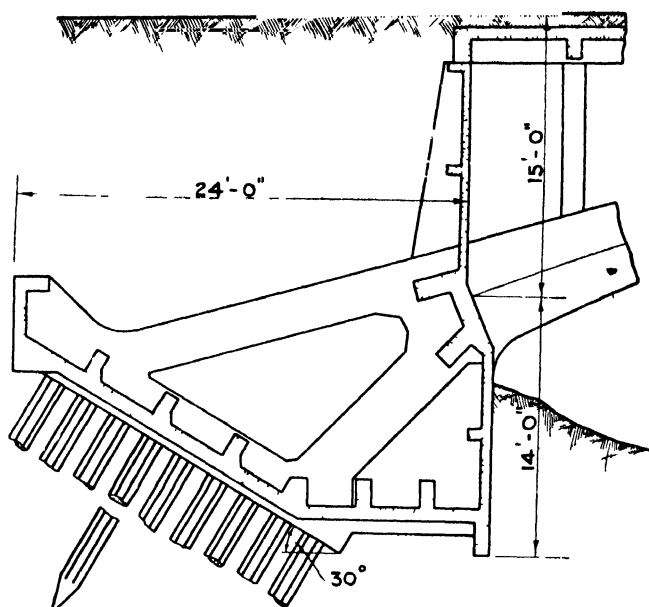



FIG. 147.—Piled Abutment for the Warrington Bridge.

Both the anchor slab and ties are designed to be flexible in the vertical direction, so that they may conform to any slight settlement of the natural or filled material upon which they are constructed.

The friction between the concrete anchor slab and earth with which it is in contact may be taken as varying between one-third and one-half of the weight of filling above the slab. The exact amount will depend upon the nature of the material surrounding it.

When anchor slabs are placed at such a distance back from the wall they support as to be below the angle of repose of the material, the friction upon both the upper and lower faces may be taken into account in computing the resistance of these slabs to sliding. If

the slab is placed above this angle, then only the friction upon the lower face may be taken.

134. Abutment Supports to Arched Bridges.—For the abutments of an arch bridge subject to flooding the method outlined for intermediate bridge supports can be employed, or in cases where the excavation can be kept clear of water with ordinary pumping abutment foundations can be constructed in a trench of the required size, which may be timbered in the customary manner.

If neither of the above methods is suitable, it is necessary to employ coffer-dams that will effectively enable the necessary work to be carried out " in the dry."

Abutment supports for arched bridges may be constructed either of mass or reinforced concrete, and either of these types may be in addition supported upon piles.

Fig. 147 illustrates the reinforced concrete piled abutments employed for the Warrington Bridge. This structure replaced an older bridge and was erected in two halves in order that traffic should not be stopped during the time of construction. Fig. 148 gives a view of the piles to the second part of one of these abutments, and also the counterforts to the portion already constructed and in use. Figs. 44 to 47 show types of mass concrete abutments for arch bridges of moderate spans.

The stability of an arch abutment, whether of mass or reinforced concrete, is investigated in a similar manner. The detail design of each of the members forming the latter (see Fig. 147) is straightforward when the external forces coming upon them have been ascertained.

Having determined the forces and moments imposed upon an arch abutment, the two remaining factors influencing their design are as follows :—

(1) Depth of suitable stratum upon which to found ;

(2) Safe bearing capacity of stratum.

These factors will, of course, vary for almost every bridge, and it is therefore practically certain that no two bridges will present the same problem so far as abutment design and construction are concerned.

In order to illustrate the forces to be dealt with and the general procedure in designing arch abutments, the following example is given. This form of abutment (Fig. 149) is selected, not because it is necessarily the most economical, but because the forces acting upon it can be more clearly visualised and conveniently described.

This abutment will be designed to support the arch bridge given in the example in Art. 83. The stratum upon which it is founded is presumed to be capable of safely sustaining a pressure of 2½ tons per square foot.

FIG. 148. VIEW OF ABUTMENT COUNTERFORT AND INCLINED PILES FOR WARRINGTON BRIDGE.

PLATE XI.

This information is required before the abutments can be designed and is obtained by means of trial pits or bore-holes.

When determining the most suitable form of abutment, which must be done by trial, it is desirable to arrange the width of base so that the normal resultant force from all the external permanent loading or pressure passes approximately through the centre of gravity of the base. This ensures an evenly distributed pressure upon the ground under the condition of loading causing settlement. The effect of the live load, so far as foundation pressures are concerned, is of secondary importance, since the assumed maximum

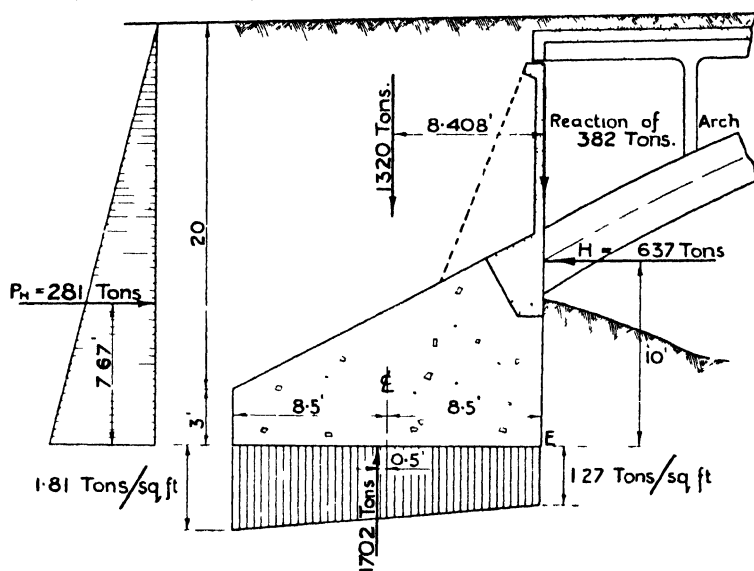



FIG. 149.

superimposed loading rarely, if ever, comes upon the structure. The proportion actually doing so is present for so short a period as to make it doubtful whether its action appreciably affects this aspect of the problem.

In any case, the whole of the assumed superimposed loading is much smaller than the dead weight of the structure, which is permanently present. Nevertheless it is customary to include the effect of the maximum superimposed loading and to ensure that the maximum pressure under this condition does not exceed the safe bearing capacity of the ground.

135. Design of Mass Concrete Arch Abutment.—Referring to Fig. 151 and the example in Art. 83, it will be seen that the forces

coming upon the abutment from the arch consist of the horizontal thrust, vertical reaction, and bending moment at the springing.

On the abutment itself the principal load is the weight of earth above it and its own weight. The earth filling behind the abutment also acts upon it in the opposite direction to that of the horizontal arch thrust. The magnitude and position of its resultant may be found by Rankine's theory for the pressure of earth on a vertical face, the surface of the filling being level.

It should be noted, in ascertaining the stabilising forces resultant from the filling immediately behind the abutments, that it is customary to calculate the active pressure of the earth filling upon the abutment; the resistance, however, is considerably greater, being in actual fact a passive resistance and therefore representing a much greater figure so far as the stability of the bridge abutment is concerned.

The above forces will now be calculated and moments taken about point E (Figs. 149 and 151) of the abutment in order to find the position of the upward resultant of the pressure from the ground under the abutment.

The bridge will be treated as a whole, which ensures a more accurate estimation of the pressure when the live load is on the bridge.

(a) *Foundation Pressure under Dead Load only.*

Total Dead Load of Bridge.

Weight of road metal and footpaths (say, average 9″ thick) .	80′	× 60′	× 0·75 × 110 lbs.			396,000 lbs.
Weight of deck slab . .	80′	× 60′	× 0·67 × 150	,,	=	480,000 ,,
Weight of deck beams . .	16	× 27′	× 0·75 × 150	,,	=	48,600 ,,
Weight of columns	16	× 8′	× 0·5 × 150	,,	=	9,600 ,,
Weight of parapets	80′	× 2′	× 500	,,	=	80,000 ,,
Weight of arch ribs . .	84·66′	× 2·5′	× 22′ × 150	,,	=	698,400 ,,

Total 1,712,600 lbs.
or $Wd = 764{\cdot}5$ tons.

The total dead load horizontal thrust is

$$\frac{Wd.l}{8f} = \frac{764{\cdot}5 \times 80}{8 \times 12} = 637 \text{ tons.}$$

Fig. 149 gives a tentative outline of the proposed abutment, which is 65 feet long. It is necessary to calculate the weight of the

abutment and the earth above it, and to find the moment of vertical forces about the inside edge of the abutment, as follows :—

		Weight	Distance of C G	Moment
Earth	$17 \times \left(\frac{20 + 11{\cdot}5'}{2}\right) \times 65 \times \frac{110}{2{,}240}$	855 tons	9·26	7,917 tons feet.
Abutment	$17 \times \left(\frac{3' + 11{\cdot}5'}{2}\right) \times 65' \times \frac{130}{2{,}240}$	465 ,,	× 6·84	3,181 ,,
	Total	1,320 tons	—	11,098 tons-feet.

The total active pressure Ph of the earth is calculated from the appropriate formula, *i.e.*, $\frac{w.h^2}{2} \times \frac{1 - \sin\theta}{1 + \sin\theta}$ where

w = weight of earth per cubic foot = 110 lbs. ;
h = maximum height of earth = 23 feet (see Fig. 149) ;
θ = angle of repose of earth = (say) 30° ;

$$Ph = \frac{110}{2\,240} \times \frac{1}{2} \times 23' \times 23' \times 65' \times \frac{1 - 0{\cdot}5}{1 + 0{\cdot}5} = 281 \text{ tons.}$$

This horizontal pressure acts at $\frac{23}{3} = 7{\cdot}67$ feet above the bottom of the abutment.

Moment about base of abutment = 281 × 7·67 = 2,155 tons feet.

To find the position of the resultant upward component, take moments about point E (Fig. 149). The magnitude of the upward component is equal to the weight of the abutment and earth above it plus the reaction of the arch, or

$$W = 1{,}320 + 382 = 1{,}702 \text{ tons}$$

$$\therefore\ 1{,}702 \times x = (637 \times 10) - 2{,}155 + 11{,}098$$

$$x = \frac{15{,}313}{1{,}702} = 9 \text{ feet.}$$

The eccentricity e of the resultant from the centre line of the base is thus $\left(9 - \frac{17}{2}\right) = 0{\cdot}5$ feet.

The maximum pressure at the outside edge of the abutment is

$$\frac{W}{A} + \frac{W.e.y}{I} = \frac{W}{A}\left(1 + \frac{6\,e}{b}\right)$$

where e is the eccentricity of the force and b is the breadth of the abutment

$$= \frac{1{,}702}{17 \times 65}\left(1 + \frac{6 \times 0{\cdot}5}{17}\right)$$

$$= 1{\cdot}54 \times 1{\cdot}176 = 1{\cdot}81 \text{ tons per square foot.}$$

The minimum pressure at the inside edge is

$$\frac{W}{A}\left(1 - \frac{6e}{b}\right)$$

$$= 1{\cdot}54 \times 0{\cdot}824 = 1{\cdot}27 \text{ tons per square foot.}$$

It will be seen from the above figures that the distribution of pressure under the abutment is approximately uniform. If a more exact uniformity of pressure is desired, the width of the abutment must be increased.

(b) *Foundation Pressures, taking into account Superimposed Loading.*—The width of the roadway is sufficient to accommodate four trains of the Ministry of Transport's loading. These will be placed on the bridge to give the maximum possible horizontal thrust and thus produce the maximum overturning moment.

The footpaths will at the same time be assumed to be covered with a superimposed load of 1 cwt. per square foot, evenly distributed.

The position of the rolling load causing the maximum horizontal thrust is given in Fig. 150.

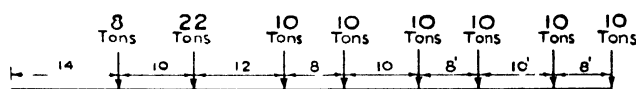



FIG. 150.

From the influence lines in Tables F, J, and H, the following total thrust, vertical reaction, and bending moment, at the left-hand abutment are obtained :—

Horizontal Thrust Tons	Vertical Reaction Tons	Bending Moment at Springing Tons-feet
$4 \times 8 \times \frac{80}{12} \times 0{\cdot}075 = 16{\cdot}0$	$4 \times 8 \times 0{\cdot}92 = 29{\cdot}4$	$4 \times 8 \times 80 \times (-0{\cdot}067) = -171{\cdot}5$
$4 \times 22 \times \frac{80}{12} \times 0{\cdot}1654 = 97{\cdot}1$	$4 \times 22 \times 0{\cdot}784 = 69{\cdot}0$	$4 \times 22 \times 80 \times (\ 0{\cdot}0367) = 258{\cdot}4$
$4 \times 10 \times \frac{80}{12} \times 0{\cdot}23 = 61{\cdot}3$	$4 \times 10 \times 0{\cdot}58 = 23{\cdot}2$	$4 \times 10 \times 80 \times (+0{\cdot}0174) = +55{\cdot}7$
$4 \times 10 \times \frac{80}{12} \times 0{\cdot}23 = 61{\cdot}3$	$4 \times 10 \times 0{\cdot}43 = 17{\cdot}2$	$4 \times 10 \times 80 \times (+0{\cdot}042) = +134{\cdot}4$
$4 \times 10 \times \frac{80}{12} \times 0{\cdot}182 = 48{\cdot}5$	$4 \times 10 \times 0{\cdot}25 = 10{\cdot}0$	$4 \times 10 \times 80 \times (+0{\cdot}0483) = +154{\cdot}6$
$4 \times 10 \times \frac{80}{12} \times 0{\cdot}118 = 31{\cdot}5$	$4 \times 10 \times 0{\cdot}13 = 5{\cdot}2$	$4 \times 10 \times 80 \times (+0{\cdot}0373) = +119{\cdot}4$
$4 \times 10 \times \frac{80}{12} \times 0{\cdot}030 = 8{\cdot}0$	$4 \times 10 \times 0{\cdot}028 = 1{\cdot}1$	$4 \times 10 \times 80 \times (+0{\cdot}0113) = +36{\cdot}2$
Total 324 tons.	Total 155 tons.	Total 70·4 tons-feet.

The superimposed load on the footpaths is

$$\frac{112}{2,240} \times 10' \times 2 = 1 \text{ ton per lineal foot of bridge,}$$

which produces a horizontal thrust of

$$\frac{p.l^2}{8.f} = \frac{1 \times 80 \times 80}{8 \times 12} = 67 \text{ tons.}$$

The bending moment at the springings produced by this superimposed load is negligible.

Total vertical reaction from arch

$$= \frac{764}{2} + 155 + \frac{80}{2} = 577 \text{ tons.}$$

LIVE AND DEAD LOAD.

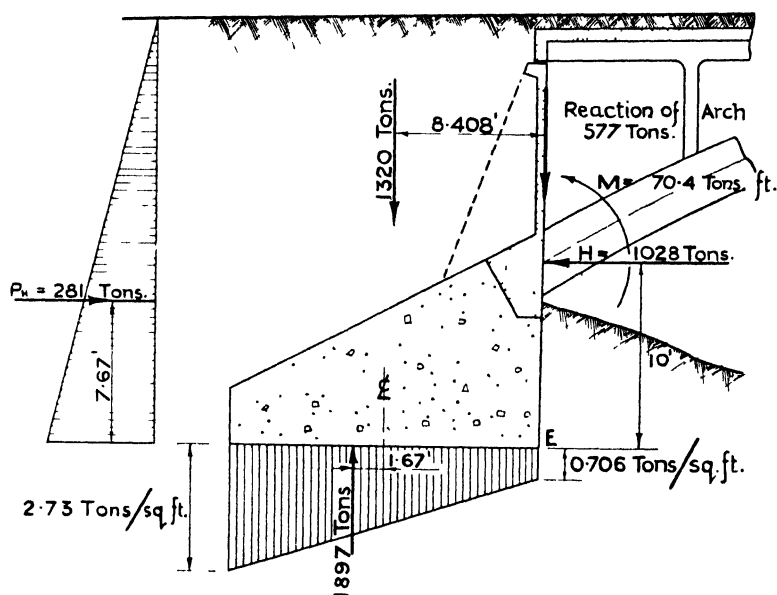



Fig. 151.

Position of resultant upward pressure on base of abutment (see Fig. 151) =

$$(1,320 + 577)\,x = (637 + 324 + 67) \times 10 - 2,155 + 11,098 + 70{\cdot}4 = 1,897\,x = 10,280 - 2,155 + 11,098 + 70{\cdot}4$$

$$x = \frac{19,293{\cdot}4}{1,897} = 10{\cdot}17 \text{ feet.}$$

The eccentricity of the resultant from the centre line of the base is $\left(10{\cdot}17 - \frac{17}{2}\right) = 1{\cdot}67$ feet.

The maximum pressure at the outside edge of the abutment is

$$\frac{W}{A}\left(1 + \frac{6e}{b}\right)$$

$$\frac{1,897}{17 \times 65}\left(1 + \frac{6 \times 1{\cdot}67}{17}\right)$$

$$1{\cdot}717 \times 1{\cdot}589 = 2{\cdot}73 \text{ tons per square foot,}$$

and the minimum pressure

$$1{\cdot}717\,(1 - 0{\cdot}589)$$

$$1{\cdot}717 \times 0{\cdot}411 = 0{\cdot}706 \text{ tons per square foot.}$$

136. Piling. Where the specified or safe founding level exceeds 15 or 20 feet below the surface, piling may be resorted to. Where piles are employed, they are usually covered by a capping piece placed at ground level in the form of a reinforced or mass concrete slab, upon which is constructed the bridge support proper. The latter may comprise an *in situ* reinforced concrete trestle, or alternatively a number of reinforced concrete columns, each resting upon a cap embracing a group of piles.

Piling is occasionally resorted to in the case of arched bridge abutments where all or some of them require to be driven on the batter (see Fig. 147).

In cases where only vertical piling is employed and a horizontal thrust is taken by the member supported by them, as, for instance, an abutment retaining wall, it is essential to prevent this lateral thrust coming upon the heads of the piles. This may be accomplished by providing ties and an anchor slab or slabs, as shown in Fig. 146.

When piles are employed they may be either driven down to a sufficient depth to enable them to carry their loads as columns, or alternatively the condition of the ground may make it economically necessary to treat them as friction members capable of carrying a limited load which may be verified by test. In either case it is unsafe to presume the heads of the piles capable of taking a lateral thrust without appreciable movement.

When designing foundations in which piling is introduced, it is a frequent assumption to consider the capping slabs as capable of taking a proportion of the total load to be carried, this latter being dependent upon the area and the assured safe ground pressure under the bases in question.

Although in most cases this may appear to be justified by results, it cannot be recommended, since there is considerable doubt as to whether the proportion of loading assumed to come upon the bases does so in reality, and it is probable that, beyond possibly the weight

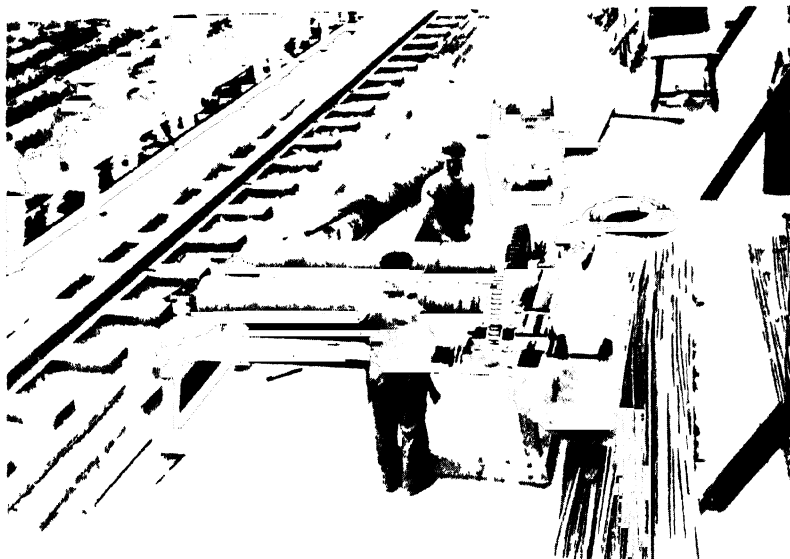



Fig. 152. Method of Forming Spiral Reinforcement for Piles.

Fig. 153. Pre-cast Octagonal Pile 87 ft. 6 ins. Long.

PLATE XII

of the concrete base deposited directly upon the ground, no loading is carried in this manner.

In the event of any of the piles settling under load, of course the capping slabs would immediately become loaded, but, in view of the large factor of safety assumed in determining the safe-carrying capacity of piles, the requisite settlement of these members necessary to bring about the above condition is unlikely.

It might be added that when designing under-water supports of this nature the buoyancy of the permanently submerged portion of the foundation is deducted from the load to be carried, no allowance being made for bearing upon the ground of the reinforced concrete slabs or capping pieces.

137. Reinforced Concrete Piles.- Reinforced concrete "plug" or "king" piles may be classed under two heads, according to the manner in which they are reinforced, the first being those, usually of octagonal section, reinforced, in addition to longitudinal rods, by a continuous closely pitched steel spiral. This form of lateral reinforcement increases the resistance of any concrete compression member in the most efficient and economical manner known, and is particularly suitable for piles of all kinds. The method of forming the spiral reinforcement by winding on a mandril of appropriate diameter is shown in Fig. 152. Piles reinforced in this manner are employed where the loads required to be carried are considerable, say exceeding 50 tons per pile, or where the length of the members is such as to require a degree of flexibility for handling otherwise unobtainable.

It may be of interest to state that piles of this type, 17 inches octagonal section, 87 feet 6 inches long, have been cast and handled, and driven to "refusal" in boulder clay with a 4-ton hammer falling 48 inches. Fig. 153 shows one of these piles being lifted into position. Similar piles 18 inches octagonal section are employed in an adjacent bridge and viaduct foundation, these being nearly 100 feet long.

The second type of pile is the ordinary square section member reinforced with longitudinal rods and links in the usual manner.

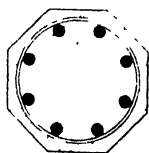



Fig. 154.- Octagonal Section Pile (Considère Type).

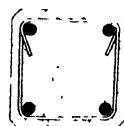



Fig. 155.—Square Section Pile.

This type is usually suitable for carrying light loads or for short piles of the friction type.

The above types are shown in cross section in Figs. 154 and 155 respectively.

138. Safe Load on Piles.—The formula most commonly employed in this country for calculating the safe-carrying capacity of reinforced concrete piles when driven to a specified set is

$$P = \frac{R^2.h}{6p\,(R + W)}$$

where
R = weight of ram or hammer;
h = height of drop in inches;
p = penetration per blow in inches;
W = weight of pile.

The frictional resistance of reinforced concrete piles may be taken to vary between 1 cwt. and 10 cwts. per square foot of surface area, the exact figure depending upon the material with which they are in contact.

With few exceptions piles are in fact supported almost wholly by friction, and a higher calculated safe load can be arrived at by using the above formula, but taking an average of say three successive blows after a period of rest from the previous driving. Usually a period of rest of an hour in driving a pile will cause a considerable diminution of penetration for the first few blows after driving is recommenced.

This method of obtaining a safe load is usually permissible, but where very occasionally piles are driven through bad material such as waterlogged silt, the momentary resistance of driving after the pile has penetrated some distance may give a very misleading indication of the load that can safely be imposed upon it.

Although the particles of silt offer a certain resistance to the penetration of the piles whilst they are being driven, apparently due to the water present, they soon adapt themselves to new positions, and cease to exert any appreciable lateral pressure upon the pile, which in consequence is free to settle under the load it is presumed to be capable of carrying.

It is obviously desirable that piles should penetrate such material and be driven into harder strata below.

Where there may be doubt as to the capability of piles to carry certain loads, due to the driving records being unsatisfactory, an actual test is sometimes made. Unless the loads in question are moderate, such a test, which usually involves at least three piles, becomes inpracticable. Loading tests on piles when carried out must be of sufficient duration to ensure that the supporting capability of the strata is permanent and not of the temporary nature mentioned above.

139. Safe Pressure upon Foundations.—The following table, taken from "Cylinder Bridge Piers," by J. Newman, gives the permissible pressure upon foundations :—

Description of Ground.	Approximate safe Load in tons per square foot.
Bog, morass, quicksand, peat, moss, marsh land, mud, hard peat turf, silt	0 to $\frac{1}{4}$
Soft, wet or muddy clays and alluvial deposits of moderate depth in river beds	$\frac{1}{4}$ to $\frac{1}{3}$
Diluvial clay beds of rivers	$\frac{1}{3}$ to 1
Soft clay and wet sand	1
Alluvial earth, loams and loamy soil (clay with 40 to 70 per cent. of sand) and clay loams (clay with about 30 per cent. of sand)	$\frac{3}{4}$ to $1\frac{1}{2}$
Ordinary clay and dry sand mixed with clay	2
Loose sand in shifting river beds, the safe load increasing with the depth	$2\frac{1}{2}$ to 3
Dry sand and dry clay	3
Silty sand of uniform and firm character in a river bed, secure from scour, and at depth greater than 25 feet	$3\frac{1}{2}$ to 4
Hard clay mixed with very coarse sand	4
Sound yellow clay, containing only the normal quantity of water	4 to 6
Solid blue clay, marl and indurated marl, and firm boulder gravel and sand	5 to 8
Soft chalk, impure and argillaceous	1 to $1\frac{1}{2}$
Hard white chalk Ordinary superficial sand beds	$2\frac{1}{2}$ to 4
Firm sand in estuaries, bays, etc.	$4\frac{1}{2}$ to 5
Firm compact sand and gravel foundations at a depth not less than 20 feet	6
Firm shale, protected from the weather, and clean gravel	6 to 8
Compact gravel	7 to 9

CHAPTER XI

NOTES ON MATERIALS AND THEIR USE IN CONSTRUCTION

140. General.—The materials used, and the methods of construction for reinforced concrete bridge work are, in most respects, the same as, or very similar to, those used in other concrete engineering structures and in building construction. These are dealt with fully in the many text-books upon the subject, and the following notes are therefore mostly confined to the description of new developments of constructional interest in the building of reinforced concrete bridges.

With the improvements that are taking place in the manufacture of cement, and the greater attention that is being paid to the improvement of concrete, with regard to the mixing and the selection and grading of aggregates, greatly superior concretes are being produced. These allow an even greater precision to be used in the design of bridges than was previously the case, and with the increase of loading due to modern traffic, and the larger structures that are now being erected in concrete, it is necessary, in some instances, to make the most precise calculations.

In such cases, it is imperative that the materials used are consistently reliable and of the highest quality, so that the calculated stresses may safely be imposed anywhere in the work.

Frequently structures are erected with an excessive reserve of strength to offset the possible presence of questionable materials and workmanship. In bridge work of any importance this has the effect of detracting from its appearance, and often defeats its object by the resultant increase in dead weight.

The question of cost is also important, and the improved concrete referred to above, when properly taken into account, makes for economy.

141. New Cements.—The most important development that has taken place for many years in connection with concrete is the introduction of improved cements. These are known as rapid hardening cements, and are primarily of two kinds, viz., aluminous cements and rapid hardening Portland cements. The former is a new class of cement having characteristics entirely its own, whilst

the latter, as its name indicates, is a Portland cement conforming to the requirements of the British Standard Specification, No. 12 (1925), differing but slightly in its chemical composition, and deriving its rapid hardening qualities almost wholly from refinements made in the various stages of its manufacture.

Although these new cements are rapid hardening, they do not set more quickly than ordinary Portland cement. In this respect, both classes conform to the requirements of the above British Standard Specification.

142. Aluminous Cement Concrete.—For a number of purposes the suitability of this concrete is unique. It possesses remarkable rapid hardening qualities, and attains strengths unknown prior to its discovery. It is possible safely to place it under stress within a few hours of deposition, and it enables such operations as piling and tidal work to be expedited considerably, with a consequent saving of time and cost. In practice there are often occasions where the advantage of using a concrete that can be put into use within a few hours is not confined to the actual operation in question, but assists the progress of the work generally.

The following table gives the result of a laboratory test on a series of 3 : 1 standard sand and aluminous cement cubes :—

Age	Crushing Strength
3 hours	43 lb. per square inch
5 ,,	1,410 ,, ,, ,,
7 ,,	3,700 ,, ,, ,,
12 ,,	5,700 ,, ,, ,,
21 ,,	7,430 ,, ,, ,,
3 days	7,580 ,, ,, ,,

Experience has shown that certain precautions require to be taken in using this kind of concrete. It is vitally important to prevent the mixing of aluminous with Portland cement, whether intentionally or unintentionally. The two kinds of cements should, as far as possible, be stored separately, and concrete mixers, chutes, etc., should be thoroughly cleaned of one kind before being used for the other. In cases where the two kinds of concrete adjoin, care should be taken to ensure that the concrete placed first is set before the other variety is deposited against it.

Owing to the rapidity with which aluminous cement concrete hardens, the temperature of the material rises to a much greater extent than is the case with Portland cement concrete. This averages about 100° F., and is reached about ten hours after mixing. In consequence, it is important that, immediately the concrete has set, and during the early hardening period, the surfaces should be kept well flooded with water.

For large masses having minimum dimensions exceeding about 24 inches, it is difficult, in practice, to supply a sufficiency of water to the centres of such masses, with the result that the strength of the interior concrete suffers. A compensation for this disadvantage is the fact that aluminous cement concretes can be mixed and placed at temperatures much lower than is possible with ordinary concrete, without special precautions being taken, and without harmful effect.

In reinforced concrete bridge construction comparatively rich mixtures are frequently specified, the proportions of 1 : 1½ : 3 being common, and where for expediency it is proposed to employ a rapid hardening cement the change is sometimes made without alteration to such proportions. Where the adoption of rapid hardening concrete involves aluminous cement, however, a richer mix than 1 : 2 : 4 is uneconomical, and may, in fact, be detrimental.

The rise in temperature of the material, being entirely due to the cement, increases with the cement content, and will probably cause the interior of the concrete to require more moisture than it may be practicable to supply. Shrinkage is also greater with rich concrete.

An aluminous cement concrete of a 1 : 2 : 4 mix will be found to give adequate strength for almost any construction.

143. Rapid Hardening Portland Cement Concrete.—Owing to the small increase in cost of rapid hardening Portland cement over the ordinary kind, and to the great advantages obtained by its use, it seems evident that eventually it will be adopted generally. The increased strength and rapid hardening properties of this new product are derived from the general improvements in manufacture in all its stages. A slightly higher percentage of combined lime is present in the finished product, but the quantity still conforms to the requirements of the latest British Standard Specification.

The following table gives the result of a laboratory test on a series of 6-in. cubes with a 1 : 2 : 4 mix of rapid hardening Portland cement concrete :—

Age	Crushing Strength
1 day	1,750 lb. per square inch
2 days	2,910 ,, ,, ,,
3 ,,	3,853 ,, ,, ,,
5 ,,	4,840 ,, ,, ,,
7 ,,	5,360 ,, ,, ,,
28 ,,	6,810 ,, ,, ,,
6 months	7,260 ,, ,, ,,
12 ,,	7,490 ,, ,, ,,

Although this rapid hardening concrete rises in temperature during setting and hardening to a lesser extent than aluminous cement concrete, the increase is sufficient to permit work to be carried on

under wintry conditions, the only precautions necessary being to heat the mixing water and to protect the concrete in the usual manner after deposition.

Only the precautions necessary with ordinary Portland cement concrete need be taken when using this material, excepting that the amount of water used in mixing should be reduced to a minimum, consistent with workability This point should receive constant attention during the progress of the work, as excessive water greatly reduces the rapid hardening qualities and ultimate strength of the concrete.

144. Increased Stresses for Rapid Hardening Concretes.—A word of caution is necessary regarding the use in design of higher working stresses, where rapid hardening concrete is to be used. While increased stresses can be adopted—some authorities now admitting higher compressive stresses—these increments cannot be taken directly proportional to the improvement in crushing strengths given above over strength of ordinary concrete.

The figures stated are taken from laboratory tests and are generally higher than would be produced from similar concretes used in construction.

The principal reason, however, for using only moderately increased stresses is that the improvement in resistance to crushing of these new concretes is apparently not accompanied by an improvement to the same extent in adhesive and other qualities.

In the past, the crushing strength of concrete obtained from test cubes has been regarded as a criterion of the general strength and quality of the material in question, the tensile strength and other values varying as a rule in reliable ratios.

Until more evidence is available in this respect for the new concretes, it is wise to be conservative in adopting higher working stresses.

145. Concrete Aggregates.—There are two classes of coarse aggregates suitable for concrete in bridges, viz., ballast and crushed stone. The adoption of one or the other usually depends upon the locality of the work to be executed. Ballast makes an excellent aggregate provided it is clean and properly graded. Of the stone aggregates, traps, granites, and the harder kinds of sandstone are the most commonly used in Great Britain. Occasionally blast furnace slag and broken brick are employed for mass concrete work.

All the aggregates suitable for general reinforced concrete work can be used for reinforced concrete bridge work. The use of limestone is rightly forbidden in cases where there is any likelihood of the structure being subjected to fire, owing to the well-known disabilities of limestone under this condition. For reinforced concrete bridges, however, such an objection does not hold, and there is no

reason why good quality limestone should not be used, provided that very high stresses are not likely to be realised. Limestone aggregate has already been used quite successfully in the construction of large bridges abroad.

The maximum size for coarse aggregate depends upon the size of the concrete member or members for which it is to be used, and the amount and spacing of the steel reinforcement.

Should the amount of reinforcement be high and the spaces between the rods restricted, the maximum size of the aggregate may require to be reduced, but not necessarily to the extent of enabling all the stones to pass between all the reinforcement. It is quite practicable to obtain a thoroughly dense mass with the rods properly embedded, even though, in some cases, the larger stones exceed in size the spacing of the closest reinforcement and the amount of concrete cover provided.

Fine aggregate consists of quartz sand, or other material similar in character. The greatest care should be taken to ensure it being free from contamination, especially vegetable matter. Carelessness in this respect is probably responsible for more unsatisfactory concrete than any other cause.

Superficial examination is not sufficient. There have been many instances where organic impurities in sand, quite unsuspected during progress of the work, have caused serious trouble. A simple and reliable chemical test for organic impurities in sand, and one which is equally applicable to coarse aggregate, is the caustic soda test. This test consists of thoroughly shaking a sample of sand or stones in a test-tube, or clear glass bottle, with a 3 per cent. solution of caustic soda, and allowing it to stand for twenty-four hours. The quantity of sand or stones should come about half way up the fluid in the container. If, after standing for the prescribed period, the colour of the solution is a clear pale yellow, the material may be regarded as fit for use. Should the solution be markedly yellow or brownish, or should it appear turbid and cloudy, the presence of organic matter in undesirable quantities is indicated, and the material should be rejected.

All fine aggregate should also be graded. Ordinary sand should be graded as stated on p. 251, but where the residue of stone crushing is employed, a proportion of particles, finer than usually specified, can be allowed.

Many engineers in the past have rightly refused to allow stone dust to be used as sand in the mixing of concrete. With the modern plants now installed at many quarries, however, the use of stone dust as fine aggregate becomes practicable, since it can be suitably graded, and if of a hard and durable nature, exceedingly good concrete can

be produced by its use. It is necessary, when employing such dust, to ensure that the most finely powdered particles are dispelled from the material. With modern installations, this is automatically done by blowing apparatus, and thus the likelihood of a film of such powder remaining as a coating upon the larger stones is obviated, and the proper adhesion of the cement mortar is thereby assured.

146. Water.—Provided ordinary precautions are taken to ensure the use of clean water for the mixing of concrete, no trouble need be anticipated. Small impurities have been found by exhaustive tests to be of no consequence, excepting certain acids which can never be present in the usual sources of supply. A good criterion of water satisfactory for the mixing of concrete is its suitability for drinking.

Much has been written concerning the amount of water that should be used for the mixing of concrete. It is indisputable that excessive water reduces the strength of the resultant concrete, and that the drier the concrete can be mixed the stronger it will be. When the material is reinforced concrete, the question of workability arises, and in almost all cases a greater quantity of water is required to produce a properly compacted mass than is necessary for hydration of the cement. Nevertheless, it is found generally that excessive water is used, and it should be the constant endeavour of those supervising the work to prevent this occurring. The handling and deposition of concrete become so much easier with a wet mix that such is always the tendency. The necessary quantity of water can only be determined practically by trial, since it depends not only on the amount of cement used, but also upon the proportions, quality, and moisture content of the aggregates.

147. Steel Reinforcement.—In general, the best type of steel rods for use in reinforced concrete bridge construction are those of plain round section, of ordinary mild steel, conforming in all respects to the British Standard Specification, No. 15 (1930).

There are no technical objections to the use of high carbon steel or deformed rods, provided the former is thoroughly tested, and the strength of the latter calculated upon their minimum cross-sectional areas. No increased stresses should be taken upon either of these, and, as there are several practical objections to their use, the type of reinforcement recommended above is employed for the vast majority of bridges.

The diameters most commonly used are $\frac{5}{16}$ inch, $\frac{3}{8}$ inch, rising by $\frac{1}{8}$ inch to $1\frac{1}{4}$ inch and $1\frac{1}{2}$ inch. Smaller diameters than $\frac{5}{16}$ inch become easily distorted in the work, and it is doubtful whether in practice, such wire becomes sufficiently well embedded for the surrounding concrete properly to adhere to it. Larger diameters

than 1¼ inch are seldom necessary and cannot be employed economically unless used in very long lengths, this being necessary so that the adhesive resistance shall be equal to the working strength of the bars under direct stress.

The maximum length usually adopted for the larger diameter rods is about 45 feet, this being the greatest length obtainable without specially rolling. The smaller diameter rods can be obtained in coils of long lengths up to about 2 cwts. in weight. Reinforcement is usually delivered in this way where it is required for use as spiral reinforcement, the longest lengths obtainable being an advantage in this case.

Generally, the reinforcement is ordered and delivered in straight rods cut to the lengths required, the bending being done upon the job.

Most bending can be done " cold," which is preferable, although for large diameter bars this is difficult. In such cases, there is no objection to the rods being gently heated before bending. This, however, should be carefully done, and the temperature limited to that indicated by a dull red.

When steel reinforcement has to be kept for a considerable period on the job it should be protected from the weather to prevent undue corrosion. Racks are usually made which keep the bars off the ground and enable them conveniently to be sorted. A light roof over these racks gives the requisite protection, and usually proves economical in the long run, as the cleaning of the steel to remove excessive rust is avoided.

Any rods which are coated with rust scale, or become contaminated by grease, oil, or paint, must be cleaned before fixing, and thereafter proper precautions should be taken to keep them clean and free from the oil or other substance that may be applied to the shuttering.

There occasionally appears to be some doubt as to what amount of rust is permissible upon steel reinforcement. A bright red rust which forms when steel is exposed to rain is not injurious—in fact, it is preferable to perfectly smooth surfaces or mill scale. Where, however, exposure is such that deep rusting occurs, causing formation of rust scale which can be flaked off the rods, the reinforcement must be thoroughly cleaned. This is best done by wire brushes. In rare cases, the rusting may be sufficient to cause an appreciable reduction in cross-sectional area, more particularly with small diameter rods, although it is found in practice that a surprising amount of oxidisation can take place without serious diminution in area.

This is, nevertheless, a point to be watched, and where excessive rusting has occurred, the bars should either be rejected or suitable allowance made for their reduced strength.

The most suitable wire for binding the intersections of rods to keep the reinforcement correctly assembled in position is No. 16 S.W.G. It should be annealed soft iron, and if of the diameter recommended, single strands are usually sufficient and better than double strands of smaller gauge material.

Many bridge members are of such a length that it is necessary to provide rods subject to the same form and intensity of stress in more than one length. A typical example of this occurs in the tie-beams of a bow-string girder, although many beam bridges of large span present the same problem. Most of the standard building regulations specify that, in such cases, the bars shall be hooked at their ends, and a lap provided, the latter being calculated as so many diameters of the rods in question, according to the kind of stress being brought upon them.

The longitudinal reinforcement in such long members is usually of large diameter, and it is often found, in practice, inconvenient to provide lap joints of the kind mentioned above. Frequently there is a considerable number of rods to be accommodated, and the space in which to place them somewhat restricted. The following arrangement is one that gives the required continuity of strength, is economical, and enables the rods to be placed accurately in position without trouble.

Consider the number of rods necessary to be continuous from end to end. Sever these at intervals and introduce additional rods of the same diameter, so that at any cross-section the full number required is present. The number of rods to be added depends upon the number of severances, or butt joints, which, in turn, depend upon the lengths of rods and the size of bars used. For example, it is possible to provide an effective continuous tensile section of five rods, 1½ inch diameter, using lengths of 45 feet with the addition of one rod of the same size and length. It must be understood that the rods are placed so as to give the standard overlap between each butt joint. In tension, this lap is sixty diameters, and in compression, forty diameters.

Fig. 156 shows such an arrangement, and illustrates how accurately the rods can be placed by its adoption.

148. Temporary Staging.—The temporary staging for reinforced concrete bridges varies so much according to the type of bridge to be erected and the natural conditions encountered that it is not possible to lay down any method that should be followed. It is customary to leave the exact type and arrangement to the contractor, and for the proposed details to be submitted to the engineer for his approval.

It is necessary, however, in all work of importance, for the

contractor to be supplied with the exact ordinates to the curves forming the soffits of the various spans. These should be calculated from an easily established datum, and at frequent intervals. In addition, an allowance at each of these ordinates must be made for the probable settlement of the staging, due to its own weight and due to the dead weight of the work it must carry. This allowance should include additionally the estimated elastic deformation of the structure after removal of the staging, so that the final curve of each of the span soffits coincides with that desired.

To compute the above allowances with accuracy is frequently difficult, owing to the many variable factors to be considered, some of which can scarcely be known. In such cases, covering figures greater than those to be realised are usually preferable.

For large spans, the temporary staging is usually designed scientifically, and is only made strong enough for the actual load it will have to carry.

The time at which staging should be eased or removed depends upon the span, and the manner in which the bridge has been designed. Occasionally, weather conditions also affect the time for removal.

For ordinary Portland cement concrete it is usual to allow from four to six weeks to elapse between the time of depositing the last batch of concrete in the main supporting members and the striking of the centering. If rapid hardening concrete is used, the time can be reduced to about two weeks, although such a short period is not always useful from the practical standpoint.

In considering this question, the state of the construction in adjacent spans (if any) must be taken into account, and care should be taken to make sure that the conditions assumed in the design are completely realised in the work.

It does not follow in all cases that to leave the centering in until the last moment is to add to the security of the structure, by relieving the concrete of stress until it has hardened as long as possible. In many instances it is an essential part of the design to cause the principal members to be load carrying before completion of the work. Such is frequent in the case of arch bridges of large span and structures faced with stonework.

The periods stated above allow of the members in question carrying loads beyond their own weight such as may come upon them from superimposed construction. The total designed load, however, is never likely to be realised until the concrete has had a much longer period in which to harden.

Immediately prior to the time decided upon for the easing or removal of the temporary staging, it is advisable for the concrete lastly deposited in the portion of the work affected to be examined

by the resident engineer or inspector. Should the surface of the concrete, at this time, appear to be hard and ring satisfactorily when struck with a hammer, instructions can safely be given to the contractors to release the staging.

149. Examples of Temporary Staging.—The following notes briefly describe some typical examples of staging for various types of bridges.

Fig. 157 illustrates the steel centering employed for the main span of the Queen Margaret Bridge, Glasgow. This was designed to carry the dead weight of the arch vault and ribs, the concrete for these being deposited symmetrically so as to prevent undue distortion of the steelwork staging. Upon the completion of the arch the steel centering is slackened and can be removed if required, the concrete arch ribs being strong enough to carry the stone-faced spandril walls, superimposed roadway, etc. The centering has a clear span of 140 feet, and is supported from reinforced concrete brackets built out from the mass concrete abutments on either side of the river.

A typical timber truss for a large rise-span ratio arch constructed over a valley is given in Fig. 166.

Fig. 136 illustrates the method employed for the Warrington Bridge (see Frontispiece) so as to provide, during construction, the clear waterway specified. A brief description of this staging is given on p. 231, and another view is given in Fig. 164.

The type of staging employed for a three-hinge arch, 230 feet span, to carry main-line railway traffic over the River Marne, on the outskirts of Paris, is illustrated in Fig. 158.

The method of staging adopted for the George the Fifth Bridge, Glasgow, is illustrated in Figs. 159 and 160. This staging was of the ordinary type, composed of rows of 12 by 12 inch pitch-pine piles, carrying loads of from 10 to 12 tons per pile.

The rows of piles were 8 feet apart and were driven parallel to the river flow. The spacing of the piles in the various rows was about 7 feet centres.

The piles varied from 32 to 42 feet in length, and they were cut off a few feet above low-water level. Superimposed on this piling was the usual arrangement of timber bracing, etc., formed to the required curve of each span. Accommodation was essential in the centre span for a waterway during construction, and to provide this, rolled steel joists of the required section were employed to cantilever the staging out, so as to take the ends of the cantilevered projections of the centre span.

Fig. 160 illustrates the suspended centering employed for the concreting of the centre gap to the centre span, which was tem-

porarily omitted, so as to conform to the conditions assumed in the design of this bridge.

Originally known as the Oswald Street Bridge, this structure is briefly described on pp. 238 to 240.

Of the greatest ingenuity is the method of construction employed for the temporary staging of a bridge between Annecy and Geneva in the south-east of France.

This is a road bridge 27 feet wide between parapets, crossing a gorge 480 feet deep, with a central main span of 450 feet and a short approach viaduct on either side. Firstly, an aerial ropeway transporter was erected, having a span of 600 feet, and capable of carrying a load of 2 tons. The approach viaducts and main piers over the springings of the main arch were then constructed. (See Fig. 161.) Superimposed upon these piers timber-framed towers were erected, and from these a series of steel cables were slung, spanning the gorge. These cables rested upon concrete saddles on top of the timber towers, and were anchored back to concrete blocks embedded in the ground on either side of the roadway. Along these cables were vertical suspenders, having at their lower ends a series of timber cross-joists arranged roughly to the intrados curve of the arch, but about 15 feet below it. (See Fig. 161.) These cross-joists were connected and a light working platform built up. The work so far described was only strong enough by itself to support a limited load amounting to two lines of longitudinal timbers, consisting in each case of three balks about 14 by 9 inches, separated by chocks, to allow for the insertion of the diagonal shear members. The addition of this longitudinal stiffening strengthened the work sufficiently to enable it to carry two further timber longitudinals which, when braced laterally, form the complete bottom boom to the arched timber stage.

The ends of all the longitudinals were kept apart a few inches, and in most cases were filled with aluminous cement mortar, but at intervals the ends were shod with metal, and large folding wedges introduced. This latter precaution enabled the thrust taken by any member to be controlled.

At this point in construction the stage was sufficiently strong to carry the remainder of the timber arch which was built upon it. When complete, it was found to be extraordinarily strong and rigid. The top surface was shuttered, and to prepare a smooth curved surface upon which to erect the permanent arch, the boarded surface was screeded with a thin layer of weak concrete to the exact contour required for the stage at this time.

The above description does not cover the many points of detail that had to be considered, but it illustrates in principle how the

Fig. 156. Arrangement of Main Rods without Laps in Deck Beams.

PLATE XIII.

[*Plates XIII to XVI.*

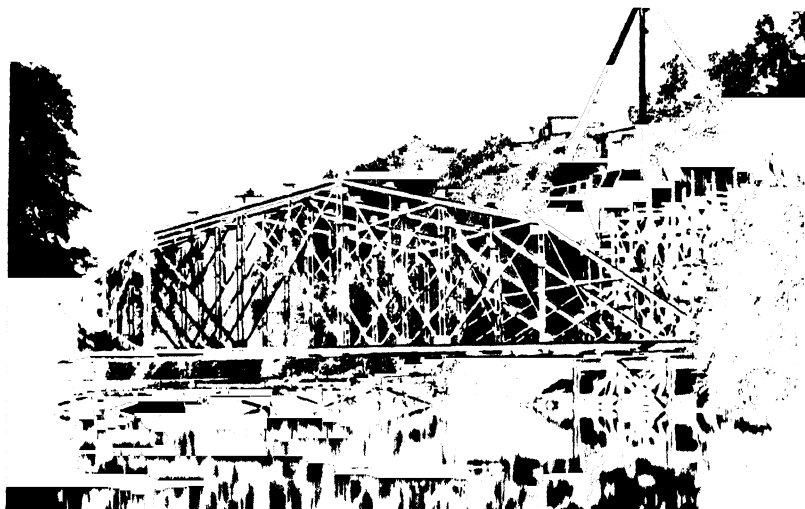



Fig. 157. View of Steel Centering for Queen Margaret Bridge, Glasgow.

Fig. 158. Staging for Arch Vault 230 ft. span.

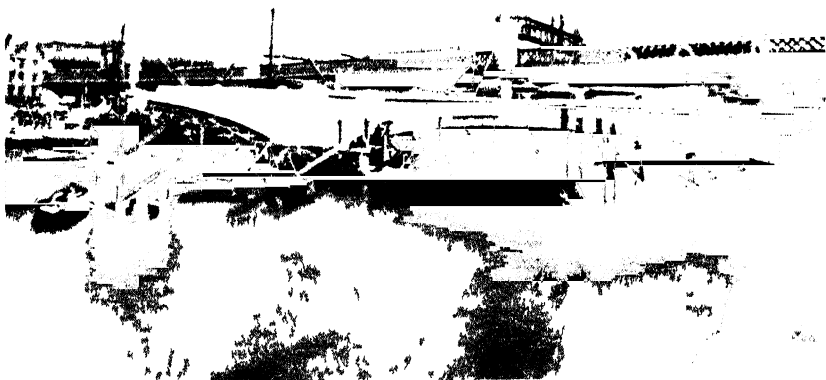



PLATE XIV.

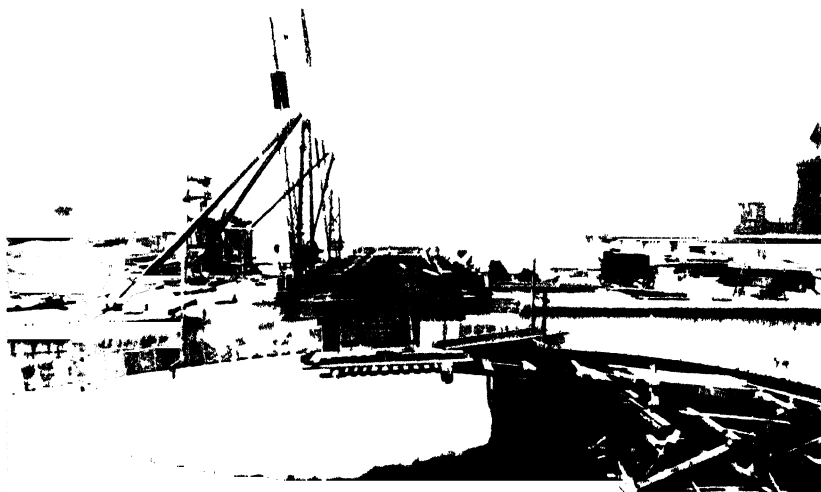



FIG. 161. SUSPENDED SKELETON STAGING FOR 450 FT. SPAN ARCH OVER DEEP GORGE.

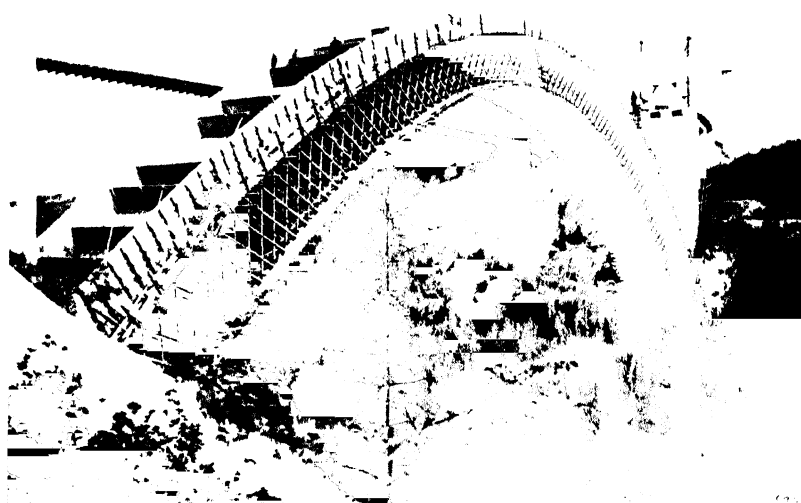



FIG. 162. COMPLETE TIMBER TRUSS STAGE FOR 450 FT. SPAN ARCH ERECTED FROM STEEL CABLES.

PLATE XVI.

staging for this enormous structure was erected from cables only strong enough by themselves to carry about one-third of the weight of the timber arch.

Incidentally, the complete stage was only sufficiently strong to carry the bottom slab of the box-vault which forms the main permanent arch. The concrete bottom slab was, in turn, only capable of carrying the concrete immediately above it, and so on.

Certain lateral stiffening was provided by means of steel cables in the early stages of erection. Fig. 162 gives a view of the completed staging.

Figs. 179 and 180 show the floating timber stage employed for the Plougastel Bridge over the River Elorn. A description of this is given in Art. 162.

150. Formwork or Shuttering.—Formwork is usually constructed of steel or timber, or the combination of these two materials. The timber commonly used for shuttering work in Great Britain is that known in the trade as yellow deal, although any other kind, which is satisfactory when subjected to alternate wetting and drying, may be employed.

Suitable methods for shuttering the ordinary members in reinforced concrete bridges, such as columns, beams, slabs, etc., are to be found in many of the text-books dealing with reinforced concrete building construction, although nearly all experienced contractors prefer their own methods.

One of the most important points in connection with shuttering work is to see that all shavings, sawdust, and other refuse has been removed from the moulds before concreting is commenced. This is best done by washing out the boxes with a hose, and temporary holes should be left in the bottom of the forms at suitable intervals to facilitate this.

The thorough cleansing of the forms although an important matter is one easily overlooked, and it requires the greatest surveillance on the part of resident engineers and contractors' representatives to see that it is done in all cases.

The internal faces of all moulds to receive concrete should be treated with a suitable preparation, such as limewash, to prevent adhesion of the concrete. A mixture of soft soap and rapid hardening cement has been used for this purpose, and has resulted in excellent surfaces being obtained on the concrete.

In hot weather, the internal faces of the shuttering should be thoroughly wetted prior to the deposition of concrete. This is especially necessary in thin work such as walls, etc.

The following are suitable thicknesses for timber shuttering :—

Deck and vault slabs	$1\frac{1}{4}$ inches
Beam and wall sides	$1\frac{1}{2}$ to 2 inches
Beam soffits	2 inches

For slab work some contractors prefer to use groove and tongue boarding, 2 inches thick, reversing the faces against the concrete upon each re-use.

All shuttering should be made watertight to prevent loss of liquid from the concrete. It should be supported in such a way that it can be easily removed in the manner required without shock or injury to the concrete.

In the case of external vertical surfaces requiring to be rubbed down whilst the concrete is in a " green " condition, the shuttering should be erected in such a way that it may be removed in sections working from the bottom upwards.

In constructing the formwork for beams a camber of 1 inch in 25 feet should be given to the beam soffits, to allow for settlement before and after the formwork is struck. Before removing the supports to the secondary members, it is advisable for the concrete to be tested in the manner already described for the main supporting members, to make sure that it has thoroughly set.

A general guide of the minimum times for the removal of shuttering and supports is given on p. 257.

151. Surface Treatment.—With bridges, as with other concrete structures, the attention given in the past to the actual appearance of the visible surfaces, as distinct from the general form and ornament, has been very meagre. In bridges of other materials proper care is always given to their finished appearance, and in the case of metal structures, surface treatment in the form of painting, with the consequent expenditure, is continuous.

Formerly there appears to have been only one surface treatment adopted for concrete bridges—that of rubbing down with cement grout, which in many cases produced an appearance even less attractive than that given by the concrete as left from the shuttering. The operation of rubbing down originated with the necessity of making good the surface imperfections of the concrete, and was never intended to provide an appearance having any merit in respect either to colour or surface texture. Little, if any, expenditure was directed to the special treatment of concrete surfaces, and the somewhat drab results can be seen when traversing the roads of Great Britain to-day.

As a result of inattention to this question, there is a tendency, when concrete bridges occupy important sites, to employ granite or stone as a facing material in the form of a veneer a few inches in thickness. This, however, is costly, and by devoting only a fraction

of the cost so involved to surface treating the concrete itself, a pleasing result can be obtained.

The parapets of a concrete bridge are difficult to treat satisfactorily, and are, in many instances, practically all that the public ever see of the bridge. For this reason many engineers prefer to form the parapets, where they are non-structural, in stone or brickwork. Especially is this the case in country districts where a suitable local stone is obtainable.

In considering the question of surface appearance there are two difficulties, viz : concrete construction joints and large uninterrupted surfaces. The former are unavoidable, and if occurring in a plain surface are almost impossible to conceal. Large plain surfaces, even where the shutter marks can be avoided by careful timber work, or by the use of plywood, are usually uneven in colour and surface texture.

Much can be done to eliminate the trouble caused by construction joints by breaking the surface horizontally in such a way as to permit of the concrete being cast continuously between the breaks. In cases where this is not convenient, another material, such as brick tiles, can be introduced, forming a distinct line or break upon the concrete face.

For the treatment of plain surfaces bush-hammering is effective in producing uniformity in texture and colour. This consists of abrading the entire surface to a depth of about $\frac{1}{8}$ inch, usually by means of a pneumatic hammer of suitable size.

In all cases where the surface finish is obtained by revealing the aggregate, the appearance is obviously affected by the kind of aggregate employed.

Broken granite, marble, and certain other stone aggregates give satisfactory results, and where bush-hammering is resorted to are more easily treated than ballast concrete. The latter, however, has been successfully treated, but care is required and skilled workmen should be employed.

If the concrete is only to be rubbed down, this should be done as soon as it is possible to remove the shuttering, and while the concrete is still green. The surfaces should be thoroughly scrubbed with wire brushes and well washed down with water from a hose. Where it is desired to reveal the colour of the aggregate stones, a weak solution of hydrochloric acid should be applied to the concrete surfaces, the latter afterwards being thoroughly cleansed down with water.

Concrete may be coloured by means of tinted cements, special coloured aggregate, or a combination of the two, but discretion should be used in regard to the amount and nature of such colouring when applied to an engineering structure such as a bridge. By the admixture of a small quantity of lamp-black or other dark colouring

matter, a slight difference in the colour or shade of the natural concrete is economically obtained, and can be applied effectively to recessed portions of the walling, parapets, etc. Pleasing effects can also be obtained by using concrete made with one of the white cements now upon the market, but care must be exercised in all cases to guard against over elaboration.

An effective appearance may be produced by casting the concrete against an indented or reeded surface, usually formed by nailing to the plain boards strips of timber of a sectional shape to produce the required degree and frequency of indentation. Upon removal of the shuttering the projecting strips are hacked off so as to produce a surface with alternate bands, rough and smooth. A fluted or grooved finish is obtainable by suitably prepared shuttering, this providing an effective and comparatively inexpensive means of surface decoration

If it is desired merely to produce a uniform colour, one of the compositions prepared for the purpose and applied as paint to the concrete surface can with advantage be employed.

Where the importance of the work warrants it, bronze ornamentation can be introduced with good effect, in which case the concrete surface should be prepared so as to be in keeping with it If bronze balustrades be employed, the elevation and general appearance of the bridge are improved and a more graceful structure results

As the parapets, and any terminal or intermediate pilasters to same, are more in evidence than any other part of a bridge, it is desirable that particular attention be given to them, both as regards design and finish.

Where these members are not structural, and are required to be in reinforced concrete, they are frequently pre-cast in sections, and certain portions can usually be pre-cast, even in cases where the parapets act wholly or partially as girders, by the provision of suitable reinforcement left projecting from the previously moulded units for the subsequent connection with the *in situ* concrete work.

A superior surface finish is obtainable in pre-moulded work, with the additional advantage that all units are identical, and if carefully set in position and properly made good with the *in situ* work, a very satisfactory job results.

In certain cases special aggregate can be used for the concrete to the visible surfaces such as spandril walls and arch rings. If the thickness of the members is sufficient to make it practicable, a special concrete can be employed in the form of a veneer about 2 or 3 inches in thickness deposited at the same time as the ordinary concrete forming the bulk of the work, the two qualities being temporarily separated by wooden or steel forms which are lifted as the work is brought up.

CHAPTER XII

DESCRIPTION OF BRIDGES

The following selection of reinforced concrete bridges illustrates the various types of construction referred to in the preceding chapters :—

152. Warrington Bridge.—This bridge, which forms a good example of a flat arch bridge with independent ribs of rectangular section, has a clear span of 134 feet and is 80 feet between parapets.

The deck platform is carried by eight parabolic arch ribs, 45 inches by 30 inches deep at the crown, spanning from piled abutments.

The rise-span ratio of 1/10 is low, but was unavoidable in view of the road level being fixed and the necessity of providing a full waterway.

Owing to the low rise-span ratio, the thrusts upon the temporary hinges and abutment supports were relatively high, the former being about 240 tons and the latter 350 tons maximum.

This thrust upon the abutments necessitated special care in their design, and reinforced concrete counterforts were placed (see Figs. 147 and 148) opposite each arch rib, these distributing the thrust to reinforced concrete piles driven into hard clay. These piles were arranged in rows of nine in line with each of the ribs, and were driven to an inclination approximating very closely to the resultant thrust, taking into account the weight and resistance to lateral displacement of the abutments.

A difficulty met with in construction was in connection with the centering. In order to allow navigation to be maintained, the allowance permitted below the soffit of the finished ribs at the crown could not exceed 6 inches.

In order to overcome this difficulty, composite girders of steel and timber were constructed and superimposed over the central 50 feet. From these girders, shown in Fig. 164, the staging for this portion of the work was suspended.

An allowance of 4 inches was made in setting out the timber centering for the deflection of the ribs at the crown due to settlement of centering, etc., and it is interesting to note that the actual settlement during construction amounted to 3 inches, which increased to 4 inches after the bridge had been open to traffic for eighteen months.

The test loads employed were as follows :—

(1) Five tramcars, weighing 12 tons each, and two steam rollers one weighing 20 tons and the other 15 tons, were crowded as closely as possible at the centre of the bridge. Under this total load of 95 tons the deflection at the crown was $\frac{1}{32}$-inch.

(2) A 20-ton steam roller, closely followed by five loaded tramcars, were run as fast as possible over the bridge. Under the total load of 80 tons the deflection was about $\frac{1}{64}$-inch with a maximum reading due to vibration of $\frac{1}{32}$-inch.

Great care was taken by the consulting engineers to obtain a pleasing elevation as the frontispiece (Fig. 163) shows.

The total quantities of materials used in the construction are as follows :—

Concrete	2,400 cubic yards.
Steel reinforcement . .	242 tons.

Figs. 135 and 136 illustrate the temporary hinges employed in this structure.

153. St. Jean la Rivière Bridge.—That a reinforced concrete bridge can be successfully designed from an æsthetic, as well as an engineering, standpoint is seen from Fig. 165, which shows an elevation of the above structure.

This bridge is one of three similar arch bridges constructed along this valley and has a span of 150 feet.

Owing to the natural conditions obtaining at the site a liberal rise-span ratio of 1/3·7 was adopted.

There is no feature of any particular importance to be noted, the type of bridge economically suited to the site being perfectly straightforward in design.

The temporary staging required some consideration in view of the height of this structure, and the arrangement employed is illustrated in Fig. 166.

The deck platform is 16 feet wide between parapets and accommodates a line of tramway vehicles each weighing 15 tons.

154. Red Bridge, Ilford.—Illustrated in Fig. 167, this bridge is an ordinary arch slab or vault, earth filled.

It is 40 feet span and carries a roadway 44 feet wide, with footpaths on either side of 8 feet, giving it an overall width between parapets of 60 feet. The rise-span ratio is 1/7·3.

The abutments are of reinforced concrete, comparable in type with those employed for the Warrington Bridge described above. Piling was, however, unnecessary, since the pressure imposed by the abutments upon the foundations did not exceed the permissible figure.

The thickness of the arch slab at the crown is 12 inches, increasing to 15 inches at the springings.

The whole of the work, including the parapets, is constructed in reinforced concrete.

The total quantities of materials employed are as follows :—

Concrete	330 cubic yards.
Steel reinforcement . .	29½ tons.

155. Arch Bridge over River Vesubie.—This structure forms a good example of a bridge in which the deck platform is suspended from the main arch ribs. It is one of the largest bridges in the world of its class, having a total length of roadway of 344 feet 6 inches, with a clear span of 315 feet. The width between parapets is 24 feet and the rise-span ratio 1/6.

The reasons for the adoption of this type of bridge were as follows :—

Firstly, as the river is subject to severe floods it was not considered desirable to have any intermediate supports, since the failure of the older structure was due to these being carried away by an exceptional flood.

Secondly, hard rock exists on either side of the valley capable of taking the thrust from arch abutments at the required level.

Thirdly, the bridge is relatively narrow, enabling an arch rib to be placed over each parapet without necessitating heavy cross beams.

The bridge is shown in whole and part elevation in Figs. 168 and 169 respectively.

Due to the locality and exposed position the range of temperature is high, and in the design a variation of from 11° Fahr. to 107° Fahr. was allowed for.

The resultant movement, due to this variation, in an uninterrupted length equal to the roadway of the bridge, would result in stresses far in excess of those permissible on the materials employed. The introduction of a transverse expansion joint therefore was essential, and was accordingly provided.

This transverse joint cuts the roadway, platform and parapets completely in two. It is arranged near one end of the bridge and is placed midway between two of the transverse beams supporting the roadway slab and which connect the suspenders.

It is formed by a notch in plan. This is provided in order to form a key in the longitudinal direction of the deck construction, giving the necessary transverse support against lateral movement due to wind pressure.

This expansion joint is only provided in the reinforced concrete work, no interruption being made in the material forming the road surface itself.

The question of wind pressure was serious, owing to the peculiar position of the bridge and its somewhat narrow construction, and a figure of 50 lbs. per square foot of surface area was allowed for.

The resistance offered by the bridge against the total wind pressure is provided by the deck construction acting as a horizontal girder, and the two curved ribs, lattice braced and supported near each end of the bridge by a stiff diaphragm carried down to the roadway and supported at this level.

The curved braced girder was calculated for its full developed length, but was assumed to be horizontal so far as bending was concerned.

The deck construction was designed and constructed as follows :—The short length of roadway on one side of the expansion joint was considered as a cantilever and was rigidly fixed to its adjacent abutment.

It was calculated to take the distributed wind load coming upon it, in addition to the reaction from the longer portion of the roadway, which was assumed to be freely supported at the rebated expansion joint and rigidly fixed at its opposite end. In this manner all the factors which may be regarded as a source of danger in a bridge of this type constructed with concrete, were taken care of, and all risk of any defects resulting from them eliminated.

Regarding the construction of the bridge, the temporary staging had to be designed with very great care, in view of the necessity for absolute rigidity against any movement due to settlement or to possible lateral displacement due to wind pressure when the concrete was being placed.

Transverse trestles of timber piling, diagonally braced, were formed across the bed of the river except in the permanent waterway, where, owing to the rapid flow, reinforced concrete sheet piling was employed.

The under staging was essentially composed of five longitudinal timber trusses resting upon the transverse trestles described above. These trusses were braced in such a manner that each of the vertical suspenders were diagonally strutted from the above transverse trestles.

In this manner the decking and parabolic ribs were rigidly supported, and the possibility of any sag in the temporary staging during the placing of the concrete prevented.

Illustration No. 170 shows the trestles and above-described longitudinals, in addition to the somewhat elaborate staging necessary for the superstructure.

The whole of the temporary staging was of timber, with the

exception of the two transverse trestles composed of the concrete sheeting mentioned above.

The road surfacing material was composed of ordinary macadam, about 8 inches thick, laid over the area required. This figure of 8 inches was a constant thickness, and the necessary transverse camber was formed in the deck slab itself, this member being constructed to the required curve on both its upper and lower faces.

Tests were made upon the completion of the bridge. These consisted of passing backwards and forwards over the bridge, two 18-ton traction engines, side by side, followed by several lorries, each weighing 8 tons. The total rolling load amounting to 100 tons. The maximum recorded deflection was $\frac{3}{8}$-inch.

156. Biel Bridge.—This structure illustrates a method of dealing with ordinary small span girder bridges of narrow width.

The architectural treatment (Fig. 171) is both simple and economical, yet a pleasing elevation has been obtained.

The clear span of this bridge is 36 feet, and it is 12 feet wide between parapets.

The roadway slab is carried by six transverse beams, which, in turn, are carried by the main longitudinal beams shown in the illustration.

157. Bowstring Girder Bridge at Nantes.—This bridge is typical of a modern bowstring girder bridge of large span. Figs. 172, 173 and 174 show the general arrangement employed and the clear waterway of 180 feet obtainable with this class of structure.

As will be seen from the illustrations, the footpaths have been cantilevered. This was done in order to reduce, as far as possible, the depth of the transverse roadway beams.

The rise-span ratio is 1/6, and the overall depth of the curved ribs is 5 feet at the springings, increasing to 6 feet 6 inches at the crown. The maximum width of these members is 3 feet 9 inches. They may be regarded as of H girder section, in which two pairs of spirals form the top and bottom flange reinforcement. These flanges are connected by a thin web, 8 inches thick, stiffened at intervals in the usual manner.

The total quantity of steel reinforcement provided in the above bridge amounts to 212 tons, which is made up in the following manner :—

Bowstring girders	139 tons.
Deck construction	67 ,,
Abutment supports (not including reinforced concrete piling)	6 ,,
Total	212 tons.

A large proportion of the steel to the bowstring girders is placed in the horizontal ties.

The variations in the length of the bridge, due to rise and fall of temperature, have been provided for; and at one end of the bridge provision is made for angular movement, whilst at the other end roller bearings are introduced, which permit longitudinal, as well as angular, movement to take place, thereby eliminating all possibility of secondary stresses being set up from expansion or contraction due to temperature variations.

The above bridge has an overall width of 65 feet 7 inches, with a roadway of 32 feet 9 inches carrying a double tramway track.

The lateral spacing of the bowstring girders is 39 feet 5 inches centre to centre.

The whole of the deck construction, including the ties, is formed with a longitudinal camber of approximately 1 in 50, and the transverse deck beams and vertical suspenders are spaced at 9 feet 9 inches centres throughout the span.

The latter members are 9 inches thick in elevation, tapering in width from 2 feet 8 inches at the deck level to 1 foot 9 inches at their upper ends.

As the reader will see from the illustrations in Figs. 173 and 174, these members are perforated, as are also the cantilevered portions of the cross beams at deck level. The material omitted is not structurally necessary, and, although relatively small, reduces the dead weight of the structure, the latter providing, of course, by far the most serious loading to be carried.

The temporary staging for a bridge of this class, if constructed in the ordinary manner, would be somewhat expensive. With the method used for this particular bridge, and others of its type, however, the cost of the centering is greatly reduced. The method employed is briefly as follows:—

Arch trusses, constructed in two halves, are erected and supported on temporary brackets built on the abutment walls. Suspended from these by rods are horizontal platforms at the level of the underside of the horizontal ties. These trusses are designed to carry the weight of half of the curved ribs and one half of the horizontal ties. These portions of the girders are then concreted and allowed to set, after which they are capable of carrying their own weight and the additional weight of the remaining concrete necessary to complete the girders.

When the girders are completed, the deck platform is constructed, the staging for this being supported on the girders themselves. By the adoption of this method of centering, one of the objections to the bowstring girder type of bridge, that is, the

relatively high cost of centering, is obviated, and this type of structure rendered advantageous, both from the economical as well as the accommodation standpoint.

158. Riverford Road Bridge, Glasgow.—This, as specified, is an arch bridge of the spandril-filled type and is designed to carry very heavy rolling loads, viz., a vehicle of 100 tons on four wheels at 9-foot centres, drawn by a number of heavy tractors.

The bridge is on the " skew," the longitudinal axis of the roadway making an angle with the abutments of 56°.

The skew span is 80 feet, the rise 12 feet, and the width between parapets 60 feet.

The arrangement adopted is an arched slab 9 inches thick. stiffened by superimposed ribs, hinged at the springings.

These permanent hinges are of concrete, being of the type illustrated in Fig. 137.

The ribs, which project into the filling, are parallel to the parapets and are seven in number. They are 12 inches thick and have a variable depth which is a maximum at the haunches of 6 feet 3 inches.

The spandril walls and parapets are separated from the abutments by expansion joints.

An elevation of this bridge is given in Fig. 175.

Approximately 400 cubic yards of concrete and 44 tons of steel reinforcement were used in constructing this work.

159. Bowstring Railway Bridge at Beautor.—This structure (Fig. 176) is interesting in several respects.

Firstly, it is a stiffened " bowstring girder bridge "—that is, the parapet members take a considerable proportion of the total bending moments.

Secondly, it is one of the few examples of this type of bridge carrying railway traffic ; and

Thirdly, the support at one end is designed as a rocker bearing in reinforced concrete.

The main span is 128 feet and the width between parapets 12 feet.

The overall depth of the parapet girders is 6 feet 7 inches, and the rise of the curved member is 21 feet 4 inches from the centres of the parapet girders to the centres of the curved ribs.

160. Bridge at Aubervilliers.—The total length of this bridge is 264 feet, comprising a main central span of 132 feet and side spans over the canal paths and roadway.

The width between parapets is 54 feet, accommodating a roadway 32 feet between kerbs and two footpaths which are cantilevered from the main bowstring girder upon either side.

It will be seen from Fig. 177, illustrating this structure, that the maximum waterway is provided, and also a minimum obstruction to traffic along either of the canal banks.

The whole of the work is of reinforced concrete and is so arranged that the appearance of the bridge, as a whole, is not ungraceful.

The method of providing for the expansion and contraction of the deck platform is of interest. A transverse expansion joint is formed at the abutment retaining wall on the extreme left and flexible column supports of the type indicated in Fig. 20 introduced. The deck platform is again completely severed by a transverse expansion joint to the left of the right arched bowstring support, and the ends of these girders are carried upon cast steel roller bearings at this point.

With this arrangement, the inevitable variations in length of the bridge are provided for, and in such a way that neither of the main arched supports are subjected to appreciable strain.

It will be noted that the centre of expansion for the length of roadway between the expansion joints is only a few feet from the left arch support.

In order to illustrate the extreme lightness of the bridge, which carries heavy vehicular traffic, it may be stated that the total concrete employed, including the foundations, is equivalent to a slab 2 feet thick over the entire deck area.

161. George the Fifth Bridge, Glasgow.—The design for this bridge, a model of which is illustrated in Fig. 178, is, in several respects, unique.

There were many difficulties to be contended with in the design, and it was only after investigating several alternatives that the accepted design illustrated, was evolved.

The specified conditions imposed the following limiting dimensions :—

A clear central span of 146 feet and side spans of 110 feet.

The width of the structure is 80 feet between parapets, and covered accessible accommodation had to be provided for gas and water mains, electric, post office and tramway cables. The whole of the bridge, excepting the soffits of the spans and of course the foundations is faced with granite.

The depth of reinforced concrete construction was limited to 4 feet 8 inches at the crown of the central span, 5 feet 5 inches at the centre of the side spans, and 14 feet 6 inches at the river piers.

Moreover, the highest recorded tide level is within a few inches of the level of the arch springings.

The width of the centre piers could not exceed 20 feet and the maximum depth for these piers was also specified.

A brief study of the above conditions will make clear the serious difficulties encountered in the design.

Arches were found to be impracticable and uneconomical, owing to the low rise-span ratios of 1/12 for the centre span and 1/19 for the side spans, and also owing to the width of the river pier foundations which would have been required to take the unbalanced thrust from the centre span.

The adopted design, which complies with the whole of the above conditions, consists of a cellular platform comprising vertical ribs, stiffened at the top by the deck slab and at the bottom by curved vaults. This platform is entirely separated from the supports and rests upon cast steel rollers. These bearings are arranged at the abutment supports, and upon either side of the river piers, and in the latter case so inclined that the thrusts from the bridge superstructure always pass through the centres of the bases of the supports. By this means uniform pressure on these foundations under any condition of loading is ensured, and the whole of the superstructure permitted to expand and contract freely with variation in temperature.

The river pier foundations consist of circular caissons, 20 feet external diameter, founded approximately 61 feet below the under side of the bridge platform. The tops of these caissons are at about low water level (ordinary spring tides). The water at this level is about 16 feet deep and the average penetration of the caisson below the river bed is therefore 45 feet.

The abutment foundations also comprise circular caissons, their external diameter being 16 feet 6 inches, and their founded level 47 feet below the underside of the bridge platform.

Connecting the upper ends of the river pier caissons are massive transverse reinforced concrete lintols extending in height from L.W.O.S.T. to a few feet above H.W.O.S.T. The abutment caissons are connected by transverse sleeper beams also in reinforced concrete and extending to a few feet above high water level.

The superimposed rolling loads for which the bridge is designed are specified as follows :—

(*a*) 120 tons on four wheels, 9-feet centres, drawn by five tractors 15 tons each.

(*b*) Two lines of tramways occupying central 18 feet of roadway. Each tram weighs 13 tons and has an overall length of 30 feet.

Remaining area of roadway and footpaths to be covered with a distributed load of 1 cwt. per square foot.

The following are the total materials required for the work :—

Concrete	4,800 cubic yards.
Steel reinforcement . . .	660 tons.
Mass concrete filling to caissons .	4,670 cubic yards.

162. The Plougastel Bridge.—Primarily on account of its size, this bridge, which spans the Elorn River near Brest, received during its construction the attention of engineers in all parts of the world. At the time of its erection it was by far the largest reinforced concrete structure of its kind. It consists of three main arches spanning the river, each about 613 feet, centre to centre, with an approach viaduct on either side, making a total overall length of approximately 2,650 feet. It carries a single track railway with a superimposed roadway 26 feet wide between parapets and at an elevation of about 135 feet above low-water level.

The general design of the bridge is not unusual, excepting that provision was made for the use of hydraulic jacks to eliminate the secondary stresses resulting from arch shortening. The main arches are of box form, comprising a top and bottom slab with two external and two intermediate walls.

Toward the crown of each arch the central portion of top slab is omitted, or rather, raised, enabling the railway track to pass through the arch, and the roadway to pass immediately over the arch at these points. The deck construction is carried by two longitudinal girders of N form, about 15 feet apart. These main trusses are situated between the roadway and railway track, the top and bottom flanges being integral with the upper and lower decks respectively. Supports from the arches at about 60-foot intervals are in the form of cross-walls stiffened externally under the above girders (see Fig. 181). A continuation of this construction forms the approaches.

The abutments and river piers are of mass concrete carried down to rock foundations.

While it may be said that the design of the finished structure is not unusual, this cannot be said of its construction. The methods employed revealed the greatest ingenuity on the part of M. Freyssinet, who was responsible for the design, as well as for the construction. Throughout all the stages of erection there was apparent the most amazing originality, directed in every instance to realising economy without sacrifice of either quality or efficiency, and it was in this way that the cost of the bridge in question was so much lower than the alternative designs in structural steelwork.

It is not possible in a description of this scope to touch on many

of the constructional devices referred to above, but these extend from the works testing machines, which were made almost wholly of reinforced concrete, to the main staging, which was composed of timber of ordinary scantlings. Two very able descriptions in English * give the more important and interesting methods employed.

In the case of the river piers, the foundations were constructed by means of a reinforced concrete caisson and compressed air. This caisson, which was provided with the necessary top gear comprising air locks, compressors, etc., was constructed on shore, completed, launched, towed in position, and grounded upon a prepared bed for the first river pier. The mass concrete foundation was then constructed within the lower portion of the caisson, using compressed air, the caisson being jacked up as the work proceeded. Eventually it was tilted in a most ingenious way, and floated off into position for the other pier. As the caisson described above could have served no useful purpose after the construction of the second pier, it was so designed that it could be left in position and filled solid with mass concrete, the temporary superstructure of course being demolished.

The abutments were constructed within a circular caisson built *in situ*, this extending above high water, enabling the abutments to be concreted "in the dry."

The outstanding constructional device was the arched staging, which was employed successively for each of the three main river spans. In its completed state this consisted of a floating timber arch, lattice braced, with the springings tied together horizontally by a number of steel cables (see Fig. 180).

This staging was erected on the foreshore (see Fig. 179), the springings being constructed in reinforced concrete, and each resting upon a reinforced concrete pontoon, which was allowed to flood during construction.

Upon completion, these pontoons were emptied by pumping, and the staging floated out into position for the first arch. It was then attached by suspenders to projections from the river piers and abutments, and lifted off the pontoons. Concrete thrust blocks were also provided which enabled the cables to be slackened and the pontoons to be towed to mid-stream, where they were moored together and used as a dump for sand supplies arriving by barge. When the concreting of the arch was completed the above procedure was reversed, and the stage employed for the next span.

It is considered by most engineers who have visited the work that the successful building and handling of this gigantic arch truss in timber is in itself an engineering feat of first importance.

* By Leslie Turner, B.Sc., A.M.I.C.E., *Concrete and Constructional Engineering*, December, 1926, January, 1928, June, 1928, etc., and H. E. Steinberg, M.Inst.C.E., *Engineering*, October 18th, 1929.

The magnitude of the above operation can be appreciated when it is noted that this stage was nearly 600 feet span, 100 feet high, and weighed about 540 tons. As stated above, the timber used was of ordinary scantlings, and was connected entirely with 10-inch wire nails.

The timber centreing for the superstructure was built up in sections on shore and transported by cableways to the required positions (see Fig. 181). The latter were supported by timber towers 150 feet high, designed by M. Freyssinet. These towers again were of ordinary timber scantlings, nailed together, the lower ends being shod in concrete with a simple hinged device. The transporters, capable of carrying up to 4 tons, were electrically driven, the operator travelling above the load in a cabin with the motors. The current was supplied through the carrying cables and returned through small subsidiary cables.

The concreting of the main arch was carefully planned to minimise as far as possible the effects of shrinkage, and symmetrically to load the staging. The concreting of the slabs and walls was commenced by depositing small sections with large gaps between. This was done simultaneously from each springing, working towards the crown. Work thereafter proceeded in stages at each gap, progressively diminishing these gaps towards the crown.

During this time a transverse gap was left at each of the springings, the thrust of the arch being taken by a number of pre-cast struts placed in line with the bottom vault slab. After the springing gaps were filled and the concrete sufficiently hardened, the arch was opened at the crown by means of hydraulic jacks (see Art. 128). Rectangular openings were provided in slabs and walls for this purpose, and twenty-eight hydraulic jacks were introduced, having a total thrust capacity of about 7,000 tons.

About 26,000 cubic yards of concrete have been used for the entire work. The larger aggregate comprised a hard broken stone quarried nearby, and the smaller, a stone dust mixed with ordinary sand in equal parts.

The mixture varied according to the stresses imposed, aluminous cement being employed for the work below water level and ordinary Portland cement for that above.

An illustration of the completed bridge is given in Fig. 69 (Plate IV.).

APPENDIX

General Notation.—The following are the symbols principally used in the foregoing pages.

Any symbols not given hereunder are either included in the text or will be found in the relevant diagrams :—

Beams.

A, B, C, D, etc. .	Points of support.
A*m*	Area of free bending moment diagram.
F	Shearing force at any section.
F*a*, F*b*, F*c*, etc. .	Shearing forces at sections adjacent to supports A, B, C, etc.
M	Bending moment.
M.max. . . .	Maximum bending moment.
M*a*, M*b*, M*c*, etc. .	Bending moments at supports A, B, C, etc., in a continuous beam.
P	Point or concentrated load.
R*a*, R*b*, R*c*, etc. .	Reactions of loads at supports A, B, C, etc.
S	Position of section considered for which influence lines are drawn.
l, l_1, l_2, etc. . .	Spans of consecutive beams.
al	Distance of section from support for which influence lines are drawn (a being a function of the span).
xl	Distance of load P from support (x being a function of the span).
x	Distance of the centre of area of "free" bending moment diagram from one support.

Arches.

A and B . . .	Position of springings.
A*e*	Equivalent cross-sectional area of an arch (including rods).
A*v*	Equivalent average cross-sectional area (including rods.)
C	Position of crown of three-hinge arch.
E	Coefficient of elasticity of concrete.

Arches (*cont.*).

H	Horizontal thrust of the arch or horizontal component of end reactions.
I*e*	Equivalent moment of inertia at any arch section normal to its axis.
I*c*	Equivalent moment of inertia of the arch at the crown.
M	Bending moment at any section of the arch.
M*a* and M*b* . .	Bending moments at the springing points A and B in a hingeless arch.
M*c*	Bending moment at crown.
M_f	Bending moment in a freely supported beam of the same span as the arch.
M*s*	Bending moment at section S for which influence lines are drawn.
N	Normal force acting upon a cross-section of the arch.
P	Point or concentrated load.
R*a* and R*b* . .	Inclined reactions at the springings.
S	Position of section for which influence lines are drawn.
V*a* and V*b* . .	Vertical reactions of equivalent freely supported beam and of two- and three-hinged arches.
V_1 and V_2 . .	Vertical reactions of hingeless arch.
a	Distance of point load from support A.
ds	Element of length of arch axis.
dx	Element of the span.
f	Rise of the arch.
K	Constant in equations (two hinged and hingeless arches).
l	Span of arch.
n	Ratio. $\frac{\text{Change in length}}{\text{Original length}}$.
p	Load per unit of span.
u	Vertical eccentricity of the line of resistance from the arch axis.
y	*y* co-ordinate = height of arch axis above springing level.
x	*x* co-ordinate = distance of section from support A for which influence lines are drawn.
z	*z* ordinate = height of reaction locus above springing level.
α	Angle which the tangent to the arch axis makes with the horizontal.

Specification for a Reinforced Concrete Bridge.—The following clauses for the most part deal with the reinforced concrete construction. Clauses relevant to the necessary general work, together with any special clauses and those coming under the head of general conditions, require to be added. Any clauses not applicable must, of course, be deleted.

Trouble and friction have frequently arisen between client, engineer and contractor as the result of carelessly worded or inaccurate specifications being put forward; hence the advantages to be gained by the use of a carefully prepared and accurately worded specification are too obvious to require detailed enumeration.

The outstanding benefit of using such a specification as this is that all parties to the contract are adequately protected against loss, which might be incurred as the result of a misunderstanding of the exact nature of the work called for by the terms of the contract.

Such progress is now being made in the perfecting of the manufacture of cement for specific purposes that it may be necessary to introduce special clauses dealing with new cements. These clauses would state the nature of the work on which these special materials were to be used, and would also give specification of quality and proportions of aggregate as in the case of the materials used for general bridge work to-day.

1. Descrip of work. The bridge is a (*state type, such as arch*) bridge feet clear span between abutments and feet wide between parapets, with provision for (*describe accommodation of bridge platform, such as width of roadway, pavements, etc.*) all as shown on drawings Nos. and in accordance with such further details or with any modified drawings or written instructions which may from time to time be furnished to the contractor.

Describe any special features such as surface treatment.

2. Site. The site of the new bridge is as indicated on drawing No. .

The exact position must be set out by the contractor to the approval of the engineer.

3. Character of site. The sections of the ground as indicated on the drawings are approximate only, and cannot be guaranteed, nor can it be promised that the ground (*or water level*) will be as at present shown when the work is commenced.

The contractor before tendering should visit the site and satisfy himself as to the nature of the ground

(*and river bed*), and also generally in respect to the levels and dimensions of the proposed constructional work.

He shall also satisfy himself as to the nature of the pile driving that may be expected, and to any obstructions that may influence his schedule rate. It is believed that in general no greater length of pile will be required than feet, but no warranty is given in this respect.

The contractor must accept the entire site as he finds it, and any work that may be necessary to carry out the contract must be provided for in his contract price.

4. Maintenance of waterway during construction. (*For bridges over rivers and canals.*)

In order that the traffic along the river (*canal*) shall not be interrupted at any time during the construction of the bridge, a clear waterway of feet wide by feet in height measured from the water level to the lowest point of the temporary staging, shall be maintained throughout the work.

The contractor will require to mark clearly the temporary staging in such a way that it can easily be distinguished by river (*canal*) traffic.

5. Maintenance of traffic under bridge during construction. (*For bridges crossing existing railways and roads.*)

In order that the traffic on the railway (*road*) shall not be interrupted during the construction of the bridge, clear openings feet wide (measured square with the abutments) by feet in height from rail level (*crown of road*) to the lowest point of temporary staging shall be maintained throughout the construction.

6. Cofferdams.

The contractor shall construct and maintain the necessary watertight cofferdams in which to build the (*river piers and abutments*).

These cofferdams are to be carefully removed after completion of those portions of the work for which they are required.

For back faces and sides of abutments, the excavation is to be on the line of the concrete, and filled solid. The contractor's price to allow for this method of construction and for leaving in position any sheet piling, strutting, etc., that may be necessary.

7. Excavations.

The excavations for the (*river piers and abutments*) are to be completed to the lines and levels shown on the drawings.

Deposit sufficient for refilling any suitable, hard dry materials; carry remainder away, and deposit same where directed.

The sides of all excavations shall be properly supported with good, sound timber, such timber to be carefully removed as the excavations are filled in, but the removal of such timbering shall not relieve the contractor of his responsibility for the stability of the work.

Where for the safety of the work or adjacent ground the engineer wishes timber to be left in, the contractor shall have no claim for extra payment.

After the work of excavation has been completed, the surface of the ground shall be prepared to receive the concrete. Before executing reinforced concrete work in foundations a layer of mass concrete not less than 3 inches average thickness shall be applied to the earth surface and brought to a level spade finish to receive the reinforced concrete.

The contractor is to state where provided for in the annexed bill, his schedule rate for carrying the excavations to such depths below the levels indicated as may subsequently be found necessary, and including for any increased penetration of cofferdams in the river bed consequent on such deeper excavations

8. Pumping. The contractor shall provide all pumping that may be necessary in order to remove and keep the water, whether river, sub-soil water, or water from any source whatever, out of the (*river piers, abutments and other excavations*) during the construction of these portions of the work.

9. Bad Ground. If loose soil, bad ground, or cavities are met with in any part of the foundations, the contractor shall excavate same to a solid foundation and shall fill up such excavation to the proper level with mass concrete of the quality hereinafter specified, properly rammed as directed by the engineer.

For such additional excavations and mass concrete, the contractor will be paid in accordance with the schedule rates furnished by him in the annexed bill of quantities.

10. Fill and ram. The excavated areas around the foundations of all (*retaining walls*) are to be filled in to the required levels with approved material, well rammed and consolidated in layers not exceeding 6 inches at a time.

11. Piles. The reinforced concrete plug piles are to be inches (*octagonal*) section. The manufactured lengths, which vary between feet and

feet, are given on drawing No. . These lengths may be altered by the engineer after further investigation, and any difference between the total length of piles ordered and the quantity given in the annexed bill will be added to or deducted from the contract amount at the appropriate schedule rate.

The piles are to be manufactured of the quality of concrete hereinafter specified.

Piles are to be moulded in proper and efficient box moulds. Sides of the moulds may with care be stripped from the piles two days after casting, but the piles must not be disturbed on the bottom boards for at least ten days, or, if possible, a fortnight, when they may be carefully canted and rolled to the stacking ground. They must on no account be lifted or rolled at this stage, and supports or packing on which the piles are canted must not be more than 6 feet apart, and be level with each other.

The piles are to be well watered daily for fourteen days after manufacture and must not be driven in the work at less than nine weeks after manufacture, and until tests on plain concrete cubes of the same material and made at the same time as the piles indicate that a crushing resistance of at least lbs. per square inch has been obtained.

In lifting or rolling the piles after maturing, they must be slung or supported at stated distances apart, so that no excessive bending moments are developed. Particulars regarding the method of slinging the piles will be supplied by the engineer later.

All the piles that are damaged in handling or driving are to be replaced by the contractor at his own cost and without delaying the completion of the work beyond the contract time.

All piles are to be driven to the appropriate lines and levels. Should any pile be deflected from the vertical or the proper line, the resident engineer will have power to order same to be drawn and re-pitched until a reasonably perfect course is obtained. No strutting, union screw, or similar device will be allowed for the purpose of endeavouring to bring a badly driven pile into position.

During driving the heads of the piles are to be protected with a helmet of an approved type.

The hammer used for driving must not be less than tons in weight, and all the piles must be driven to a final penetration set equivalent to 1 inch for blows with a feet drop.

If this set is obtained in the case of any pile before same has reached the level shown on the detail drawings, the engineer will have the right to order driving to be continued or not at his discretion.

Should the requisite set have not been obtained when the pile head reaches (*the required*) level, driving will be continued until this set has been reached.

Such further driving is to be carried out by the contractor at the schedule rate.

A record of driving of all piles is to be kept by the contractor or his agent, and the contractor will also give every assistance to the resident engineer to enable him to keep a similar record.

NOTE.—*If rapid-hardening Portland cement is used: The ten days for rolling can be reduced to three days. The piles should be well watered until driven at four weeks instead of nine weeks, or when test cubes give satisfactory crushing results.*

12. **Stripping heads of piles.**

The heads of the piles are to finish at a level of feet inches (*above the underside of the reinforced concrete column bases which they support*) Should permission be given to stop any piles before this level has been reached, the contractor is to cut off the surplus length of pile at his own expense.

After the piles are driven, the concrete at the heads is to be cut down and the spirals stripped for a length of feet inches for the purpose of bonding the longitudinal rods (*to the superstructure*).

13. **Pile shoes.**

All castings are to be of clean, grey, tough metal, and free from sand, honeycombing, or porous places, air holes or other defects, and delivered on the works without being painted, stopped, or plugged in any part—otherwise they will be rejected.

The pile shoes are to be to the dimensions shown on plans, and to be made with "chilled-hardened" cast-iron bases and mild steel straps cast in—*i.e.*, the straps to run continuously through the cast-iron base.

The shoes are to be fitted accurately to the ends of the piles so that the cast-iron point is truly on the axis of the piles.

14. **Temporary staging.**

The contractor shall provide efficient and rigid temporary staging upon which the (*arch vaults*) shall be constructed.

The total dead weight of the (*vaults and ribs*) to

(*each river span*) to be carried by the staging is tons, equivalent to cwts. per square foot of stage surface.

The contractor must submit his proposals for this staging to the engineer for his approval.

The construction of the staging must be such as will provide for the required minimum waterways indicated on the drawings, which waterways must be maintained throughout the whole period of the contract. All necessary lighting and other requirements of the local and other interested authorities must be included for by the contractor.

The necessary staging for the construction of the (*river piers and abutments*) and for the (*roadway spans*) on either side of the river must also be provided for.

The staging to the river spans will require to be protected to prevent barges and other river craft colliding with it.

15. Portland cement.

All cement (*except Ciment Fondu*) to be used in the works shall be Portland cement of British manufacture, of the best quality, of a brand or brands to be approved by the engineer, and to comply in every respect with the conditions, analyses, and tests laid down in the British Standard Specification for Portland Cement, 1925.

The cement shall be delivered on the work in consignments of not less than tons per time, each bag being closed to the satisfaction of the engineer with a lead seal bearing the trade mark or initials of the manufacturer, the seal being of such a form that it cannot be removed without destroying it.

The engineer may cause samples to be taken from the consignments of cement, for testing by a recognised expert.

If the results of these tests, which must be final, show that the samples do not conform to this specification, the whole consignment must be rejected as unfit for use, and must forthwith be removed off the ground by the contractor at his own expense.

The various consignments must be brought to the ground in ample time to allow the above tests to be carried out before the cement is required for use.

No cement must be used which has become damp or deteriorated in any way.

Should any cement be rejected owing to its

failure to pass the tests, the cost of testing any new consignments to replace the rejected cement must be borne by the contractor.

NOTE.—*20-ton lots is minimum. If total is less, specify in one consignment. Largest consignment,* 100 *tons.*

16. Rapid-hardening cement. In the event of rapid-hardening Portland cement being used for the whole or any part of the work, it must conform in all respect to the above Specification for Portland cement, excepting that it should be delivered in small and frequent quantities to ensure freshness, and the contractor must make the necessary arrangements with the suppliers for this to be done.

NOTE.—*If the entire job is of rapid-hardening cement, this clause can be incorporated with the previous clause.*

17. Stores for cement. The contractor must provide a suitable water-tight store for the cement, with an elevated floor, to the satisfaction of the resident engineer.

Any cement which is found to be damaged on its removal to the mixing boards will be rejected and must immediately be removed from the site by the contractor.

18. Sand. All sand used upon the work must be thoroughly clean and free from earthy or deleterious matter.

It must be sharp and coarse; fine sand of uniform size will not be allowed.

All sand shall pass through a mesh three-sixteenths of an inch square measured in the clear. Sand shall not be used if it contains more than 10 per cent. of fine grains that pass a 76-mesh sieve as used for cement tests, unless the proportion of cement be increased to the satisfaction of the engineer.

Sand will be rejected if briquette tensile tests give lower results than obtainable with "Standard Sand."

19. Aggregate. All aggregate stones shall consist of good, hard stones of approved quality and perfect cleanliness, and shall be crushed to pass a three-quarters of an inch square mesh.

The size of the stones should vary uniformly from one-quarter of an inch diameter to the above maximum. All particles smaller than one-quarter of an inch diameter will be classed as sand.

For mass concrete the coarse material must be of such a size as will pass every way through a two-inch square mesh.

No slag, breeze, or coal residues of any description will be allowed.

20. Displacers. For the mass concrete forming the filling to the (*river piers and abutments*), "displacers" or "plumbs" may be used at the discretion of the resident engineer.

Such "displacers" must be perfectly clean, angular stones, not more than two cubic feet volume, and must be free from shakes or loose fragments.

These "displacers" are to be well soaked with water immediately before or at the time of depositing.

No "displacer" must be nearer to any external surface or other "displacer" than 6 inches.

The total quantity of "displacers" by volume used in any section of the filling must not exceed 10 per cent. of the whole mass of the concrete in that section.

21. Samples. Samples of sand and aggregate proposed to be used by the contractor must be deposited with the engineer for his approval.

These two samples shall be a fair indication of the general quality and shall be delivered to the engineer in 8-ounce glass bottles.

After approval, samples will be retained by the resident engineer for comparison with all deliveries which are made on the work.

22. Water. The contractor shall make the necessary arrangements and provide for the supply of fresh water, and pay all charges for same for the mixing of the concrete, such water to have temperature exceeding 40° F., and to be perfectly clean.

23. Quality of concrete. The concrete to be mixed in the following proportions for the different parts of the work, as set forth in the bill of quantities :–

Quality 1 : 1 : 2.

	If Ballast.	If Stone.
Aggregate stones . .	19 cub. ft.	20 cub. ft.
Sand	$9\frac{1}{2}$ cub. ft.	10 cub. ft.
Cement	$7\frac{1}{2}$ cwts.	8 cwts.

(Making approximately 1 cub. yard of concrete.)

Quality 1 : $1\frac{1}{2}$: 3.

	If Ballast.	If Stone.
Aggregate stones . .	22 cub. ft.	23 cub. ft.
Sand	11 cub. ft.	$11\frac{1}{2}$ cub. ft.
Cement	6 cwts.	$6\frac{1}{4}$ cwts.

(Making approximately 1 cub. yard of concrete.)

Quality 1 : 2 : 4.

	If Ballast.	If Stone.
Aggregate stones . .	23 cub. ft.	24¼ cub. ft.
Sand	11½ cub. ft.	12 cub. ft.
Cement . . .	4¾ cwts.	5 cwts.

(Making approximately 1 cub. yard of concrete.)

Quality 1 : 3 : 6 (*Mass*).

	If Ballast.	If Stone.
Aggregate stones . .	24¼ cub. ft.	25¼ cub. ft.
Sand	12 cub. ft.	12½ cub. ft.
Cement . . .	3¼ cwts.	3½ cwts.

(Making approximately 1 cub. yard of concrete.)

or 27 cub. ft. of approved, clean aggregate mixed with sand in suitable proportions to 400 lbs. of cement.

24. Proportions of concrete. The relative proportions of broken stone or ballast aggregate and sand in the mixture are variable, and the contractor must ascertain by experiments made from time to time, under the direction and supervision of the resident engineer, the percentage volume of the voids in the broken stone or ballast before the addition of sand is made.

The relative proportions of several aggregates, which may be varied from time to time by the resident engineer, are to be such that the proportion of sand is slightly in excess of that necessary completely to fill the voids in the coarser aggregate, desideratum being concrete comprised of well-graded materials, which must be watertight, dense, homogeneous, and free from voids.

25. Mixing of concrete. The mixing of the concrete is to be done in machines to be approved by the engineer. No hand-mixed concrete will be permitted unless specially ordered by the engineer. All materials when entering the mixer shall have a temperature of not less than 40° F.

If any mixing by hand is ordered, it must be done on a close-boarded wooden floor, and the materials carefully turned twice whilst in the dry state and three times after the addition of water, or until the components are thoroughly well mixed, and give a concrete of uniform quality throughout.

Only such quantities as are required for immediate use to be mixed at any one time. Sufficient water is to be added to enable the mixture to flow readily round all the reinforcement and into every part of the moulds. The state of the concrete in this

respect to be judged by the resident engineer, and any dispute to be referred to the engineer, whose decision in the matter shall be final and binding on all parties.

26. Depositing concrete. The concrete shall be carefully placed, and unless special permission is obtained it shall not be dropped freely from a height of more than 6 feet.

After deposition, the concrete is to be well rammed and speared until it has been made to penetrate and fill all the spaces between and around the steel rods and properly and completely surround them throughout their entire length in such a manner as to ensure a solid mass entirely free from voids.

It is imperative that the work be done quickly as well as efficiently, and an adequate number of hands must therefore be employed to ensure this.

Before placing the concrete, the moulds shall be cleaned of shavings, pieces of wood or other rubbish.

When placing the concrete, the fine material must be carefully worked against the moulds so that the faces of the concrete shall be left perfectly smooth and free from honeycombing upon the withdrawal of the moulds. Any defect in this respect must be dealt with by the contractor as directed by the engineer.

Where concrete is deposited against stonework, the surfaces of the stones shall be well watered immediately prior to deposition of such concrete.

When once deposited and rammed, the concrete is not to be interfered with or shaken.

27. Depositing concrete under water. The concrete (*filling*) to the abutments (*river piers*) where placed under water is to be carefully deposited through a tube not less than 6 inches in diameter, the tube being brought up gradually so that its lower end is always a few inches below the level of the deposited concrete, in order that the filling may be well consolidated.

28. Joints. All joints in slabs, beams, and horizontal members are to be formed by inserting temporary vertical boards against which the concrete deposited can be properly rammed. The position at which joints may be made will be indicated by the resident engineer.

All previous work where joints occur must be properly cleaned and washed with cement grout upon the face of the connection immediately before commencing fresh operations. Such grouting shall be held to be covered by the schedule rates for the concrete.

29. Gauge boxes. For the purpose of each and every batch of concrete, gauge boxes corresponding to the proper quantities of sand and coarse materials respectively must be used.

The cement must be proportioned by weight. All proportioning must be carried out in such a manner that the proportions of the materials may be easily and readily checked.

30. Test cubes. The contractor is to make, as soon as the materials have been approved, at least three (6-inch) test cubes of the qualities of concrete. These cubes are to be matured in damp sand.

They will be crushed at an independent testing works and must show a resistance, at the age of twenty-eight days, of not less than per square inch.

The responsibility for procuring the materials for giving this crushing resistance will rest with the contractor.

Further test cubes may require to be made during the progress of the work according to instructions issued by the engineer.

The contractor is to include the sum of £ for the testing of these cubes as set forth in the bill of quantities.

31. Protection of concrete. The contractor shall adequately protect the reinforced concrete against too rapid drying. During the first week after deposition, it shall be kept thoroughly damp by means of wet sacking or other material, and by watering daily as shall be required, Sundays and holidays included.

No Portland cement concrete shall be laid when the temperature is below 36° F. It shall be adequately protected from frost by straw or sacking or such other method as the engineer may deem effective. Frozen aggregate or sand or concrete which has been frozen shall not be used.

32. Steel reinforcement. The steel reinforcement will be in the form of plain round bars of mild steel of British manufacture, and must comply in every respect with the conditions, analyses, and tests laid down in the British Standard Specification for Structural Steel (No. 15, 1930).

The contractor can purchase the reinforcement either from the rolling mills or from merchants dealing in this class of material, the quality being as specified above.

All the steel must be entirely free from scale or loose rust before fixing in the work. It must not be oiled or painted. Welding is forbidden. The steel must be bent cold to the shapes and placed exactly as shown in the detail drawings, and the contractor shall without extra charge provide all fixings required, and shall take precautions to see that all such temporary fixings are removed as the concrete is brought up. At intersections the rods must be bound together with No. 16-gauge pliable wire at frequent intervals, so that the reinforcement may not be displaced in the process of depositing the concrete.

33. Shuttering. All shuttering must be of approved dressed timber, true to line, and not less than $1\frac{1}{4}$ inches in thickness.

The shuttering to all visible surfaces of *in situ* concrete work is to be approved tongued and grooved timber.

All arrises and hollow corners to have 1-inch chamfers and fillets, except where otherwise shown on the drawings.

The faces next the concrete of all timber, temporary moulds or shuttering are to be planed smooth except where otherwise stated in the bill of quantities.

In every case the joints of the shuttering are to be perfectly close, so as to prevent the loss of liquid from the concrete.

Before the concrete is placed, the shuttering must be coated with an approved preparation for preventing the adhesion of the concrete to the moulds, and it is to be of such a nature and so applied that the surface of the finished concrete is not stained.

All shuttering and framing must be adequately stayed and braced to the satisfaction of the engineer for properly supporting the concrete during the period of hardening. It shall be constructed so that it may be removed without shock or vibration to the concrete. All moulds for beams and allied members shall be constructed so that the sides may be removed without interference with the remainder of the shuttering.

The interior of all moulds and boxes must be thoroughly washed out with a hose pipe or otherwise, so as to be perfectly clean and free from all extraneous matter previous to the deposition of any concrete.

The following minimum intervals of time should

be allowed between the placing of the concrete and the striking of the moulds :—

Beam Sides, Walls, and Columns (Unloaded).

Ordinary concrete . . .	4 days.
Rapid-hardening concrete .	2 ,,

Slabs (Props left in).

Ordinary concrete . . .	6 days.
Rapid-hardening concrete .	2 ,,

Removal of Props to Slabs.

Ordinary concrete . . .	14 days.
Rapid-hardening concrete .	7 ,,

Beam Soffits (Props left in).

Ordinary concrete . . .	12 days.
Rapid-hardening concrete .	4 ,,

Removal of Props to Beams.

Ordinary concrete . . .	28 days.
Rapid-hardening concrete .	14 ,,

The foregoing shall be regarded as absolute minimum periods, and will not in any way relieve the contractor from the responsibility of properly supporting the various members until the concrete is sufficiently set and hardened. No centreing is to be removed without permission of the engineer.

If frost occurs during the setting of the concrete, the removal of shuttering shall be delayed to the extent of the duration of the frost.

No plugs, bolts, ties, holdfasts or any other appliances whatsoever for the purpose of supporting centreing are to be fixed into the structure or placed in such a way that damage might result to the work in removing same when the centreing is struck.

34. Finish of exposed surfaces. (*This clause applies if no special treatment is required.*)

As soon as the shuttering has been removed, the visible surfaces of the concrete shall be rubbed down to a perfectly smooth finish, free from all irregularities. The finish must be produced by using carborundum blocks and rubbing with a circular motion, or other approved means, and finally the work shall be washed down with clean water.

SOFFIT OF ARCHES : (*If required*).

The contractor's attention is drawn to the necessity of procuring a perfectly smooth and

uniform surface to the soffit of all arches. Wrought boards must be used for the shuttering and the top surface brought to a smooth curve after the boards have been placed in position.

It may be necessary to screed the surface of the shuttering or to cover it with some other suitable and approved material, in order to obtain the desired result.

In pricing the work, the contractor is to allow for carrying out the work to the satisfaction of the engineer.

35. Expansion joints in spandril walls. Vertical expansion joints will be formed in the spandril walls, string course, parapets, etc., over each abutment, as indicated on drawing No. and set forth in the annexed bill of quantities. The contractor must allow for constructing these joints in accordance with the above drawing, and such subsequent detail drawings as may be from time to time supplied.

36. Expansion joints across roadway. Expansion joints across the reinforced concrete deck platform are to be formed over each abutment as set forth in the annexed bill of quantities.

Inserted in these expansion joints are to be "T" irons of suitable lengths carried across the full width of the bridge, in order to retain the superimposed roadway surfacing material.

Alternative.

Inserted in these expansion joints are to be strips of copper, 16 S.W.G., of approved quality and in suitable lengths, bent to shape and fixed as shown upon drawing No. . The space embraced by these copper strips is to be filled with an approved bitumastic composition, brought up flush with the surface of the reinforced concrete work. These joints are to be carried across the full width of the roadway, and across the pipe ducts, including the vertical portion at the kerbs and parapets.

The contractor must allow for constructing these joints all in accordance with the above drawings, and from such other detail drawings as may be from time to time supplied.

37. Sliding plates. Over each of the bridge abutments, horizontal sliding joints are to be formed.

Under each of the main longitudinal beams, raised concrete stools are to be formed, as indicated

upon the drawings, and upon these the metal sliding plates will be placed.

These sliding plates consist of two external mild steel plates five-sixteenths of an inch thick, and two internal copper plates one-eighth of an inch thick, and inches long by inches wide.

The plates will not be attached to the reinforced concrete work in any way, but before these are placed in position they are to be protected along their edges in such a manner that the copper contact plates do not become contaminated during the deposition of the surrounding concrete

38. Temporary hinges. In order to eliminate all stresses due to contraction of the concrete in setting, etc., temporary hinges are to be formed at the springings and at the crown of the arch as shown on the detailed drawings.

This will necessitate a transverse strip of the concrete decking at the crown being omitted until the filling in of the hinge sections is permitted.

In pricing the relevant items in the schedule of quantities, the contractor is to allow for forming (and for the subsequent filling in) of these temporary hinges.

39. Permanent hinges. Continuous hinges of reinforced concrete are to be formed as indicated on drawings Nos. at both abutments and at the crown.

These hinges may be formed as follows :—

The abutment and river pier supports can be screeded to the required curve and bars connecting the arched vaults to the supports left projecting. The convex curve at the ends of each of the arch vaults can be formed by screeding to the required curve either in plaster of Paris or by inserting soft wood timbering to the required shape.

The concrete in the arched vault and ribs can then be placed in the ordinary manner, and the material used for forming the convex surfaces can subsequently be removed by hacking out.

In the case of the abutment hinges, the portion of hinge in contact with the abutment filling is to be carefully filled with bitumastic asphalt.

In pricing the relative items, the contractor must allow for forming and filling in these hinges.

Testing of bridge upon completion.—The following gives a suitable form of test for an arch or beam bridge having a span or spans up to 100 feet :—

" A dead load of 1 cwt. per square foot is to be superimposed

over the footpaths. The bridge roadway is to be tested with a rolling load consisting of six steam road rollers 12—15 tons weight: 6 rollers to be taken over the bridge first, in line from (*east to west*) and then from (*west to east*); secondly: three abreast, and thirdly: two lines of three rollers meeting in the centre of the bridge. Finally the rollers shall be taken over the bridge at their maximum speed."

This test load should not be applied to any part of the structure until at least ten weeks have elapsed from the last day after the laying of the last concrete in that portion, or until the specified thickness of surfacing material has been added.

Where it is required the contractor includes for the cost of carrying out the test, instructions should be given in the specification clause and an item included for it in the schedule of quantities.

In connection with the testing of reinforced concrete bridges generally, it is found that the maximum deflection would be given by placing the whole of the specified superimposed loading upon the bridge and maintaining the bridge in this condition for some hours.

It is not usually convenient, however, to prolong the test in this manner and provided that the bridge satisfactorily carries the specified loading or a reasonable alternative loading, as indicated above, without undue deflection or other evidence of defect, it is generally assumed that the work has been properly carried out. Fundamental defects in design may nevertheless give no evidence of their existence for several years.

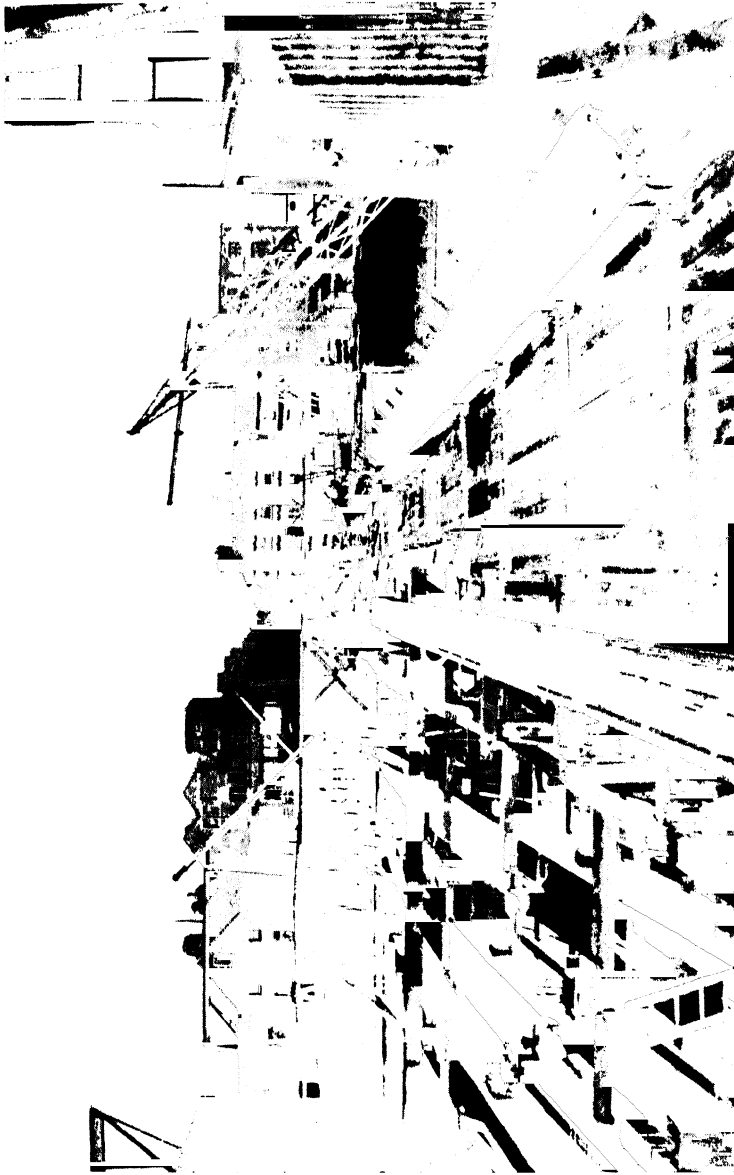



Fig 164 Warrington Bridge under Construction

PLATE XVII

[*Plates XVII – XXXIII following page 260.*

FIG. 165. ST. JEAN LA RIVIÈRE BRIDGE. 150 FT. SPAN.

PLATE XVIII.

FIG. 166. TEMPORARY STAGE FOR ST. JEAN LA RIVIERE BRIDGE.

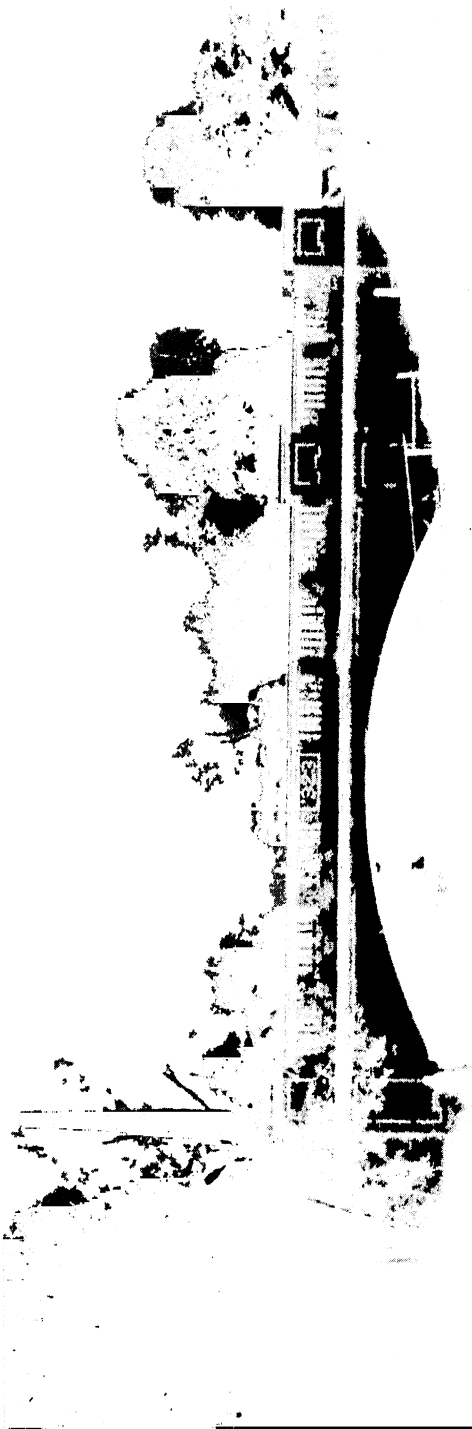



PLATE XIX.

Fig. 167.—Red Bridge, Ilford. 40 ft. Span.

PLATE XX.

Fig. 168. Visupie Bridge. 315 ft. Span

PLATE XXI

Fig 169 —Part Elevation of Vesubie Bridge

Plate XXII

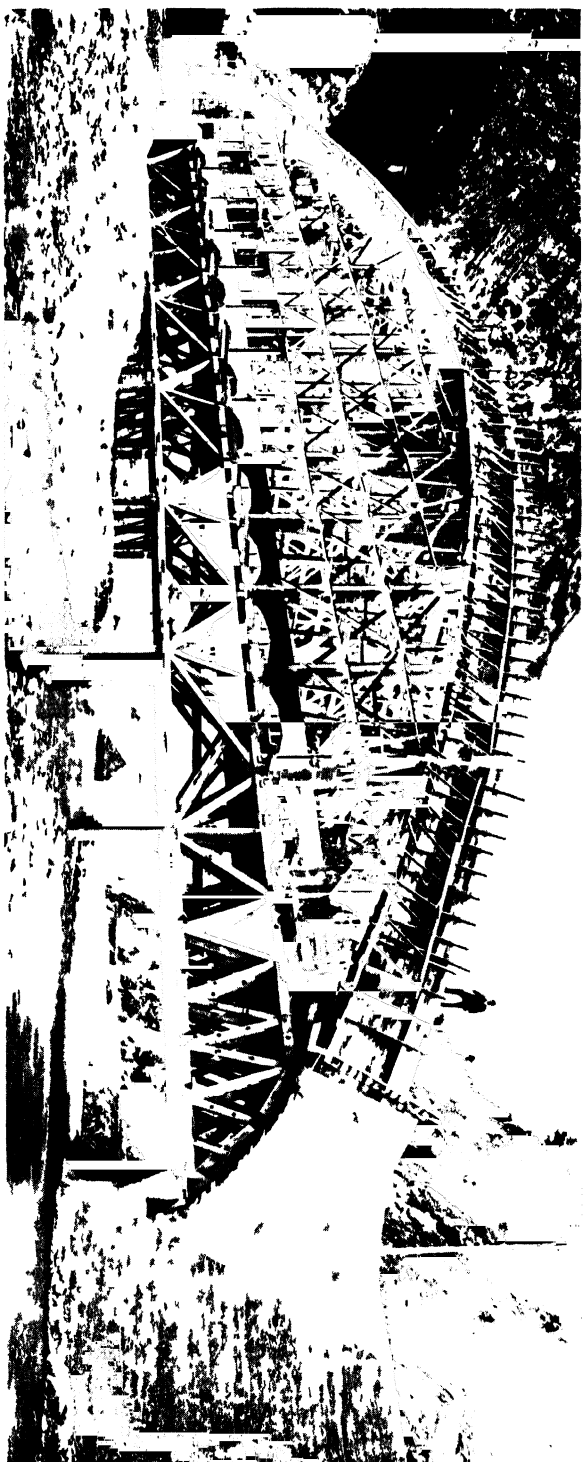



FIG. 170. VESUBIE BRIDGE UNDER CONSTRUCTION, SHOWING TEMPORARY STAGING.

PLATE XXIII.

Fig. 171. — Girder Bridge. 36 ft. Span.

PLATE XXIV.

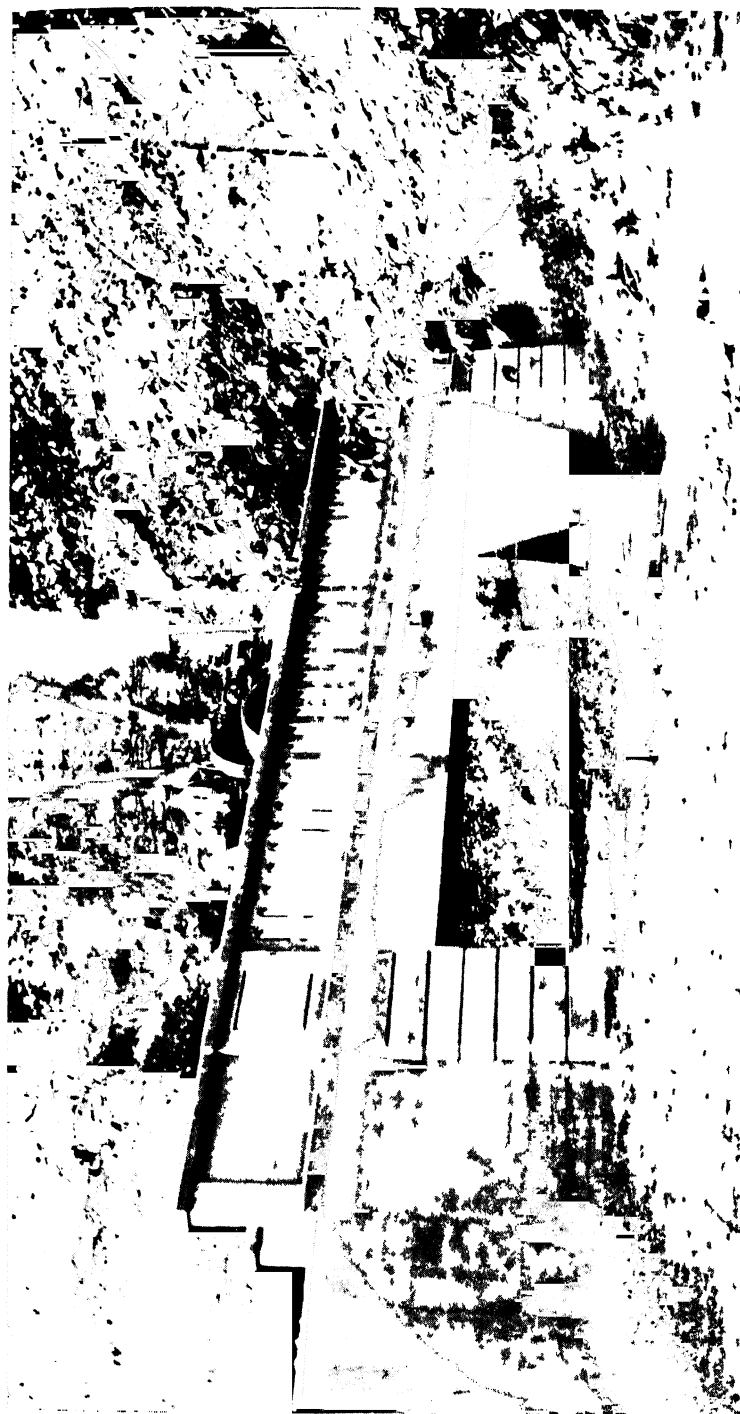
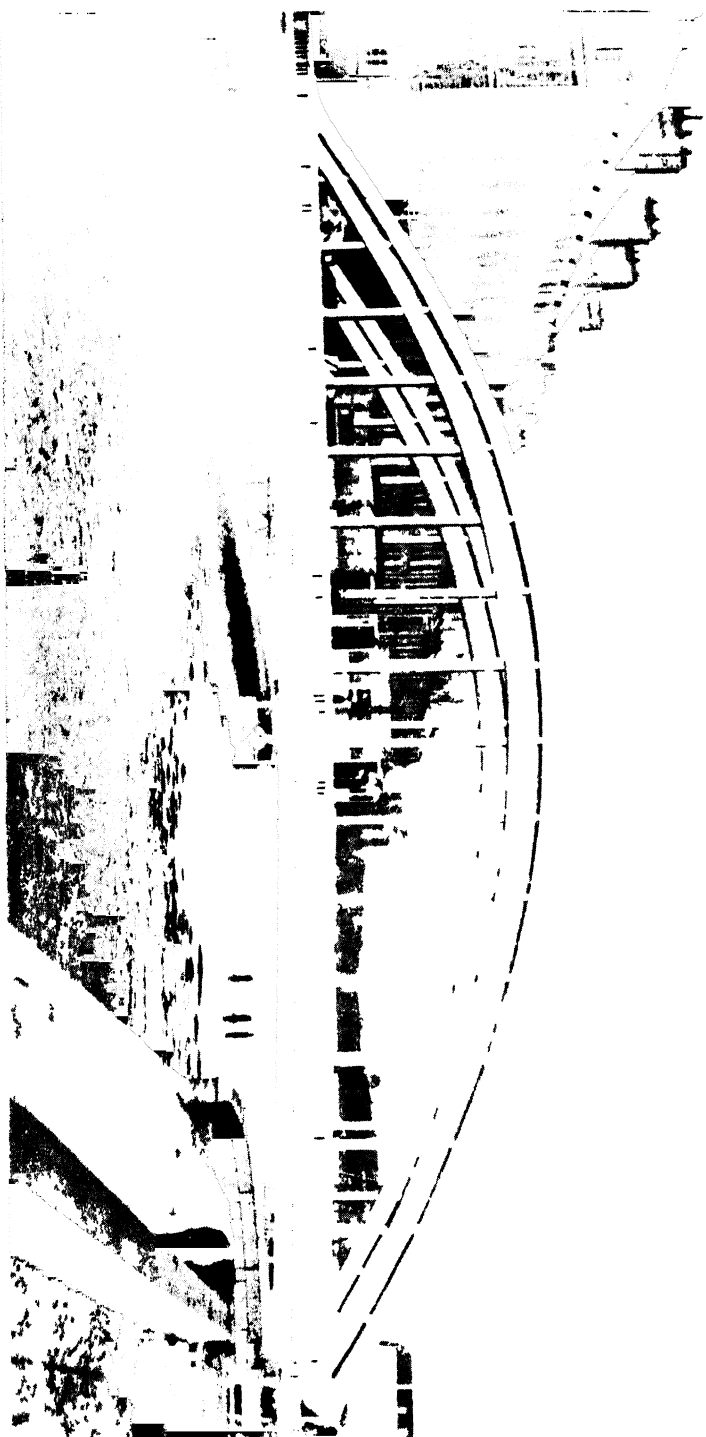



FIG. 172. BOWSTRING GIRDER BRIDGE, NANTES. 180 FT. SPAN

PLATE XXV

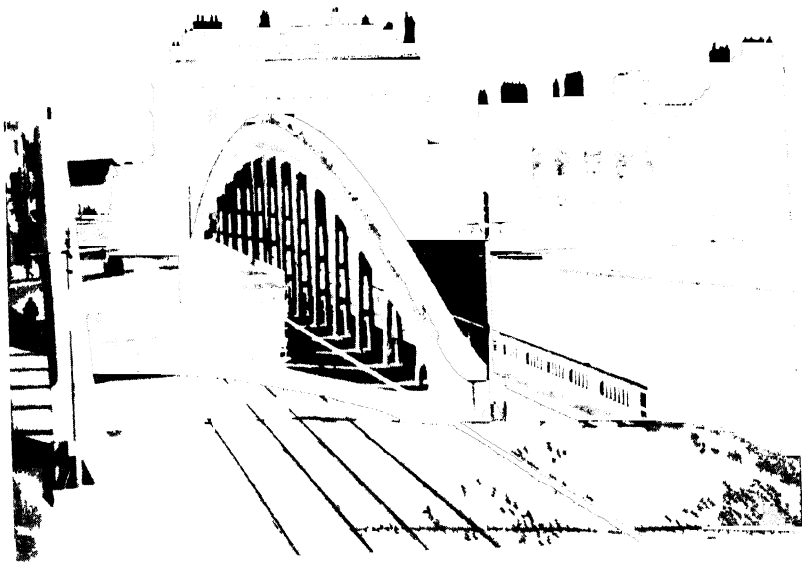



Fig. 173. Bowstring Girder Bridge, Nantes. View showing Roadway Accommodation.

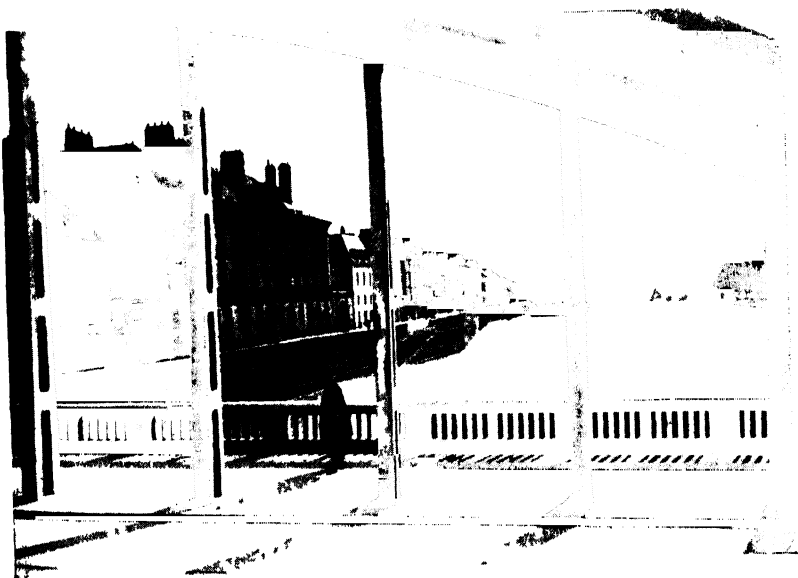



Fig. 174. Bowstring Girder Bridge, Nantes. View through Suspenders showing Footpath Accommodation.

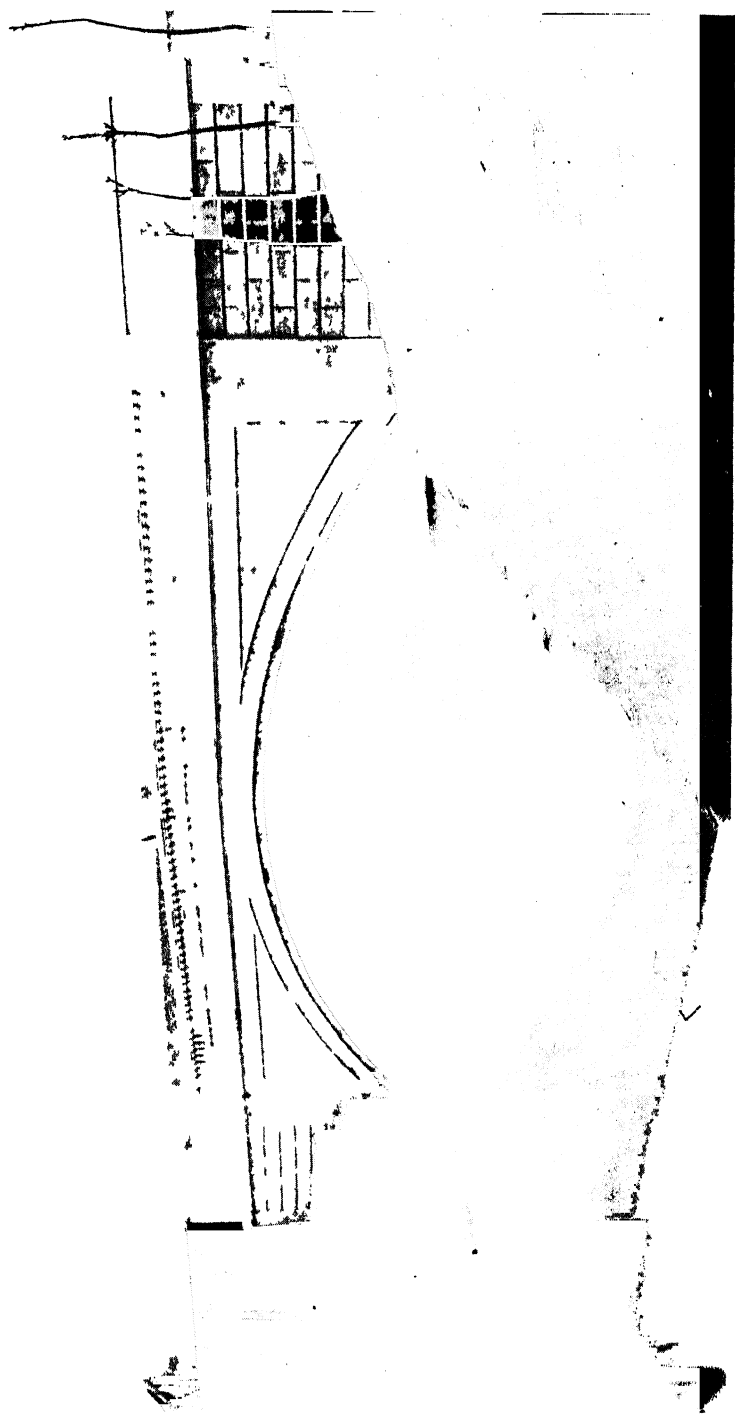



PLATE XXVI.

FIG. 175. RIVERFORD ROAD BRIDGE, GLASGOW. 80 FT. SPAN.

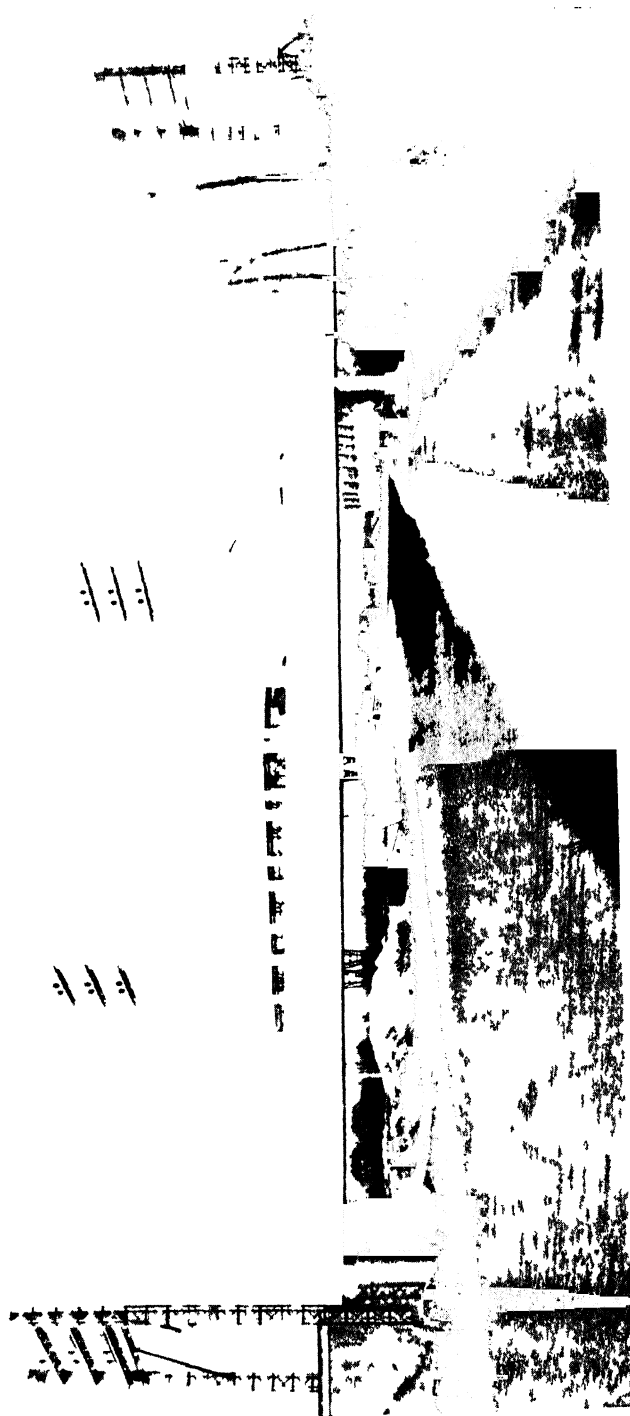



PLATE XXVII.

FIG 176 BOWSTRING RAILWAY BRIDGE BEAUFORT 128 FT SPAN

PLATE XXVIII

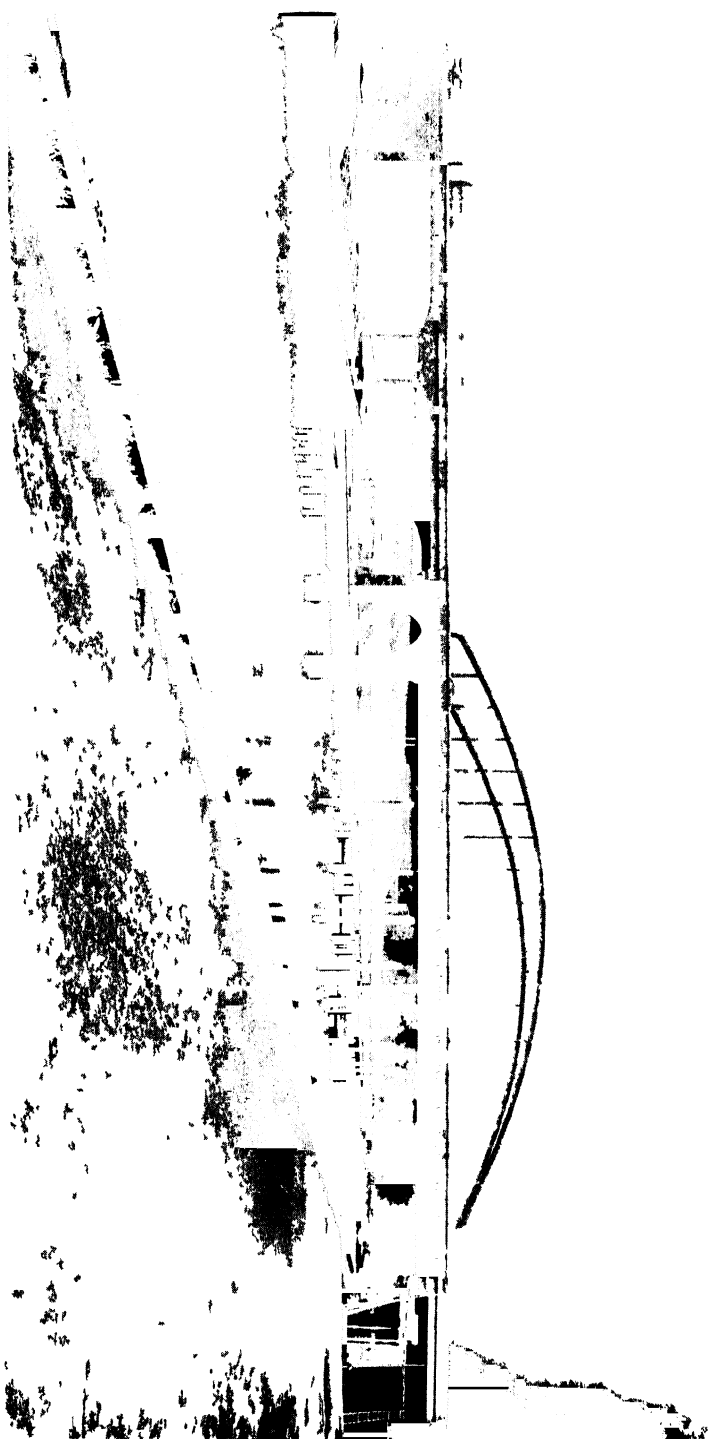



Fig. 177. Bridge at Aubervilliers. 132 ft. Centre Span.

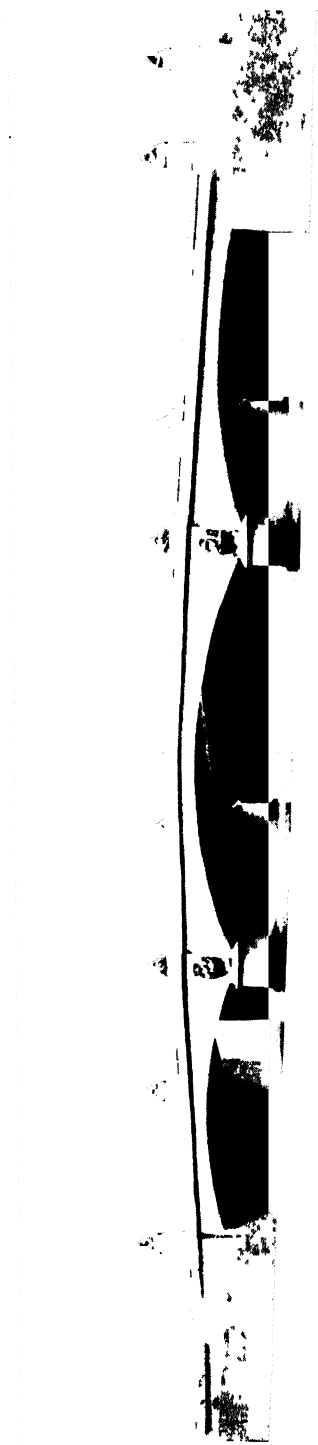



PLATE XXIX

Fig. 178.—George the Fifth Bridge, Glasgow. 166 ft. and 120 ft. Spans.

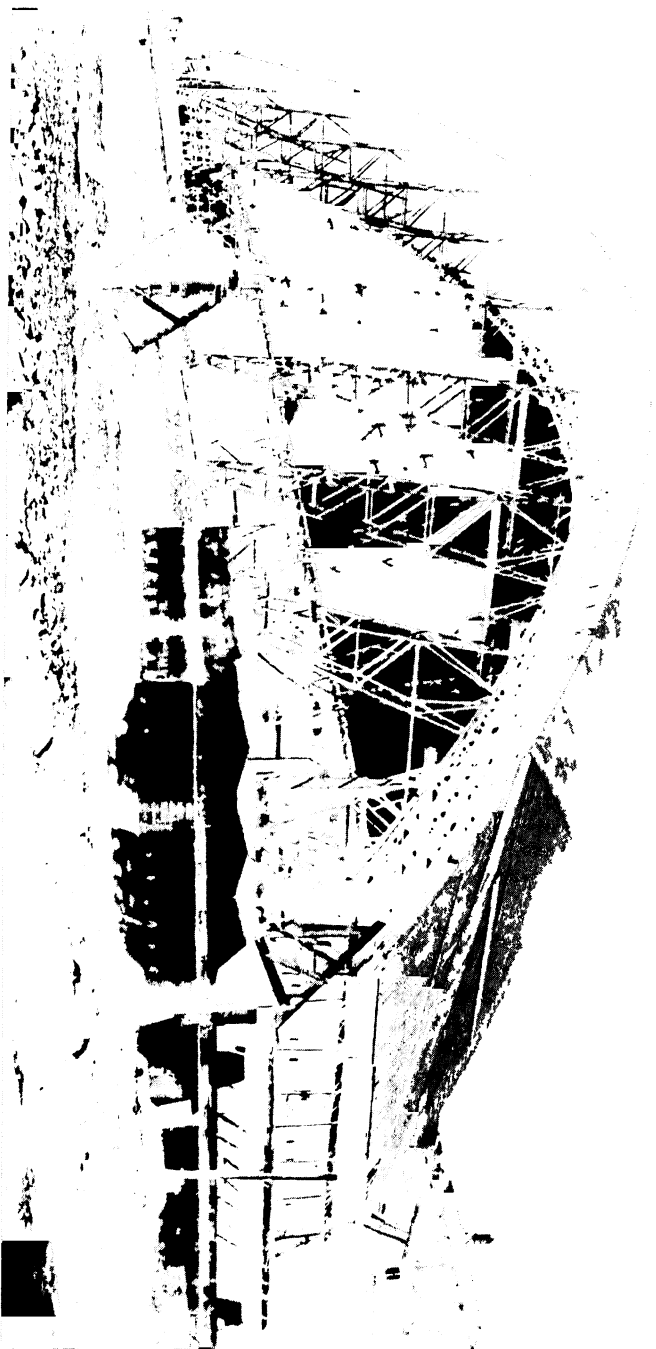



PLATE XXX.

FIG. 179 PLOUGASTEL BRIDGE VIEW OF TEMPORARY STAGE DURING CONSTRUCTION

PLATE XXXI

Fig. 180.—Plougastel Bridge. Temporary Stage in Position.

PLATE XXXII

Fig. 181. P[illegible] Bridge. Construction of Superstructure in Progress

Plate XXXIII

INDEX

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